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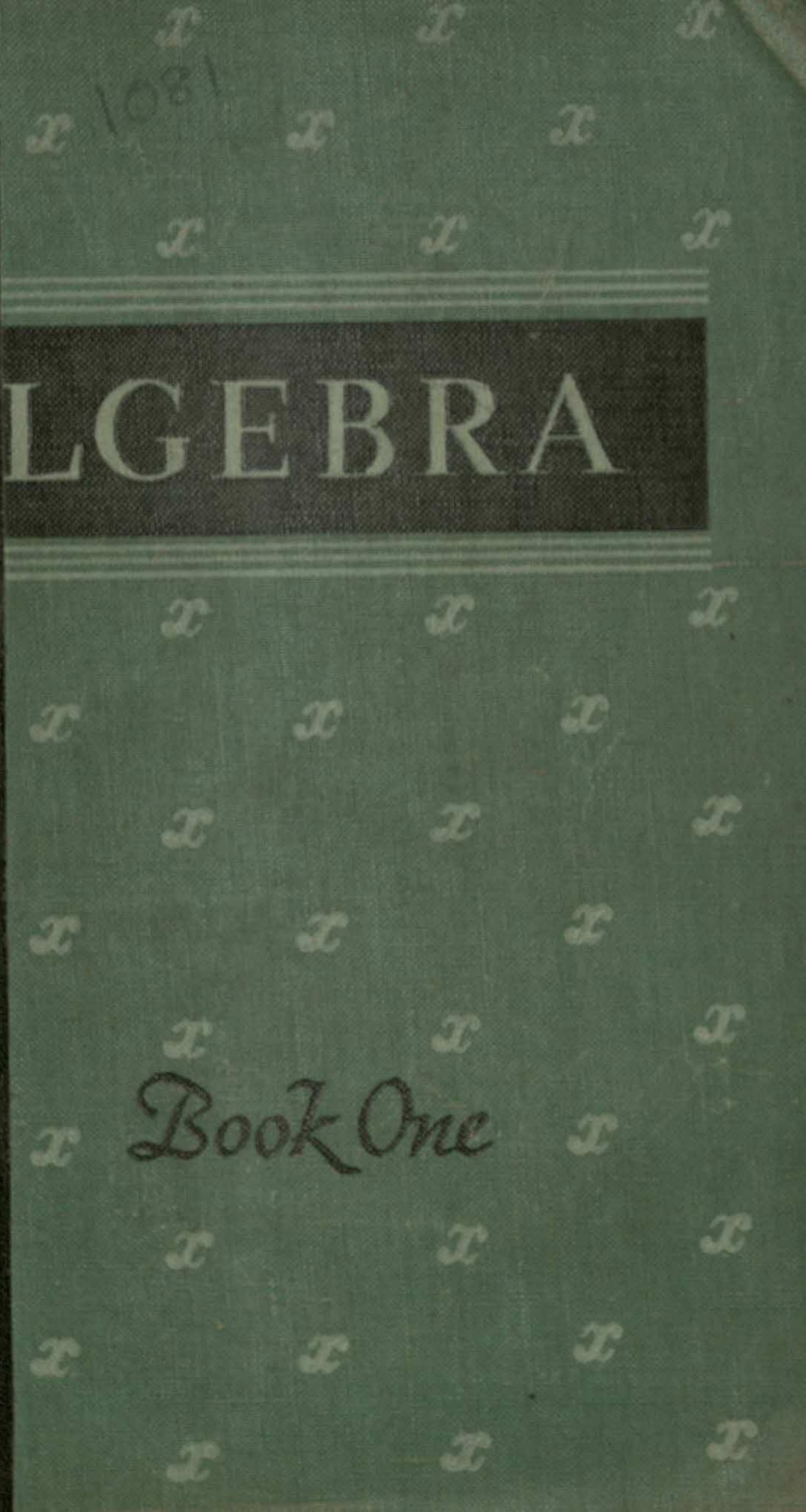

LGEBRA

Book One

Harold Glenn

ALGEBRA · BOOK ONE

ELEMENTARY COURSE

2 copies.

By A. M. WELCHONS *and* W. R. KRICKENBERGER

The Arsenal Technical High School
Indianapolis, Indiana

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TO THE TEACHER

In this elementary algebra the authors have used the basic methods of teaching and such aids as have been found to be the most effective. While preparing the text they have had five main objectives in mind:

1. To create in the pupil a desire to learn mathematics.
2. To offer the teacher all possible aids in presenting the subject.
3. To provide for individual differences in the abilities of pupils.
4. To stress the relationship of quantities.
5. To develop the pupil's power to think and to work in a systematic manner.

This algebra includes the usual topics that are basic either to the average citizen's vocation or to a further study of mathematics. It meets the requirements of modern courses of study and examining bodies.

Development of Concepts and Procedures

Since algebra is an extension of arithmetic, the concepts and procedures of arithmetic have been used as a foundation upon which the structure of algebra is built.

By carefully prepared questions and problems the pupil is led to form conclusions and generalizations which enable him in many cases to formulate his own rules and procedures.

It should be the aim in algebra to teach the most general procedure. It is better to have a rule covering many cases than to have a separate rule for each of them. The authors have followed this plan when possible.

Simplicity

The book is written for the pupil and to the pupil who is beginning the study of algebra. The language is direct, concise, and simple. New words are defined, illustrated, and reviewed. Frequent word lists keep the new words fresh in the pupil's memory.

Motivation

New subject matter is presented when possible as an extension of arithmetic. The articles on the need for mathematics in various occupations and professions serve to create in some pupils a desire to continue the study of mathematics. Problems relating to science, commerce, trades, and other fields make algebra a living subject.

Individual Differences

To provide for individual differences among pupils, three levels of work are presented. The topics and exercises marked A are for all pupils. The B topics and exercises offer additional work for pupils with more interest and ambition. The C topics and exercises are a challenge to the exceptional students.

Minimum and Maximum Courses

The amount of subject matter covered during a school year depends upon the pupil's ability in mathematics and his mathematical preparation.

As a guide in determining the progress of a class the following courses are suggested:

A Minimum Course. First semester: Chapters I–VII including A topics and exercises.

Second semester: Chapters VIII–XIV including A topics and exercises.

A Maximum Course. First semester: Chapters I–IX including A topics and exercises and selected B and C topics and exercises.

Second semester: Chapters X–XVIII including A topics and exercises and selected B and C topics and exercises.

Sequence

The sequence of topics is based upon the pupil's learning ability. Since the whole structure of algebra is based on signed numbers, they are presented early and used throughout the book.

Each new topic makes use of the preceding ones and leads to the next.

Problem-Solving

The technique of problem-solving as presented in this book has been used by the authors and has proved most effective.

Drill Material

In order that all pupils may acquire mastery of manipulation and understanding of subject matter, an abundance of drill material has been included. The amount of drill needed by any pupil or class depends upon the pupil's learning ability and preparation. It is assumed that the teacher will adjust the amount of drill to the needs of the class, and not be misled by a generous supply of exercises into spending too much time on any particular topic.

Reviews

Constant repetition is a necessary procedure in teaching mathematics. Each of the first eighteen chapters concludes with a chapter review, and most of them contain a cumulative review. Chapter XIX is a review by topics.

Tests

The testing material is adequate. It includes eighteen chapter tests (A), sixteen chapter tests (B), and five cumulative tests.

The A tests cover the topics and exercises marked A and the B tests cover the topics and exercises marked A and B. The cumulative tests are in chapters VI, VII, VIII, XVII, and XVIII.

These tests may be used in various ways. They may be used as diagnostic, practice, or mastery tests. If they are used as practice tests, the teacher may use them as guides in preparing the mastery tests.

The cumulative tests in chapters VI, VII, and VIII are based upon the subject matter preceding them. The cumulative tests in chapters XVII and XVIII are based upon the work of the second half-year.

A. M. WELCHONS
W. R. KRICKENBERGER

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TO THE STUDENT

Algebra is one of the most important subjects in your high-school curriculum. It is a subject which most educated people have studied and which is necessary in our modern civilization.

Since you wish to do well in your algebra, you should be certain that you have proper methods of study. If in any way you can improve your methods of study, you will save time in your homework and will have a better mastery of the subject matter.

The following suggestions are offered in the hope that they may be helpful to you.

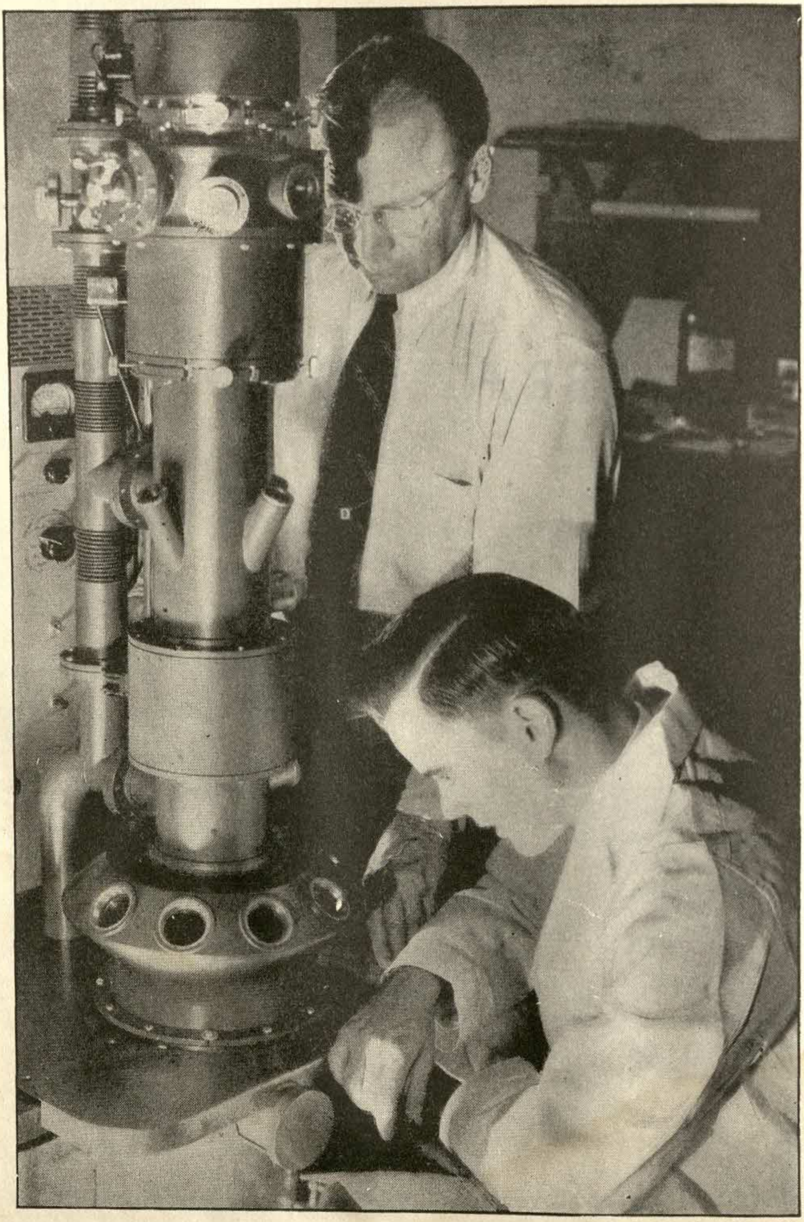
1. Budget your time for study and recreation so that you may have a definite time for study. Learn to concentrate on the subject you are studying and do not permit others to distract you. Allow one hour for the preparation for each algebra assignment, even though you may often do it in less time.

2. Read the subject matter carefully, being sure that you understand each sentence. If, however, you are unable to understand any part of your assignment, form a definite question about it and ask your teacher for an explanation. Consult the index in the back of the book or look in the dictionary for the meanings of words you do not understand.

3. Before you attempt to solve a set of exercises, study the examples which precede the exercises. While studying use your pencil and paper to follow each step of the solution, doing as much of the solution as you can without referring to the solution in the book. Test your understanding of the example by doing the solution unaided.

4. Form the habit of reviewing previous lessons. Five minutes of review each day will be very valuable to you.

ALGEBRA, BOOK ONE



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CHAPTER I

Literal Numbers

What Algebra Is [A]

As you start the study of algebra, you naturally wish to know what algebra is and how it is like arithmetic, which you have already studied.

In arithmetic the numbers are represented by the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, and by their combinations. In algebra the numbers are not only represented by these symbols but are also represented by letters. This is one difference between algebra and arithmetic.

In algebra we use all the numbers that are used in arithmetic and, besides, we use the opposites of these numbers. These opposites are called *negative* numbers. This is a second difference between algebra and arithmetic.

In algebra we use equations to solve problems. Equations enable one to solve problems more easily and to solve many problems which could not be solved by using only the methods of arithmetic. This is a third difference between algebra and arithmetic.

Thus you can see that algebra is only an extension of arithmetic. Since algebra is an extension of arithmetic, any principle or rule that is used in algebra must apply equally well in arithmetic.

Why We Study Algebra [A]

At this time you should know why algebra is taught in high school. You no doubt wish to know what value the study of algebra will be to you after you have left school.

The culture and progress of any nation can usually be measured by the mathematical development of its people. Mathematics and civilization go hand in hand; and whenever there is marked progress in a nation's life, you may be sure that mathematics is one of the causes of it.

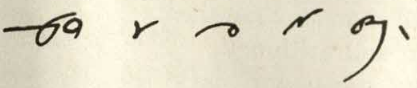
Algebra is studied for its practical value. It is used in many of the trades and industries, and is a requirement for entering many colleges and universities. Before one can progress very far in the study of electricity, light, heat, sound, radio, insurance, statistics, accounting, or television, he must have a knowledge of algebra.

Much mathematics is used by those in military service. In the Second World War a young man's advancement in the army, navy, or air service depended to a great degree upon his knowledge of mathematics.

The Shorthand of Mathematics[A]

Shorthand was invented to enable people to write rapidly and save time. An expert stenographer can write words in shorthand as rapidly as they are usually spoken. Compare the longhand and shorthand ways of writing the same words:

Mathematics is the key to the universe.

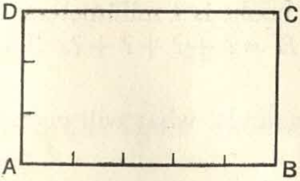


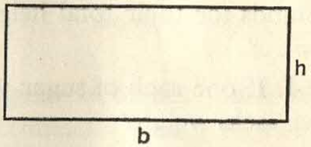
Mathematics has a shorthand of its own. In your study of mathematics you learned to use the symbol × for "times," the symbol ÷ for "divided by," the symbol + for "added to," the symbol − for "less," the symbol = for "equals" or "is equal to," and the symbol % for "per cent." In your study of algebra you will use all these symbols and many more. You will also represent numbers by letters and by expressions containing letters.

How Letters Represent Numbers[A]

Using letters to represent numbers is not new. Centuries ago the Romans let the seven capital letters I, V, X, L, C, D, and M stand respectively for the numbers one, five, ten, fifty, one hundred, five hundred, and one thousand. By combining these letters and using the bar (—) to connect them, they were able to represent any whole number. In the Roman notation the number 1948 is written MCMXLVIII.

The figure $ABCD$ below is a *rectangle*. It has four right angles and its opposite sides are equal. Its length is divided into five equal parts and its width contains three of these equal parts. If we let l stand for the length of the rectangle and w for its width, then the letter l stands for the number 5 and the letter w stands for the number 3. We say that $l = 5$ and $w = 3$. Note that the equals sign is here used for the words " stands for."





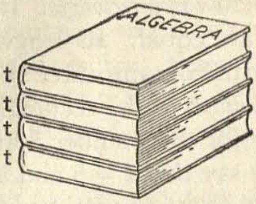
Even when we do not know the measurements of a figure, we may represent its dimensions by letters. In the figure at the right above b stands for the number of units in its base and h stands for the number of units in its height.

In your arithmetic you learned that many rules were more easily remembered when they were abbreviated. For example, you learned that $i = prt$ is a short way of stating "Interest equals the principal times the rate times the time." In this abbreviated rule (called a *formula*) i stands for the number of dollars in the interest, p for the number of dollars in the principal, r for the rate, and t for the number of years in the time. Notice that the multiplication signs are omitted. As you know, these letters may stand for one set of numbers in one problem and another set of numbers in another problem. The following exercises are intended to show other ways of representing numbers by letters.

EXERCISES [A]

1. The formula for the area of a rectangle or parallelogram is $A = bh$. In this formula, A stands for the number of square units in the area and b stands for the number of linear units in the base. For what does h stand? Change this formula into a rule.

2. $A = \frac{1}{2}\,bh$ is a formula for finding the area of a triangle. For what does each of its letters stand? Change the formula into a rule.



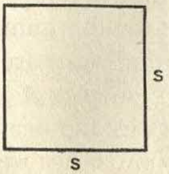
3. The thickness of each of these books is t millimeters. If H stands for their total height, then $H = ? + ? + ? + ?$. Then $H =$ --?-- t.

4. If one sack of sugar weighs p pounds, what will eight of these sacks weigh?

5. A house is twice as high as a garage. If h stands for the height of the garage, how many h will stand for the height of the house?

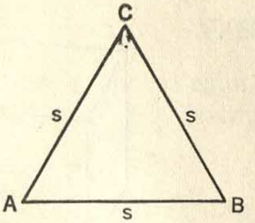
6. The height of a Saint Bernard dog is $3\,m$ and the height of a dachshund is m. Which dog has the greater height? What is the sum of their heights?

7. $d = rt$ is a formula for finding the distance when the rate and time are given. What sign is omitted between r and t? Change the formula into a rule.



8. This figure is a square. How long is each of its sides? Let p stand for its perimeter. (The perimeter of a figure is the sum of the lengths of all its sides.) Then $p = s + s + s + s$. This formula may be written $p = 4\,s$. What is the value of p when $s = 3$? What is the value of p if $s = 5$? Will p increase or decrease in value as s increases in value?

9. Triangle ABC (abbreviated $\triangle ABC$) has equal sides each s units long. Let p stand for its perimeter. Then $p = ? + ? + ?$ and $p =$ --?-- s. If $s = 6$, what is the value of p?



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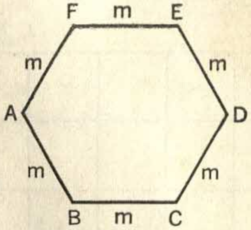


Ex. 10

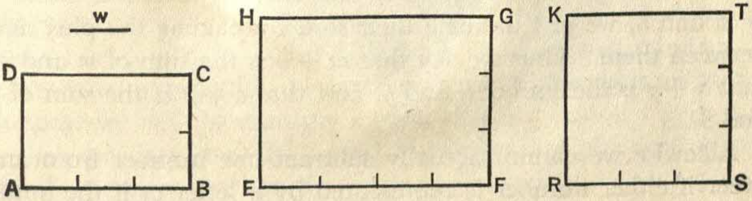
10. The figure $ABCDEF$ is a regular hexagon. Each of its sides is m units long. Let p be its perimeter. Then $p =$ --?-- m.

On your paper complete the following table based on $p = 6\,m$.

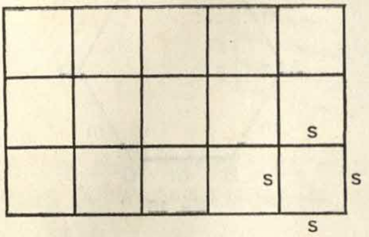
When m =	1	2	3	4	5	6	$\frac{1}{2}$	$\frac{1}{3}$	.25	1.5	2.5	?
then p =	6	12	?	?	30	?	?	?	?	?	?	72

11. Find the perimeter of a regular hexagon each of whose sides is x inches. Write the formula for the perimeter of a regular hexagon each of whose sides is x inches.

12. Look at these three rectangles. Which one of them is a square? Above rectangle $ABCD$ is a line segment whose length is w. If p stands for the perimeter of the rectangle $ABCD$, then $p =$ --?-- w. Now suppose that the rectangle $ABCD$ is enlarged, forming rectangle $EFGH$. Then $p =$ --?-- w. Now suppose that the rectangle $EFGH$ is reduced to form the square $RSTK$. Then $p =$ --?-- w.



13. The rectangle below is divided into equal squares. If a side of one of the small squares is s, what is the perimeter of the rectangle? ($p =$ --?-- s.) If the area of each square is s^2 (meaning $s \times s$), what is the area (A) of the whole rectangle? Then $A =$ --?-- s^2.



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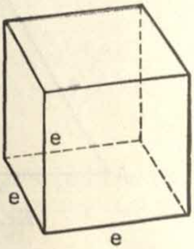


Ex. 14

14. The figure at the right above is a cube. Each of its edges is represented by e. What is the sum of all its edges? What is the area of each of its six faces? Write a formula for the total area T of the cube. Do you know what the formula $V = e^3$ says?

15. In the formula $C = \frac{22}{7}\,d$, C stands for the circumference of a circle and d stands for the length of the diameter of the circle. State the rule in words. Find C if $d = 14$ feet.

Symbols of Addition and Subtraction [A]

In algebra the signs of addition and subtraction are the same as in arithmetic, the sign of addition being + and the sign of subtraction being −.

In arithmetic the additions and subtractions can always be performed, but in algebra many of these operations can only be indicated. We can always add two numbers like 7 and 9, but we cannot add two numbers like m and n until we know their values. In algebra if we need to add two numbers like x and y or m and n, we just indicate their sum by placing the plus sign between them. Thus we say that $m + n$ is the sum of m and n, that $x + y$ is the sum of x and y, and that $c + 3$ is the sum of c and 3.

Likewise we cannot actually subtract one number from another if either number is represented by a letter or if the numbers are represented by different letters. Thus we cannot actually subtract n from m, but we indicate the subtraction by writing $m - n$. When the values of m and n are known, then the subtraction can be performed.

EXERCISES[A]

Using the + and − signs, indicate the following additions and subtractions. For example, "x added to h" can be written $h + x$.

1. m added to x
2. s less 4
3. x plus c
4. p increased by 10
5. m more than 4
6. the sum of h and k
7. 4 more than y
8. 10 increased by p
9. h decreased by 4
10. 4 decreased by c
11. 10 less y
12. y less 10
13. 9 added to x
14. w plus y
15. p minus h
16. 10 less than x

In exercises 17–32 the additions and subtractions are only indicated. Now perform these operations when $a = 6$, $b = 2$, $x = 4$, and $k = \frac{1}{2}$.

17. $a + b$
18. $b + x$
19. $x + k$
20. $a + x$
21. $a - x$
22. $x - b$
23. $11 - a$
24. $8 - k$
25. $x - k$
26. $a - 4$
27. $10 - b$
28. $4 - x$
29. $a + b + x$
30. $x + b - a$
31. $8 - b - x$
32. $a + k - b$

The Signs of Multiplication[A]

The multiplication sign (×) has the same meaning in algebra as in arithmetic. In arithmetic the product of two numbers can always be found. We can always multiply two numbers like 4 and 6, or $\frac{3}{4}$ and $\frac{1}{2}$; but if either, or both, of two numbers is represented by a letter, we can only indicate the multiplication. Suppose we wish to multiply x by 4. We say that $4 \times x$, or $4\,x$, is their product, but the actual multiplication of x and 4 can only be performed when the value of x is known.

The dot (·) is sometimes used to denote multiplication. Thus $6 \cdot 8$ means 6 times 8. When using the dot, you must be careful to place it so that it is not mistaken for a decimal point; 7 times 8 should be written $7 \cdot 8$ and not 7.8.

In algebra the multiplication sign is often omitted. It is needed in the indicated product of two arithmetic numbers, as in 4×7 or 3×5; but it is not needed in an indicated product such as $4 \times h$ or $m \times n$. (Why is the multiplication sign needed in 4×7?) So when no sign is written between two numbers (other than two numerical numbers), the multiplication sign is understood. Thus $6y$ means "6 times y" and ab means "a times b." Remember that the multiplication signs are usually omitted.

Factors[A]

When two or more numbers are multiplied together, the numbers themselves are called the *factors* of the product. Each factor of a number is an exact divisor of the number. Thus the factors of 15 are 3 and 5; and the factors of $5m$ are 5 and m.

Coefficients[A]

Any factor, or factors, of a product is the *coefficient* of the remaining factor or factors. Now let us see what this means. In the product 3×5 the number 3 is the coefficient of 5 and the number 5 is the coefficient of 3. In the product $4ab$, the number 4 is the coefficient of ab, $4b$ is the coefficient of a, $4a$ is the coefficient of b, and ab is the coefficient of 4.

If a letter has no coefficient written before it, the coefficient 1 is understood. Thus x means $1x$, y means $1y$, and mn means $1mn$.

ORAL EXERCISES[A]

1. In which of the following indicated products can the times sign be omitted?

a. 4×5
b. $a \cdot b$
c. $8 \times r$
d. $3 \cdot h$
e. $x \cdot y$
f. $4 \cdot m$
g. 3×6
h. 8×7
i. $9 \times p$

2. What is the value of $4a$ if $a = 1$? if $a = 3$? if $a = 6$? if $a = \frac{1}{2}$? if $a = 1.5$?

3. What is the value of $7x$ if $x = 6$? if $x = 10$? if $x = \frac{1}{2}$? if $x = 3\frac{1}{7}$? if $x = .2$?

4. What is the value of xy if $x = 3$ and $y = 6$? if $x = 7$ and $y = 8$? if $x = 2$ and $y = .5$? if $x = \frac{1}{2}$ and $y = \frac{1}{3}$?

5. What is the value of st when $s = 10$ and $t = 5$? if $s = 8$ and $t = \frac{1}{4}$? if $s = 7$ and $t = 9$? if $s = 1.3$ and $t = .8$?

6. Name the factors of 6; of 10; of 21; of bc; of abc; of $8hk$.

7. In the expression $4abc$ what is the coefficient of abc? of bc? of $4a$?

8. In the expression $4a + 5b + c$ what is the coefficient of a? of b? of c?

Signs of Division [A]

In algebra division is indicated by the division sign (÷), by the colon (:), and by writing the dividend as the numerator of a fraction and the divisor as the denominator of the fraction. Thus "10 divided by 2" can be indicated by $10 \div 2$, $10 : 2$, or $\frac{10}{2}$ and "x divided by y" can be written $x \div y$, $x : y$, or $\frac{x}{y}$.

The Use of Parentheses [A]

In mathematics it is often necessary to group two or more numbers to show that some operation is to be performed on all of them. The expression $10 - (6 + 2)$ means that the sum of 6 and 2 (which is 8) is to be subtracted from 10. The expression $4(x + y)$ means that the sum of x and y is to be multiplied by 4. Since there is no sign of operation between the 4 and the $(x + y)$, the multiplication sign is understood.

Besides the parentheses ( ), other signs of grouping are used. These are the brackets [ ], the braces { }, and the bar ——. The bar may be used above or below the numbers to be grouped. $7 + \overline{x + 6}$ means that the sum of x and 6 is to be added to 7 and $\frac{8 + 10}{2}$ means that the sum of 8 and 10 is to be divided by 2.

The Meaning of Exponents[A]

The short way of writing $a \times a$ or aa is a^2. The small 2 written to the right of and a little above the a in the expression a^2 is called an *exponent.* An exponent tells how many times a number is used as a factor. The expression a^2 is read "a to the second power" or "a square." a^3 means $a \times a \times a$ and is read "a to the third power" or "a cube." Notice that a^3 means that three a's are to be multiplied together. If a figure or letter has no exponent written to the upper right of it, the exponent 1 is understood. Thus x means x^1 and 6 means 6^1.

In a product such as ay^3 the exponent 3 applies only to the letter or figure which it follows; that is, only the y is to be cubed. Thus ay^3 means $ayyy$.

When dealing with exponents ask yourself this question: "What does this exponent mean?"

ORAL EXERCISES

A

1. What is the meaning of x^2? of 5^2? of m^2? of x^3? of b^4? of c^5? of 8^3? of 6^4?

2. In the expression $4y^3$, which is to be cubed, the y or the $4y$?

3. What is the coefficient of x in the expression $4ax$? What is the exponent of x in this expression?

4. Tell how the following expressions can be written using exponents:

a. xxx
b. aa
c. mm
d. rrr
e. $ffff$
f. $2kk$
g. $cccc$
h. sss
i. $4xx$
j. $2ww$
k. $3xxx$
l. $ccccc$

5. What do the following expressions mean?

a. c^2
b. m^3
c. h^4
d. p^4
e. k^5
f. y^3
g. m^2n
h. mn^2
i. m^2n^2
j. $3y^4$
k. $5y^5$
l. $7 \cdot 10^3$

6. If $a = 2$, $b = 3$, $x = 4$, $d = 1$, and $e = \frac{1}{2}$, find the value of

a. x^2
b. a^2
c. b^2
d. e^2
e. d^2
f. a^3
g. b^3
h. x^3
i. d^3
j. $4a^2$
k. $5b^2$
l. $8e^2$

7. If $a = 1$, $b = 2$, and $c = 3$, find the value of

a. $4a$
b. $5c$
c. $4b$
d. $\frac{1}{3}c$
e. $\frac{1}{4}b$
f. $\frac{1}{2}a$
g. ab
h. ac
i. bc
j. $2ab$
k. $6bc$
l. $10ac$

B

8. If $x = 3$ and $y = 4$, find the value of

a. x^2
b. $3x^2$
c. $5x^2$
d. y^2
e. $4y^2$
f. $6y^2$
g. x^3
h. $2x^3$
i. $\frac{1}{3}x^3$
j. x^2y
k. $2x^2y$
l. xy^2

Terms [A]

When an algebraic expression is made up of certain parts connected by + and − signs, these parts with the signs immediately preceding them are called the *terms* of the expression. The algebraic expression $4a + 5b - 4c$ has three terms, which are $4a$, $+5b$, and $-4c$.

An algebraic expression of one term, such as $4x$ or 7, is called a *monomial;* one of two terms, such as $x^2 + 4x$, is called a *binomial;* and one of three terms, such as $3a + 2b - c$, is called a *trinomial*. Binomials, trinomials, and all algebraic expressions of more than three terms are called *polynomials.*

Like Terms and Unlike Terms [A]

You know that 2 books + 5 books = 7 books and that 4 ft. + 7 ft. = 11 ft. You also know that 5 times a number plus 4 times that number is equal to 9 times that number. We can simplify the expression $7c + 4c$ and obtain $11c$. These examples illustrate the fact that

Like terms can be added or combined.

There is no way of combining terms that are unlike. We cannot simplify the expression $4x + 5y$ because we cannot add the unlike terms $4x$ and $5y$. We cannot simplify the expression $5x^2 + 7x - 3$ because we cannot combine the unlike terms $5x^2$, $7x$, and -3. The expression 3 ft. + 8 in. has unlike terms. It can be simplified by changing both terms to feet or both terms to inches.

A polynomial can be simplified if two or more of its terms are like terms. For example, $3x^2 + 5x + 4x^2 + 8x$ can be simplified to $7x^2 + 13x$ by combining the $3x^2$ with the $4x^2$ and the $5x$ with the $8x$. Similarly, $3a + 2b + 7a - b + 6$ can be simplified to $10a + b + 6$.

EXERCISES

A

1. Show that $4a + a = 5a$ when $a = 3$; when $a = 6$.

2. Show that $x^2 + 3x + 2x^2 + 8x = 3x^2 + 11x$ when $x = 1$.

3. $3x^2 + 7x + 8 + 5x^2 + 6x - 4 =$ --?-- $x^2 +$ --?-- $x +$ --?--

Simplify by combining like terms:

4. $4h + 5h$

5. $2f + 3f$

6. $9k - 2k$

7. $6m - 2m$

8. $5c - c$

9. $4g + 5g - 6g$

10. $3xy + 6x^2 + 10x^2$

11. $x^2 + 7x^2 + 10$

12. $3b + 5p + 6b$

13. $a + b + c - b$

14. $2a + 5b - 2a + 5b$

15. $4x^3 + 5x^2 - 2x^2 + x^2$

16. $10 + m - 6 + m$

17. $5a^2 + a + 1 - a$

18. $x^2 + 3x^2 - 4x^2$

19. $x^2 + 16x + x^2 - 10x$

B

Simplify:

20. $12x^2 + 3xy + 7y^2 - 3y^2 + x^2$

21. $1.2x + 5.6x + 13.2x - 4.5x$

22. $\frac{1}{2}x + \frac{1}{3}y - \frac{1}{3}x + \frac{5}{6}y$

23. $\frac{2}{5}x + \frac{1}{15}y - \frac{1}{10}x + \frac{7}{15}y + .8x$

24. $40a^3 + 6a^2 - 25a^3 + 10 - 4a^2$

25. $1.3a - .3a - .7a + a + 6a$

Order of Operations[A]

Suppose your class were asked to find the value of $8 + 6 \div 2$. Someone in the class might do the division first and get 11 as the answer. Some other pupil might do the addition first and get 7 as the answer.

Again suppose your teacher should ask the class to simplify the expression $2 + 2 \times 2$. If you do the multiplication first, you get 6 as the result. If you do the addition first, you get 8 as the result.

From these two examples one can see that it is very important that we agree upon some order of performing the indicated operations. Mathematicians agree upon the following rule:

1. **In a series of operations first find the value of each term by performing the multiplications and divisions in the order given.**
2. **Then combine the terms.**

Example 1. Simplify: $12 + 8 \div 4$.

Solution. The first term is 12 and the second term is $8 \div 4$.

Then

$$\begin{aligned} &12 + 8 \div 4 \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

Example 2. Simplify: $6 + 10 - 12 \div 4$.

Solution. This expression has three terms. The first term is 6, the second is 10, and the third is $-12 \div 4$.

Then

$$\begin{aligned} &6 + 10 - 12 \div 4 \\ &= 6 + 10 - 3 \\ &= 13 \end{aligned}$$

Example 3. Simplify: $\frac{10 - 3 \times 2}{4}$.

Solution. There is only one term in this expression. This fraction means that $10 - 3 \times 2$ is to be divided by 4. We simplify the numerator according to the preceding rule and then we divide this result by 4.

Then $$\frac{10 - 3 \times 2}{4} = \frac{10 - 6}{4} = \frac{4}{4} = 1$$

Example 4. Simplify $4 + 3(7 - 2)$.

Solution. This expression has two terms. The first term is 4 and the other one is $3(7 - 2)$. We first simplify the expression within the parentheses, obtaining 5, and then multiply this result by 3, obtaining 15.

Then $$\begin{aligned} &4 + 3(7 - 2) \\ &= 4 + 3(5) = 4 + 15 = 19 \end{aligned}$$

EXERCISES

A

Simplify the following by performing the indicated operations:

1. $4 \times 3 + 1$
2. $3 + 6 \div 2$
3. $10 \div 5 + 1$
4. $4 + 4 \div 4$
5. $4 \div 4 + 4$
6. $5 \times 7 \cdot 3$
7. $6 \div 3 + 2$
8. $5 - 2 \times 2$
9. $6 \div (3 + 2)$
10. $6 \div 3 - 2$
11. $9 + 5 \div 2$
12. $18 + 9 \div 3$
13. $100 - (5 \times 12)$
14. $80 \div (12 + 4)$
15. $8 \div 2 + 6 \times 2$

B

Simplify:

16. $40 - 3 \times 7 + 4$
17. $12 + 2(3 + 4)$
18. $24 \div 6 - 10 \div 5$
19. $(7 + 8 + 10) \div 5$
20. $30 - 2(11 - 7)$
21. $\frac{8 + 7 - 5}{3}$
22. $\frac{15 \div 3 + 6}{2}$
23. $\frac{40 - 10}{9 + 6}$

Evaluating an Algebraic Expression [A]

Evaluating an algebraic expression means finding the value of the expression when numerical values have been substituted for the letters in the expression.

You have already learned how to evaluate in simple formulas on pages 6 and 7 and in simple expressions on page 9. We shall now consider more difficult evaluations. If you will study the following examples, you will have no difficulty in solving the exercises which follow.

Example 1. Find the value of $3x^2y$ if $x = 5$ and $y = 2$.

Solution. $3x^2y = 3(5)^2(2) = 3 \times 5 \times 5 \times 2 = 150$

Example 2. Find the value of $26 - y^2$ if $y = 3$.

Solution. $26 - y^2 = 26 - 3^2 = 26 - 9 = 17$

Example 3. Evaluate $4x^3 - 2xy$ if $x = 3$ and $y = 5$.

Solution.

$$\begin{aligned} &4x^3 - 2xy \\ &= 4(3)^3 - 2(3)(5) \\ &= 108 - 30 = 78 \end{aligned}$$

Notice that we first find the value of each term and then combine the terms.

Example 4. Find the value of $\frac{x^2 - x + 5}{2}$ when $x = 3$.

Solution. In this example the numerator, $x^2 - x + 5$, is to be divided by 2.

Then $$\frac{x^2 - x + 5}{2} = \frac{9 - 3 + 5}{2} = \frac{11}{2} = 5\tfrac{1}{2}$$

Notice that these examples, after numerical values have been substituted for the letters, are like the examples on page 16.

EXERCISES

A

If $x = 4$ and $y = 5$, find the value of

1. $\frac{1}{4}x^2$	**4.** x^2y	**7.** $2x^2 - y^2$
2. $10y^2$	**5.** $5x^2y$	**8.** $y^2 + 10$
3. $6y^2$	**6.** xy^2	**9.** $30 - x^2$

If $a = 3$, $b = 2$, and $c = 4$, evaluate

10. $2b^2 + 10$	**13.** $a^2 + b^2 + c^2$	**16.** $3bc - 24$
11. $(ab)^2$	**14.** $29 - abc$	**17.** $3b^2 + 5a$
12. $a^3 - b^2$	**15.** $c^2 - ab$	**18.** $2c^2 - cb$

If $x = 6$, $y = 3$, and $h = 2$, find the value of

19. $\frac{2x^2 + xy}{3}$	**22.** $\frac{x}{y} + \frac{10}{h}$	**25.** $\frac{2xy}{h} + (hy)$
20. $\frac{x^2 - hy}{9}$	**23.** $\frac{h^2 + y^2}{x}$	**26.** $\frac{2x^2}{3} + \frac{4y}{3}$
21. $h(2x - y)$	**24.** $3x^2y - 5h^3$	**27.** $\frac{1}{x} + \frac{1}{y} - \frac{1}{2h}$

B

28. Solve exercises 19–27 by letting $x = \frac{1}{2}$, $y = \frac{1}{3}$, and $h = \frac{1}{4}$.

29. Solve exercises 19–27 by letting $x = .5$, $y = .2$, and $h = .1$.

Thinking with Literal Numbers [A]

Pupils may at first have difficulty in thinking with numbers that are represented by letters. If at any time you do not know how to solve a problem involving literal numbers (numbers represented by letters), proceed as follows:

1. **Make up a similar, easy numerical problem.**
2. **Solve this numerical problem.**
3. **Solve the literal problem by the same method that you used in solving the numerical problem.**

Example. Find the number of inches in x feet.

Solution. 1. Find the number of inches in 2 feet.

2. Multiply 2, the number of feet, by 12, obtaining 24.

3. Multiply x, the number of feet, by 12, obtaining $12x$. Then $12x$ is the number of inches in x feet.

EXERCISES [A]

Express the following using the symbols of algebra:

1. Seven minus two
2. Nine divided by four
3. Seven minus x
4. The product of 2 and m
5. 16 increased by y
6. a more than b
7. x square
8. Twice the square of x
9. The product of h and k
10. The sum of a and b
11. Twice the sum of a and b
12. 6 less x
13. 6 less than x
14. r diminished by s
15. 7 more than twice x
16. x divided by twice y
17. y plus the double of c
18. The cube of x
19. x square less y square
20. 4 greater than b
21. Twice the cube of x
22. 4 increased by p
23. m increased by h
24. y decreased by twice m
25. The sum of m and n
26. y more than 8

If you have any difficulty with the following literal problems, follow the suggestions for solving such problems on page 18.

27. Find the cost of x pounds of sugar at 8 cents a pound.

28. Find the cost of y oranges at 10 cents apiece.

29. How many inches are there in y feet?

30. How many feet are there in m inches?

31. How many yards are there in h feet?

32. How many dimes are there in x cents?

33. At 10 cents each, how many grapefruit can be purchased for k cents?

34. Find the number of inches in k yards.

35. Find the number of inches in (x feet $+ y$ inches).

36. How many ounces are there in y pounds?

ORAL EXERCISES [A]

1. If n stands for a certain number, how many n's will stand for a number 3 times as large?

2. If x represents the number of years in Henry's age and Frank is twice as old as Henry, how can you represent Frank's age in terms of x?

3. The length of a rectangle is five times its width. If w stands for its width, how can you represent its length in terms of w?

4. If x stands for the number of years in Amy's age, what will stand for the number of years in her age 5 years from now? For the number of years in her age 8 years ago?

5. Janice has 6 times as many stamps as Eunice. Let x stand for the number of stamps that Eunice has. What will stand for the number of stamps that Janice has?

6. Charles is 6 years older than Henry. If x stands for the number of years in Henry's age, how can you represent Charles's age?

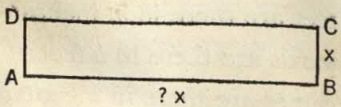
7. The difference of two numbers is 7. If y represents the smaller, how can you represent the larger?

8. The difference of two numbers is 4. If the larger is s, what is the smaller?

9. A boy has 5 times as many nickels as dimes.

Let $x =$ the number of dimes the boy has
Then --?-- $x =$ the number of nickels the boy has.

10. The length of rectangle $ABCD$ is five times its width.



Let $x =$ the number of inches in its width
Then --?-- $x = ?$

Equations[A]

An *equation* is an expression of equality between two quantities. It is a shorthand way of saying that two quantities are equal. Thus the equation $4\,n = 12$ is a shorthand way of saying "Four times a number equals 12." It asks the question "Four times what number is 12?"

The part of the equation to the left of the equals sign is called the *left member* of the equation, and the part of the equation to the right of the equals sign is called the *right member* of the equation. In the equation $4\,x + 7 = 35$, the expression $4\,x + 7$ is the left member of the equation and 35 is the right member of the equation.

Solving an equation is finding the value of the letter in the equation. If the equation is $3\,n = 15$, then $n = 5$ is the solution of the equation, because 3 times 5 is 15. In this case we say that 5 *satisfies* the equation and that 5 is the *root* of the equation. Solving an equation is finding the value of $1\,x$, $1\,y$, $1\,c$, $1\,a$, etc.

Example 1. Solve $6\,x = 12$.

Solution. $6\,x = 12$

Dividing both members by 6, $x = 2$. Notice that we divide $6\,x$ by 6, getting as the quotient $1\,x$, or x. $x = 2$ is the solution of the equation.

Example 2. Find the root of $3\,x + 2\,x = 15$.

Solution. $3\,x + 2\,x = 15$

Simplifying the left member, $5\,x = 15$

Dividing both members by 5, $x = 3$

Then 3 is the root of the equation.

Example 3. Solve $4\,x + 2.3\,x = 9.45$.

Solution. $4\,x + 2.3\,x = 9.45$

Simplifying the left member, $6.3\,x = 9.45$

Dividing both members by 6.3, $x = 1.5$

Check. Does $4(1.5) + 2.3(1.5) = 9.45$?

Does $6.00 + 3.45 = 9.45$? Yes.

In checking the solution of an equation, we substitute in the given equation the value found for the letter. Then if the left member equals the right member, we know that this value of the letter satisfies the equation and is a root of the equation.

ORAL EXERCISES [A]

Solve:

1. $3x = 15$
2. $10x = 50$
3. $7x = 21$
4. $8y = 24$
5. $9m = 18$
6. $4r = 17$
7. $9h = 21$
8. $10w = 25$
9. $7h = 32$
10. $6h = 25$
11. $x + 3x = 24$
12. $4x + x = 20$
13. $2b + 3b = 15$
14. $5p + 7p = 48$
15. $6h + h = 49$
16. $3m - m = 8$
17. $5x - 2x = 9$
18. $7x - 3x = 12$
19. $x + x + x = 30$
20. $m + 5m = 42$

WRITTEN EXERCISES

A

Solve the following equations and check each solution.

1. $3x + 4x = 105$
2. $7x + 7x = 56$
3. $7y - y + 3y = 108$
4. $11x - 5x + 2x = 104$
5. $2c + 6c - 3c = 120$
6. $8x - 3x = 1000$
7. $8m + 10m - m = 136$
8. $6w - 5w + 2w = 90$
9. $13x - x = 130 - 6$
10. $35s - 2s = 100 - 3$
11. $14h - 2h = 115 + 149$
12. $4b + 3b = 101 - 32$

B

Solve:

13. $1.2x - 0.4x = 8.8$
14. $1.8x - .7x = 99$
15. $.4m + .1m = 35$
16. $0.12a - .04a = 8.8$

17. $3.1\,p + 2.5\,p = 140$

18. $1.5\,a + 2.5\,a = 1 - .92$

19. $2\,x + 3\,x = .17 + .08$

20. $.24\,x + .16\,x = 8.6 - 3.8$

NOTE TO THE TEACHER: Remedial exercises in division of decimal fractions are given on page 574.

Solving Problems by Algebra [A]

Some problems can be solved by either algebra or arithmetic. Some problems can be solved very easily by arithmetic, and there is no need of using algebra to solve them. Then there are problems whose solutions by algebra are not difficult, but whose solutions by arithmetic are almost impossible.

Some of the problems which you will soon be asked to solve by algebra are very easy and you may wish to solve them by arithmetic. However, you are asked to solve them by algebra so that you may learn the algebraic method. You can learn the algebraic method more easily if you begin with easy problems.

Now read the following directions for solving problems by algebra. Then study the solutions of the examples on the next page. Notice that there are five steps in any solution.

1. **Read the problem** and determine what number (or numbers) you are asked to find.

2. **Represent the unknown number (or numbers) algebraically.**

   a. If you are asked to find only one number, let some letter equal it (represent it).

   b. If you are asked to find more than one number, let some letter equal one of them and then represent each of the other numbers in terms of this letter.

3. From the conditions of the problem **find two expressions or quantities that are equal.** Then connect these two equal expressions by an equals sign, forming an equation.

4. **Solve this equation for the unknown letter.**

   If you are asked to find more than one number, do this from step 2 *b* above.

5. **Check** by seeing that your answer (or answers) satisfies all the conditions of the problem.

Example 1. Five times a certain number equals 120. Find the number.

Solution. Let $x =$ the number
Then $5x = 120$
* D_5 $x = 24$, the number

Check. Does 5 times $24 = 120$? Yes.

Example 2. A number increased by six times itself equals 98. What is the number?

Solution. Let $x =$ the number
6 times the number $= 6x$
From the problem we get $x + 6x = 98$
$7x = 98$
D_7 $x = 14$, the number

Check. Does 14 increased by 6 times $14 = 98$? Yes.

Example 3. Dick's father is three times as old as Dick, and the sum of their ages is 48 years. How old is each?

We shall solve this problem in two ways.

Solution 1.

Let $d =$ the number of years in Dick's age
Then $3d =$ the number of years in his father's age
Then
$d + 3d = 48$
$4d = 48$
D_4 $d = 12$, Dick's age
$3d = 36$, his father's age

Solution 2.

Let $f =$ the number of years in Dick's father's age
Then $\frac{1}{3}f =$ the number of years in Dick's age
Then
$f + \frac{1}{3}f = 48$
$\frac{4}{3}f = 48$
$D_{\frac{4}{3}}$ $f = 36$, father's age
$\frac{1}{3}f = 12$, Dick's age

Which of these two solutions do you think is the easier? Why?

Check. 36 is 3 times 12. The sum of 36 and 12 is 48.

* D_5 is an abbreviation for "Dividing both members of the equation by 5."

Always check your answers to verbal problems by determining whether they agree with the data given in the problems.

EXERCISES

A

1. Eight times a certain number is 408. What is the number?

2. Paul said, "I am thinking of a number. If I multiply it by 11, I get 1353 as the product. What is the number?"

3. A certain number increased by 5 times itself equals 42. Find the number.

4. One number is 5 times as large as another, and their sum is 54. What are the numbers? (Why will you choose to let x represent the smaller number?)

5. One number is 8 times as large as another, and their difference is 56. Find the numbers. ($8x - x = 56$)

6. Mary is four times as old as her sister. How old is each girl if the sum of their ages is 25 years?

7. A president of a bank receives 8 times as much salary as one of the messenger boys. If the sum of their annual salaries is $9900, what is the annual salary of each?

8. Separate 276 into two parts so that one part will be twice the other.

9. Apportion $3450 among three people so that the second person will receive twice as much as the first and the third will receive three times as much as the first.

Let x = the number of dollars the first will receive
Then $2x$ = the number of dollars the second will receive
and $3x$ = the number of dollars the third will receive

Complete the solution.

10. Separate 1000 into three parts so that one part will be three times as large as another and the third part will be as large as the sum of the other two.

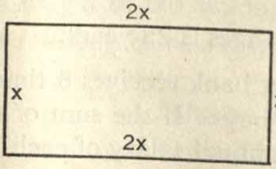
11. Patricia lives east of McKinley High School and Jane lives west of the school. Patricia lives four times as far from the school as does Jane. How far is each from the school if their homes are 18 blocks apart?

12. A board 24 feet long is to be cut into three pieces so that the longest piece will be 3 times as long as each of the other two. How long will each piece be?

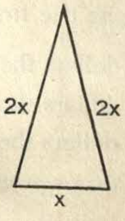
13. Bill is $\frac{1}{4}$ as old as his father and the sum of their ages is 55 years. How old is each? (Let $x =$ the number of years in Bill's age. Then $4x =$ the number of years in the father's age.)

14. If you have $10,400 to invest in a house and lot, how much should you pay for the lot, assuming that the house should cost four times as much as the lot?

15. A rectangle is twice as long as it is wide, and its perimeter (the sum of the lengths of its four sides) is 60 feet. Find its dimensions.



16. A rectangle is 5 times as long as it is wide. If its perimeter is 120 inches, what are its width and length? (Why should you make a drawing whenever it is possible to do so?)



17. The perimeter of the triangle above is 70 inches. How long is each side?

B

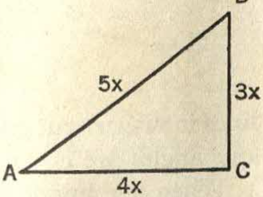
18. The quotient of two numbers is 6 and their sum is 196. Find the numbers. (Let $x =$ the smaller number. Then $6x =$ the larger number. Why?)

19. A family wishes to budget its monthly income of \$360 so that the rent, savings, and food cost shall be equal to each other and so that the remainder of the income shall equal 5 times the amount spent for food. How much can the family afford to spend for rent?

20. A certain kind of concrete contains twice as much sand as cement, and $\frac{3}{2}$ as much gravel as sand. How many cubic yards of each must be used to make 915 cubic yards of concrete, provided there is no loss in volume?

21. A certain mixture contains 1 part of glycerine, 1 part of turpentine, and 4 parts of water. How many ounces of each must be used to make 16 ounces of the mixture?

22. If you stretch a rope around three stakes driven into the ground at points A, B, and C, you form a triangle. If $AC = 4x$, $BC = 3x$, and $AB = 5x$, then $\angle C$ is a right angle. How long should each of these sides be to form the right triangle if the rope is 60 feet long?



23. An athlete wishes to run a mile in 4 minutes and 35 seconds. He plans to run the third quarter in the same time as the second quarter, to run the first quarter in $\frac{13}{14}$ of the time of the second quarter, and to run the last quarter in $\frac{12}{13}$ of the time of the first quarter. In what time shall he run each quarter?

Angles [A]

An *angle* is the figure formed by two lines meeting at a point. In Fig. 1 the two lines *BA* and *BC* meet in the point *B*, forming angle *ABC* ($\angle ABC$). The point *B* is the *vertex* of the angle and the lines *BA* and *BC* are the *sides* of the angle.

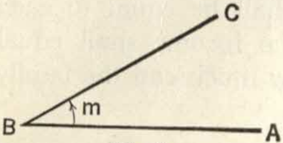


Fig. 1

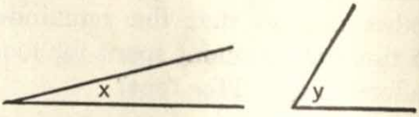
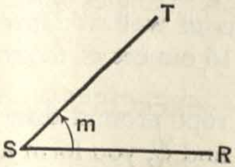


Fig. 2

The size of an angle depends upon the amount of opening between the lines and not upon the length of the sides. Of the two angles in Fig. 2, $\angle y$ is larger than $\angle x$.

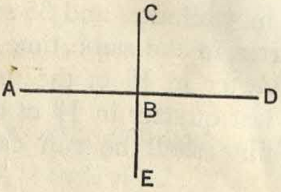
There are three common ways of naming an angle:

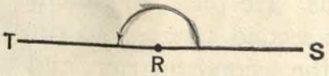


1. *By a small letter or figure written within the angle, as* $\angle m$.
2. *By the capital letter at its vertex, as* $\angle S$.
3. *By three capital letters, the middle letter being the vertex letter, as* $\angle RST$.

Kinds of Angles [A]

When two straight lines intersect (cut each other), four angles are formed. If these four angles are equal, each angle is a *right angle* and contains 90°. When one line forms a right angle with

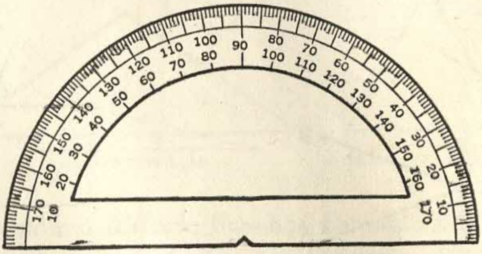




another line, the lines are *perpendicular* to each other. If the two sides of an angle extend in opposite directions forming a straight line, the angle is a *straight angle*. A straight angle contains 180°.

An angle that is less than 90° is an *acute angle*.

An angle that is larger than 90° and less than 180° is an *obtuse angle*.

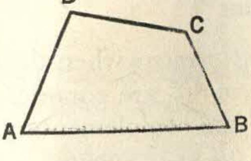


Protractor

A *protractor* is an instrument for measuring angles. You probably learned how to measure angles with this instrument in your grade school.

EXERCISES [A]

1. On your paper draw a figure which has the same shape as *ABCD* but is larger. With a protractor measure each of the four angles of the figure you have drawn. What is the sum of these four angles?



2. Draw a triangle on your paper and measure each of its angles. What is the sum of its angles?

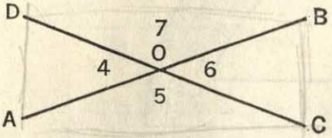
3. At 3 o'clock is the angle made by the hands of a clock an acute angle, a right angle, or an obtuse angle?

4. How large is the angle made by the hands of a clock at 2 o'clock? Which kind of angle is it?

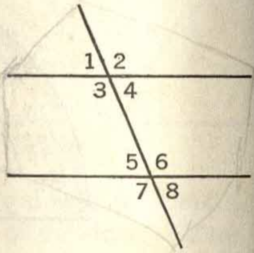
5. Which kind of angle is made by the hands of a clock at 5 o'clock?

6. Measure angles 4, 5, 6, and 7 in the left-hand figure. Which angles are equal?

7. Name ∠4 using three letters. Name ∡5, 6, and 7 using three letters.



Ex. 6, 7



Ex. 8

8. Measure the eight angles in the figure at the right above.

$\angle 1 = \angle$ --?-- $= \angle$ --?-- $= \angle$ --?--

$\angle 2 = \angle$ --?-- $= \angle$ --?-- $= \angle$ --?--

9. Copy and complete the following sentence: The size of an angle depends upon the amount of opening between its --?--.

10. Copy and complete this sentence: The vertex of $\angle BCD$ is the point --?-- and the sides of the angle are --?-- and --?--.

Complementary Angles [A]

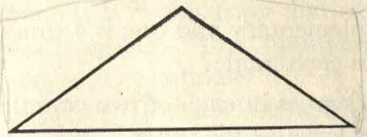
Two angles are *complementary* when their sum is 90°. Thus an angle of 50° and one of 40° are complementary; and an angle of 80° and one of 10° are complementary. Each is the complement of the other.

Supplementary Angles [A]

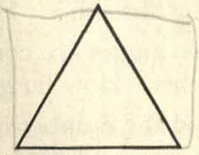
Two angles are supplementary when their sum is 180°. For example, two right angles are supplementary; and an angle of 150° and one of 30° are supplementary. Each is the supplement of the other.

Kinds of Triangles [A]

If a triangle has two equal sides, it is *isosceles.* If a triangle has its three sides equal, it is *equilateral.* You can see that an equilateral triangle is isosceles.



Isosceles triangle



Equilateral triangle

A *right triangle* is a triangle that has one right angle.

EXERCISES [A]

1. With your ruler draw three triangles having different shapes. With your protractor measure the angles of each triangle. What is the sum of the angles of each triangle?

2. Compare the sums of the angles of the triangles in Exercise 1. Complete this sentence: The sum of the angles of any triangle is --?-- degrees.

3. One angle of a triangle contains 80° and another angle contains 60°. Find by arithmetic the number of degrees in the third angle.

4. One angle of a right triangle contains 20°. Find by arithmetic the size of the other acute angle.

5. How large is the complement of an angle of 60°? of an angle of 10°? of one of 80°? of one of $x°$?

6. What is the number of degrees in the supplement of an angle of 70°? of an angle of 100°? of one of 140°? of one of y degrees?

7. How large is each angle of an equilateral triangle? (The angles are equal to each other.)

WRITTEN EXERCISES [A]

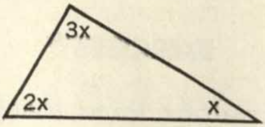
Solve the following problems according to the directions given on page 23.

1. Two angles are supplementary and one of them is twice as large as the other. How large is each angle?

2. Two angles are complementary and one is 4 times as large as the other. How large is each angle?

3. Find the number of degrees in each of two complementary angles if one is 17 times as large as the other.

4. One angle of a triangle is twice as large as one of the other angles, and the third angle is equal to the sum of the other two. How large is each angle?



Let x = the number of degrees in the smallest angle
Then $2x$ = the number of degrees in the second angle
and $3x$ = the number of degrees in the third angle

Here the three preliminary statements above and the drawing both express the same facts. Which of the two ways is the shorter? Which is the more helpful in solving the problem? Now complete the solution.

5. Of the three angles of a triangle the largest is five times the smallest, and the remaining angle is three times the smallest. Find the angles.

6. Find two complementary angles if one of them is one-ninth as large as the other. ($x + \frac{1}{9}x = 90$ or $9x + x = 90$)

7. One angle of a triangle is eight times as large as another, and the third angle is equal to the difference between the other two. Find the size of each angle.

8. One acute angle of a right triangle is four times as large as the other acute angle. How many degrees are there in each angle?

How Algebra Has Grown[A]

It may be said that algebra had its beginning when symbols were first used in problem-solving. About 1650 B.C. an Egyptian by the name of A'h-mosè, commonly called Ahmes, made a copy of a manuscript which had been written six hundred years before. It is in this copy that we find the first use of symbols in mathematics. The manuscript deals with fractions, mensuration, and equations.

Diophantus of Alexandria simplified problem-solving about 300 B.C. by allowing a letter to represent the unknown number. The idea of letting a letter stand for a number did not appeal to the thinkers of that time. The first mathematician to develop the algebraic method in problem-solving was François Viète (1540–1603). This French mathematician is sometimes called "the Father of Algebra."

John Widman, a professor in the University of Leipzig, was the first to use the + and − signs when he published a textbook in 1489. These signs were introduced into England in a textbook written in 1558 by Robert Recorde. The signs were not universally used until a hundred years later.

Oughtred, an Englishman, about 1631 used the sign × for multiplication. Almost at the same time Harriot used the dot for multiplication. (This is the same Harriot who was sent out by Sir Walter Raleigh to survey and map out Virginia.) About 1637 Descartes, the great French mathematician, expressed multiplication by omitting its sign.

Division was first denoted by expressing the quotient as a fraction. An English textbook published about 1633 expressed division by the colon (:). The symbol ÷ to express division was first used in 1659. The equals sign (=) was first used by Recorde in 1557.

Word Lists[A]

Near the close of each chapter of this book is a list of words which have appeared in the preceding pages. You should study these words so that you can spell each one of them correctly, know its meaning, and be able to use it properly in a sentence.

If you do not know the meaning of some word, turn to the page given in parentheses at the right of the word.

WORD LIST

acute angle (p. 29)
algebra (p. 3)
angle (p. 28)
binomial (p. 13)
coefficient (p. 10)
complementary angles (p. 30)
equation (p. 21)
equilateral triangle (p. 31)
evaluating (p. 17)
exponent (p. 12)
factor (p. 10)
formula (p. 5)
isosceles triangle (p. 31)
left member (p. 21)
like terms (p. 13)
literal number (p. 18)
monomial (p. 13)
negative (p. 3)
obtuse angle (p. 29)
opposite (p. 3)
parentheses (p. 11)
perimeter (p. 6)
perpendicular (p. 29)
polynomial (p. 13)
product (p. 9)
protractor (p. 29)
rectangle (p. 5)
right angle (p. 28)
right triangle (p. 31)
root (p. 21)
straight angle (p. 29)
supplementary angles (p. 30)
term (p. 13)

CHAPTER REVIEW

A

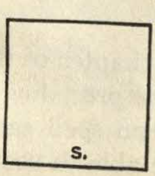
1. What are the terms of an expression?

2. How many terms are there in a monomial? a binomial? a trinomial? Name three words that start with *bi* or *by*. Name three words that start with *tri*.

3. What is the name of the 2 in the expression $5x^2$?

4. What are the like terms in the expression $4x^2 - 7x^3 + 6x - 5x^2$?

5. Write a formula to express the perimeter of this square.

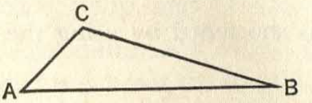


6. What is the value of the perimeter of the square when $s = 6$?

7. If $x = 7$, what is the value of $4x$? of $8x$?

8. Read $\angle A$ of the triangle below, using three letters.

9. Read $\angle ACB$ using one letter.

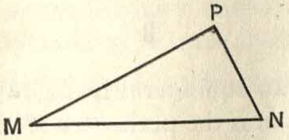


10. How many terms are there in the expression $4ab+3c^2-1$?

11. Name the factors of 10; of 15; of ab.

12. In the expression $3ab$ what is the coefficient of b? of ab?

13. In $\triangle MNP$ which is the largest angle? the smallest angle? Is $\angle M$ an acute angle or a right angle? Which of the three angles is a right angle?



14. One of two complementary angles contains 72°. How many degrees are there in the other angle?

15. Why cannot the times sign in the expression 4×5 be omitted?

16. Copy and complete: A triangle that has two equal sides is an --?-- triangle.

17. Simplify by combining like terms:

a. $4x^2 + 8x^2 + 5x^2$

b. $m^2 + 2m - m^2$

c. $2x^3 + 6x^2 + 7x^2 - x^3$

d. $2a + 3b + c + 5a$

18. Write $3a^4$ without using an exponent.

19. If $x = 6$ and $y = 4$, find the value of

a. x^2

b. $3x^2$

c. y^2

d. $4y^2$

e. x^3

f. $4x^3$

g. y^3

h. $2y^3$

i. $2x - 3y$

j. $x^2 - y^2$

k. $xy - 4$

l. $y^2 - x$

20. Write the following using the symbols of algebra:

a. The sum of a and b

b. The square of a

c. Twice the square of a

d. The product of c and 6

e. The cube of c

f. 10 less h

g. Three times the cube of x

h. a divided by m

21. A rule that is shortened by using the symbols of algebra is called a __?___.

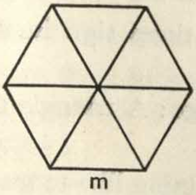
22. What is the cost of y articles at 6 cents each? at k cents each?

23. The sum of two numbers is 296 and one of them is 3 times as large as the other. Find the numbers.

24. One of two complementary angles is 5 times as large as the other. How large is each angle?

B

25. This figure is an equilateral hexagon divided into 6 equilateral triangles. What is the perimeter of each triangle? What is the perimeter of the hexagon? What is the total length of the 12 line segments?



26. If $x = 5$ and $y = 3$, find the value of

a. $\frac{8 + xy}{10}$

b. $\frac{x^2y - xy^2}{6}$

c. $\frac{2x^3 - 33}{7}$

27. If $a = 7$, $b = 1$, and $c = 2$, find the value of

a. $c + (a + b)$

b. $a + (b + c)$

c. $2b(a - b)$

d. $10 - (a - b)$

e. $(2a + b) \div c$

f. $32 \div (9b + 7)$

28. Write the following using exponents:

a. xxx	**c.** $6\ xxx$	**e.** $3\ aaaa$
b. $mmmm$	**d.** $cccd$	**f.** $4\ hh - 5\ hhh$

29. A rectangle is 7 times as long as it is wide and its perimeter is 104 inches. Find its width and length. (Remember that a rectangle has two ends, a top, and a bottom.)

30. The sum of three numbers is 6480. The second is 5 times the first and the third is 6 times the second. Find the numbers.

31. One of two complementary angles is one-fourteenth as large as the other. How large is each angle?

32. The quotient of two numbers is 8 and their sum is 27. Find the numbers.

33. The quotient of two numbers is 6 and their difference is 25. Find the numbers.

34. One of two supplementary angles is one-nineteenth as large as the other. How large is each angle?

35. Each of two angles of a triangle is half as large as the third angle. How large is each angle?

36. Each edge of a cube is 6 inches. Find the area of its six faces.

37. Find the volume of the cube in Ex. 36 ($V = e^3$).

38. How many gallons are there in q quarts?

39. A rectangular field is enclosed by a fence $\frac{3}{8}$ of a mile in length. If the length of the field is 5 times the width, how wide is the field? (1 mi. = 5280 ft.)

40. If 0.7 times a number is subtracted from 1.8 times the number, the remainder is 77. Find the number.

41. Write the following, using the symbols of algebra:

a. Twice the square of x increased by the cube of y.

b. The sum of a and b, divided by c.

c. The product of 6 and a, decreased by b.

d. The quotient of x and y.

CHAPTER TEST [A]

1. Write a formula for the perimeter p of a rectangle with base b and height h.

2. Indicate the sum of a and c.

3. Indicate "10 more than y."

4. What are the factors of 6?

5. What is the name of the 4 in the expression $4y^2$?

6. What does $a \cdot b$ mean?

7. Find the value of $6y$ if y is 9.

8. Write $4y^3$ without using the exponent 3.

9. Find the value of a^2 if a is 5.

10. Is $4a - 6b$ a monomial, a binomial, or a trinomial?

11. Combine like terms: $4a + 10 + 3a$.

12. Simplify $6 \div 2 + 4$.

13. Evaluate $x^2 + y^2 - 10$ for $x = 5$ and $y = 2$.

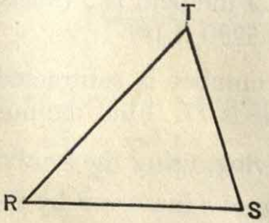
14. Find the number of inches in k feet.

15. Solve for x: $6x + 5x = 66$.

16. Solve for y: $14y - 2y = 20$.

17. One number is 5 times as large as another and their sum is 324. Find the numbers.

18. Name the angles of this triangle, using three letters.



19. If $\angle S = 72°$ and $\angle R = 48°$, how many degrees are there in $\angle T$?

20. Find the cost of 6 notebooks at c cents each.

CHAPTER TEST [B]

Indicate

1. x plus y
2. 8 less than c
3. The product of 6 and x
4. The square of h

5. Find the value of $14\ \pi$ if $\pi = 3.14$.

6. Find the value of $6\ a^2c$ if $a = 3$ and $c = 2$.

7. Simplify $8 + 8 \div 8$.

8. Evaluate $3(x - 2)$ when $x = 9$.

9. Find the weight of n bolts if each bolt weighs 4 ounces.

10. Solve $7\ x - x = 42$.

11. Solve $.5\ y - .3\ y = 8$.

12. A rectangle is 3 times as long as it is wide, and its perimeter is 200 inches. What are its width and length?

13. One angle of a triangle is three times as large as each of the other angles. How many degrees are there in each angle?

14. What is the coefficient of y in the expression $4\ y + 2$?

15. Write $7\ aaa$ using an exponent.

16. How many terms are there in the expression $a - 2\ b + 6\ c$?

17. If $x = 8$ and $y = \frac{1}{2}$, find the value of $x - 10\ y$.

Copy and complete the following statements:

18. We cannot add --?-- terms.

19. $c - d$ is the --?-- between c and d.

20. The symbols ( ) are called --?--.

MATHEMATICS USED IN PHYSICS

PHYSICS is the science which deals with matter and energy. It includes the study of mechanics, heat, light, sound, and electricity.

The study of physics is frequently begun in the third year of high school. As a preparation for high school physics, a student should have completed a year of algebra and a year of plane geometry. In algebra the student should be able to understand and use formulas, solve simple equations and verbal problems, and use ratio, proportion, and variation.

The following formulas were taken from high-school physics texts:

Formulas in Mechanics

$$d = vt \qquad f = ma \qquad v = \sqrt{2\,gd}$$

$$d = \tfrac{1}{2}\,at^2 \qquad a = \frac{v^2}{r} \qquad K.\,E. = \tfrac{1}{2}\,mv^2$$

$$W = fd \qquad \frac{t_1}{t_2} = \frac{\sqrt{l_1}}{\sqrt{l_2}} \qquad t = \pi\sqrt{\frac{l}{g}}$$

Formulas in Electricity

$$P = IE \qquad W = IEt \qquad H = 0.24\,I^2Rt$$

$$I = \frac{E}{R} \qquad I = \frac{E}{R + r} \qquad C = \frac{nE}{R + nr}$$

$$\frac{R}{X} = \frac{m}{n} \qquad R = \frac{r_1 \times r_2}{r_1 + r_2} \qquad C = \frac{E}{R + \frac{r}{n}}$$

$W = fd$ Is a Formula Expressing the Law of Work: The Work Performed Equals the Force Times the Distance through Which It Acts

CHAPTER II

Formulas

The Importance of Formulas [A]

In this chapter we shall make a further study of formulas. We shall learn how to solve more difficult problems when the rules for their solution are given as formulas, and shall practice making tables which satisfy formulas. We shall also learn how to make the formulas from rules and from tables.

You probably would like to know why so much of algebra is devoted to the formula. In the last two centuries great advances have been made in the study of electricity, light, heat, sound, mechanics, astronomy, statistics, and life insurance. Much of the progress in these different fields has been made through the means of the formula.

The formula is universally used in industry. Many large manufacturing companies maintain laboratories for research. Much of the vast amount of information gathered in these laboratories is recorded in formulas, since they save much time and energy by showing the relation of the different items involved. The handbooks of the various trades express many of their rules as formulas. Since the formula has such a wide application, a general knowledge of it should be a part of everyone's store of information.

How Formulas Are Used to Solve Problems [A]

The solution of problems through the use of the formula is not difficult if one will only take time to study the examples and carefully follow the directions, given on the next page, for solving problems. When using a formula in the solution of a problem, remember that the formula is a simplified rule or a way of stating a fact.

Example 1. Find the simple interest on \$450 for 2 years and 4 months at 6%.

Solution. The formula is $i = prt$, where i is the interest in dollars, p the principal in dollars, r the rate in per cent, and t the time in years. In this example $p = 450$, $r = .06$, $t = 2\frac{1}{3}$ or $\frac{7}{3}$. Substituting these values in the formula, we have $i = 450 \times .06 \times \frac{7}{3}$. Doing the indicated multiplications, we obtain $i = 63$. The interest is \$63. Since the sign % always means hundredths, the 6% was changed to .06 before substituting in the formula. Why is the multiplication sign needed between 450 and .06 and between .06 and $\frac{7}{3}$?

Example 2. An iron ball is dropped from a balloon and reaches the earth in 12 seconds. How high is the balloon?

Solution. The formula is $s = 16\,t^2$, where s represents the number of feet the object falls and t represents the number of seconds the object is falling.

First write the formula: $s = 16\,t^2$

Substitute the value of t: $s = 16 \times 12^2$

Perform the multiplication: $s = 2304$

The height of the balloon is 2304 feet.

Directions for Solving Problems by Formulas:

1. **Write the proper formula.**
2. **Replace the known letters of the formula by their values.**
3. **Perform the indicated operations, thus finding the value of the unknown letter.**

WRITTEN EXERCISES

A

Use the directions in the rectangle above in solving the following exercises:

1. Using the interest formula $i = prt$, find the interest on \$350 for 3 years at 4%. (See the next page.)

Your solution should appear as follows:

$$i = prt$$

$$i = 350 \times .04 \times 3$$

$$i = 42$$

The interest is $42.

The actual multiplication of 350 × .04 × 3 can either be done to the right of your solution or on a separate piece of paper. Perhaps you can do it mentally.

2. What is the simple interest on $420 for 2 years at $4\frac{1}{2}\%$?

3. Find the simple interest on $525.50 for $3\frac{1}{2}$ years at 3%.

4. Find the interest in the three cases below:

	INTEREST	PRINCIPAL	RATE	TIME
a.	?	$700	4%	3 years
b.	?	$540	$3\frac{1}{2}\%$	2 years 6 months
c.	?	$1275	6%	1 year 8 months

5. $A = p + i$ is the formula for finding the amount (A) when the principal (p) and the interest (i) are known. Find the amount when the principal is $400 and the interest is $23.

6. Using the formula $A = p + i$, find A when $p = \$130.50$ and $i = \$1.90$.

7. $A = p + prt$ is the formula for finding the amount when the principal, time, and rate are known. For what does prt stand? Find A when $p = \$325$, $r = 4\%$, and $t = 3$ years.

8. $s = vt$ is a formula in physics. If $v = 900$ and $t = 4$, what is the value of s?

9. $v = \frac{s}{t}$ is a formula which expresses the same relation of v, s, and t as the formula in Exercise 8. Find v if $s = 320$ and $t = 6$.

10. $F = \frac{wa}{g}$ is a formula for finding the force when the weight (w) and acceleration (a) are known. Find the force (in pounds) necessary to accelerate an automobile weighing 2800 pounds, 12 feet per second in one second. The value of g is about 32.2.

11. $P = .433\,h$ gives the water pressure in pounds per square inch for the height of water in feet. What is the water pressure per square inch at the bottom of a standpipe when the water column is 92 feet high?

12. $d = rt$ is the formula for the distance (d) an object will go in t units of time, when it moves r units of distance in each unit of time. If r is expressed in miles per hour, then the time must be expressed in hours, and the distance in miles. How far will an automobile travel in 5 hours, at the rate of 40 miles per hour?

13. Using $d = rt$, find the distance a train will go in 3 hours at the rate of 45 miles per hour.

14. How many feet will a boy run in 10 seconds at the rate of 28 feet per second?

15. How many yards will a girl go in 3 minutes on her bicycle if she travels at the rate of 32 feet per second?

16. The formula $r = \frac{d}{t}$ gives the rate when the distance and time are known. What is the average rate in miles per hour of an airplane that travels 420 miles in $3\frac{1}{2}$ hours?

17. The Indiana State High School cross country meet was won in 1948 by John Stayton of Anderson High School. His time for the two-mile course was 9 minutes and 59 seconds. What was his average speed in feet per second? (1 mile = 5280 feet.)

18. The formula $t = \frac{d}{r}$ gives the time when the distance and rate are known. How many hours will it take an airplane to travel 650 miles at the average rate of 125 miles per hour?

19. How long will it take a man to drive 360 miles in an automobile if he averages 45 miles per hour?

20. Using the formula $s = 16\,t^2$, which was given in Example 2, page 43, copy and complete the following table. (Do not write in your book.) When $t = 0$, $s = 16 \times 0^2 = 0$; when $t = \frac{1}{2}$, $s = 16 \times (\frac{1}{2})^2 = 4$, etc.

t (in seconds)	0	$\frac{1}{2}$	1	2	3	4	5	6	8	16
s (in feet)	0	4	?	?	?	?	?	?	?	?

Study the completed table and answer the following questions:

a. Does the distance increase or decrease as the time increases?

b. When the time has doubled, how has the distance changed?

c. Does s vary directly as the square of the time? If s and t^2 increase together, they vary directly.

21. $p = \frac{100}{v}$ is a formula stating how the pressure and volume of a given amount of air change when the air is confined in a vessel such as a bicycle pump. Copy and complete the following table:

v (in cu. ft.)	1	2	4	8	20	25	50	100	500
p (in pounds)	?	?	?	?	?	?	?	?	?

Study the completed table and answer the following questions:

a. As v increases, how does p change?

b. Is p doubled when v is doubled?

c. If p and v increase together, they vary directly. If p increases when v decreases, they vary inversely. How do p and v vary?

22. A formula for finding the normal weight of an adult who is at least 5 feet in height is $W = \frac{11(h - 40)}{2}$. W represents the normal weight in pounds and h represents the number of inches in the person's height. For example, if a man is 62 inches tall, we have $W = \frac{11(62 - 40)}{2}$. Then, $W = \frac{11 \times 22}{2}$ and $W = \frac{242}{2}$ or 121. The man's normal weight is 121 pounds. Copy the following form and complete the table:

HEIGHT	ACTUAL WEIGHT	NORMAL WEIGHT	POUNDS UNDERWEIGHT	POUNDS OVERWEIGHT
63 in.	130 lb.	?	?	?
70 in.	160 lb.	?	?	?
72 in.	176 lb.	?	?	?

B

23. $W = Fs$ is a formula for finding the work done (in foot-pounds) when the force F (in pounds) and the distance s (in feet) are known. How much work does a horse do in pulling a wagon with a force of 320 pounds a distance of 2 miles?

24. In the formula $P = \frac{N + 2}{o}$, find P when $N = 23$ and $o = 6.25$.

25. $I = \frac{N - 2}{P}$. Find I when $N = 46$ and $P = 6$.

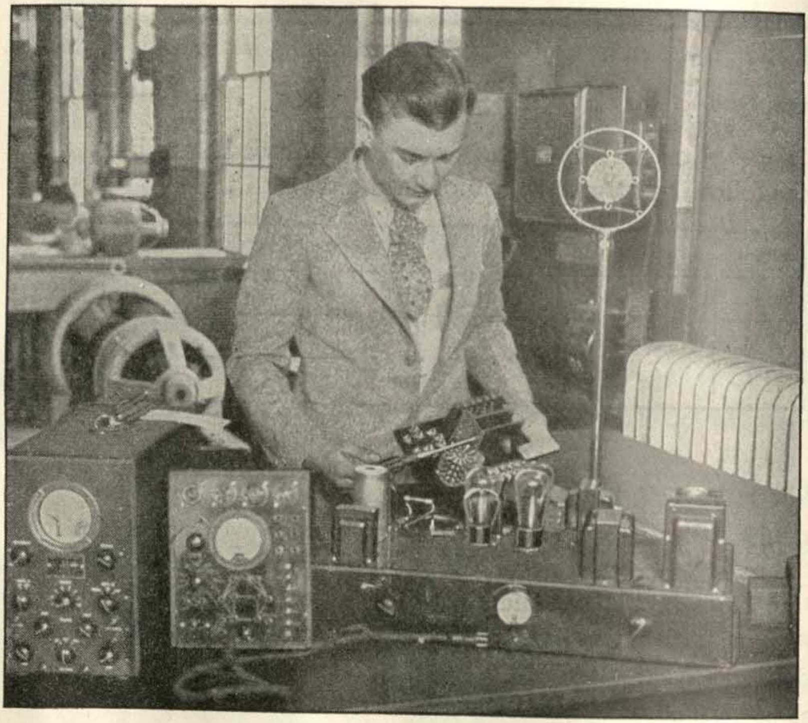
26. The volume of a sphere may be found by the formula $V = \frac{1}{6}\pi d^3$. Find V if $\pi = 3\frac{1}{7}$ and $d = 28$.

27. A formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Find V when $\pi = 3\frac{1}{7}$, $r = 14$, and $h = 20$.

28. $W = \frac{V^2}{R}$. Find W when $V = 110$ and $R = 880$.

29. $F = \frac{Wv^2}{32\,r}$. Find F when $W = 80$, $v = 96$, and $r = 1200$.

30. $W = (T - t)V$. Find W when $V = 800$, $T = 300$, and $t = 140$. Notice that $(T - t)$ is multiplied by V.



This Boy Finds Many Uses for His Knowledge of Algebra in the Study of Electricity

C

31. $I = \frac{E}{R}$ is a formula for finding I, the number of amperes of electric current, when E, the electromotive force in volts, and R, the resistance in ohms, are known. Find the number of amperes required to operate a light bulb if its resistance is 55 ohms and the line voltage is 110.

32. An electric toaster uses a current of 6.2 amperes when connected to a 110-volt line. At 7 cents per kilowatt-hour, how much does it cost to operate the toaster for 15 minutes?

The formula to use in finding the number of kilowatt-hours is $K = \frac{IEh}{1000}$, where I is the number of amperes, E is the number of volts, and h is the number of hours.

33. The horsepower developed by each cylinder of a steam engine is given by the formula $H = 2\frac{PLAN}{33{,}000}$. In this formula P is the mean pressure in pounds per square inch, L is the length in feet of the piston stroke, A is the area of the piston in square inches, and N is the number of revolutions per minute.

Find the horsepower of a steam engine with one cylinder which has an effective pressure of 90 pounds per square inch, a piston area of 65 square inches, a 12-inch stroke, and makes 200 revolutions per minute.

34. $C = \frac{nE}{R + nr}$. Find C when $n = 30, E = 7.5, R = 400$, and $r = 5$.

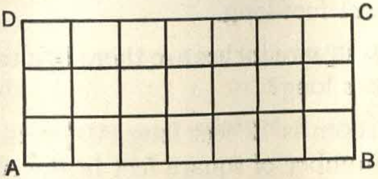
35. $H = 0.24\,I^2rt$. Find H when $I = 35$, $r = 3$, and $t = 30$.

Formulas Used in Geometry [A]

Many of the rules of arithmetic relating to geometry are simplified when changed into formulas. The rules, or formulas, for finding the circumference of a circle, the areas of a rectangle, a parallelogram, a triangle, and a circle, and the volume of a rectangular solid, should be understood and remembered.

Area of a Rectangle [A]

The rectangle $ABCD$ consists of three rows of small squares, each row containing seven of these squares, called square units.



In all, there are 3 × 7 or 21 squares. We say that the area of $ABCD$ is 21. If each of these small squares is one inch on a side, its area is one square inch and the area of $ABCD$ is 21 square inches. If each of these small squares is a foot on a side, its area is one square foot, and the area of $ABCD$ is 21 square feet. In general, the area of any figure is the number of square units it contains.

From the above discussion you can see that the area of a rectangle is found by multiplying its length by its width. The length and width are often called the base and height. The formula for the area of a rectangle is $A = bh$, or $A = lw$.

Example. Find the area of a sidewalk 4 feet wide and 40 feet long.

Solution. $A = bh$

$A = 40 \times 4$

$A = 160.$

The area is 160 square feet.

EXERCISES

A

Find the areas of the rectangles whose bases and heights are:

1. $b = 10$ in., $h = 8$ in.

2. $b = 32$ in., $h = 15$ in.

3. $b = 18$ ft., $h = 14$ ft.

4. $b = 36$ yd., $h = 21$ yd.

5. $b = 6\frac{1}{2}$ ft., $h = 7\frac{1}{4}$ ft.

6. $b = 10\frac{2}{3}$ ft., $h = 6\frac{1}{3}$ ft.

7. $b = 23.1$ in., $h = 14.4$ in.

8. $b = 10.4$ ft., $h = 7.7$ ft.

9. Find the number of square yards in the floor of a room 13 feet wide and 21 feet long.

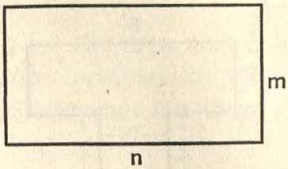
10. How many square inches are there in a table top 34 inches wide and 52 inches long?

11. A dining room is 12 feet long, $10\frac{1}{2}$ feet wide, and 9 feet high. Find the number of square feet in the surface of the four walls.

12. Find the area of a city lot 42.5 feet wide and 134.5 feet long.

13. How many square feet are there in a sidewalk 45 feet long and 54 inches wide?

14. Write a formula for the perimeter of this rectangle, letting *P* stand for the perimeter. What is the area of the rectangle?



15. How many inches equal one foot? How many square inches are there in one square foot?

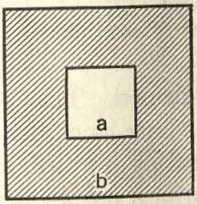
B

16. A rectangle with equal sides is a square. Write a formula for the area of a square whose sides are each equal to *s*.

17. Write a formula for the perimeter of a square, each of whose sides is equal to *s*.

18. Find the area and the perimeter of a square 11.2 feet on a side.

19. In the figure below, what is the area of the larger square? of the smaller square? Write a formula for the shaded part of the figure.



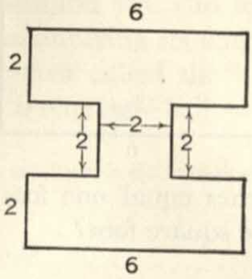
20. Find the perimeter and area of a rectangular field 181.32 rods by 94.75 rods.

21. Find the perimeter and area of a square if each of its sides is $14\frac{3}{4}$ inches long.

22. A flower bed 20 feet long and 4 feet wide is surrounded by a walk 18 inches wide. Find the area of the walk.

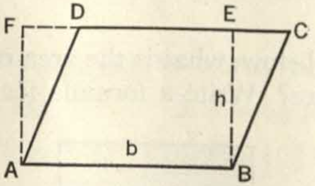
23. A square piece of cloth 18 inches on a side is cut from a strip of material a yard wide and 2 yards long. How many square yards are left in the strip?

24. If the numbers shown on the figure below stand for inches, find the area and perimeter of the figure.



Area of a Parallelogram [A]

The figure *ABCD* is a parallelogram. The symbol ▱ is used to represent a parallelogram. A rectangle is a parallelogram whose angles are right angles. If the right triangle *BEC* is cut off one end of the parallelogram and placed on the other end, the rectangle *BEFA* is formed. This rectangle and the parallelogram have the same base and altitude. Then the area of the parallelogram is found by multiplying its base by its height. The formula for the area of a parallelogram is $A = bh$.



EXERCISES [A]

Find the areas of the parallelograms whose dimensions are as follows:

1. $b = 10$ in., $h = 11$ in.

2. $b = 14$ ft., $h = 17.2$ ft.

3. $b = 10.1$ in., $h = 13.6$ in.

4. $b = 3\frac{1}{3}$ in., $h = 2\frac{2}{5}$ in.

5. $b = 7.5$, $h = 2.5$

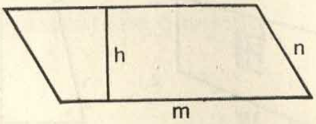
6. $b = 7\frac{1}{2}$, $h = \frac{3}{5}$

7. $b = 3\frac{3}{4}$, $h = 6\frac{2}{3}$

8. $b = 12.25$, $h = 4.50$

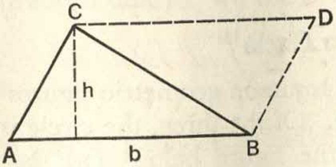
9. Find the perimeter of a parallelogram if one side is 32.3 and another side is 17.9. (The opposite sides of a parallelogram are equal.)

10. Letting A stand for the area and P for the perimeter of this parallelogram, write formulas for its area and perimeter. Is the area of the parallelogram less than, equal to, or greater than mn?



Area of a Triangle[A]

The $\triangle ABC$ is half the $\square ABDC$ and has the same base and height. Since the area of the parallelogram equals bh, the area of the triangle is $\frac{1}{2}bh$. Therefore the formula for the area of the triangle is $A = \frac{1}{2}bh$.



EXERCISES

A

Using the formula $A = \frac{1}{2}bh$, find the areas of the triangles whose bases and heights are

1. $b = 12$, $h = 9$

2. $b = 30$, $h = 20$

3. $b = 14.2$, $h = 17.4$

4. $b = 4.6$, $h = 2.3$

5. $b = 132$, $h = 17.7$

6. $b = 112.5$, $h = 37.6$

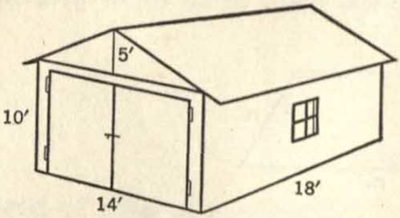
7. $b = 10\frac{1}{2}$, $h = 6\frac{2}{3}$

8. $b = 16\frac{1}{4}$, $h = 3\frac{3}{7}$

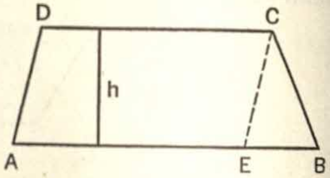
9. Find the area of a triangle whose base is x and whose altitude (height) is 8.

C

10. Find the cost of the paint for putting one coat on this garage at $2.75 a hundred square feet. (In figuring the cost add 40 square feet for painting under the eaves, and make no deductions for windows.)



Ex. 10

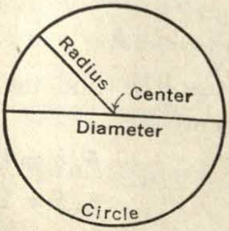


Ex. 11

11. *ABCD* is a trapezoid. It has two parallel sides, *AB* and *DC*, and two sides that are not parallel, *AD* and *BC*. By drawing *CE* parallel to *DA* the trapezoid can be divided into a parallelogram and a triangle. Find the area of *ABCD* if $AB = 16$ inches, $DC = 12$ inches, and $h = 6$ inches.

Circumference of a Circle[A]

The three most common geometric figures are the rectangle, triangle, and circle. Of the three, the circle is perhaps the most beautiful. Study the figure below and see if you remember the meaning of the words *circle*, *diameter*, *radius*, and *center*. The length of the circle is called its *circumference*.



You can perform a simple experiment to determine the relation between the circumference of a circle and the diameter. Obtain three or four circular objects and measure their circumferences and diameters with a tape measure. Then divide the circumference of each object by its diameter. Find the average of these quotients. If you are very careful in making your measurements, you will find that the circumference is about $3\frac{1}{7}$ times the length of the diameter. It is impossible to find the exact value of the fraction

$$\frac{\text{circumference}}{\text{diameter}} \text{ or } \frac{c}{d}.$$

It is customary to represent the quotient

$$\frac{c}{d}$$

by the Greek letter π, pronounced *pī.*

Then $\frac{c}{d} = \pi$

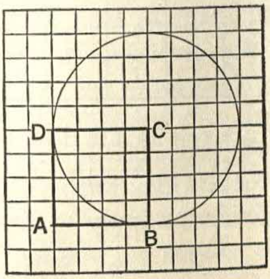
and $c = \pi d.$

Since $d = 2\,r,\ c = 2\,\pi r.$

For ordinary computations we use $3\frac{1}{7}$ or 3.14 for the value of π. Where greater accuracy is desired, we use 3.1416 for the value of π.

Area of a Circle[A]

The figure shown here is intended to make it easy for you to remember the rule or formula for finding the area of a circle when its radius or diameter is known. The radius of the circle



is equal to a side of the square *ABCD*. The area of *ABCD* is 16. You can find the approximate area of the circle by adding the small squares and parts of these squares. The sum of these small squares is about 50, which is a little more than three times the number of them in $ABCD$. To be exact, the area of the circle is π times the area of the square $ABCD$. Since the area of the square is equal to the square of the radius, the area of the circle is equal to π times the square of the radius. The formula for the area of a circle is $A = \pi r^2$.

Example. Find the area of a circle whose diameter is 14, using $\pi = 3\frac{1}{7}$.

Solution.

$$A = \pi r^2 \quad \text{and} \quad r = 7$$
$$A = \tfrac{22}{7} \times 49$$
$$A = 154$$

EXERCISES

A

Using the formulas $A = \pi r^2$ and $c = \pi d$ ($\pi = 3\frac{1}{7}$), find the areas and the circumferences of the circles whose radii are

1. 7 in. **3.** 16 ft. **5.** $5\frac{1}{4}$ in. **7.** 3.5 ft.

2. 21 in. **4.** 56 ft. **6.** $10\frac{1}{2}$ in. **8.** 7.7 ft.

9. Using $\pi = 3.14$, copy and complete the following table in which r is the radius of a circle and c and A are the respective circumferences and areas.

r	1	2	3	4	5.5	8	10	20	30	$\frac{1}{2}$	$1\frac{2}{3}$
c	?	?	?	?	?	?	?	?	?	?	?
A	?	?	?	?	?	?	?	?	?	?	?

10. If d in the formula $c = 3\frac{1}{7}\,d$ is doubled, how is c affected?

11. If r in the formula $A = \pi r^2$ is doubled, how does A change?

12. The following formulas apply to circles. Express each as a rule.

a. $d = 2r$ **c.** $c = \pi d$ **e.** $d = \dfrac{c}{\pi}$ **g.** $A = \frac{1}{4}\pi d^2$

b. $r = \frac{1}{2}d$ **d.** $c = 2\pi r$ **f.** $A = \pi r^2$ **h.** $r = \dfrac{c}{2\pi}$

B

13. Find the length of lace needed to go around the edge of a circular table cover 27 inches in diameter.

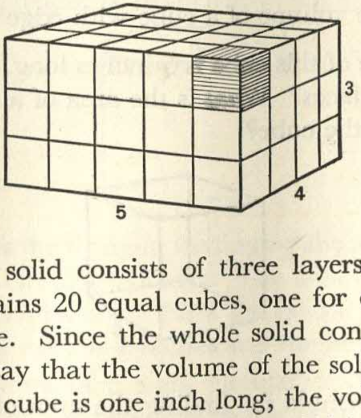
14. Find the diameter in feet of a circular half-mile race track. $\left(d = \frac{c}{\pi}\right)$

15. How many revolutions will a 28-inch bicycle wheel make in going a mile?

16. A round table top is 54 inches in diameter. What are the area and circumference of the top?

Volume of a Rectangular Solid [A]

This figure is called a *rectangular solid.* It is 5 units long, 4 units wide, and 3 units high. The shaded portion is called a *cube*. The length, width, and height of a cube are all equal.



This rectangular solid consists of three layers. Each of these three layers contains 20 equal cubes, one for each square unit in the top surface. Since the whole solid contains 60 of these equal cubes, we say that the volume of the solid is 60. If each edge of the small cube is one inch long, the volume of the cube is one cubic inch and the volume of the solid is 60 cubic inches. If each edge of the small cubes is a foot in length, the volume of the cube is one cubic foot and the volume of the solid is 60 cubic feet.

In general, the volume of any rectangular solid is found by multiplying the length by the width by the height. The formula for finding the volume of a rectangular solid is $V = lwh$.

EXERCISES

A

Find the missing letter of the formula $V = lwh$ in each of the following exercises:

1. $l = 8$, $w = 10$, $h = 16$
2. $l = 18$, $w = 7$, $h = 25$
3. $l = 10.2$, $w = 9$, $h = 6$
4. $l = 16.4$, $w = 8.6$, $h = 6.4$
5. $l = 20\frac{1}{2}$, $w = 10\frac{3}{4}$, $h = 8$
6. $l = 40$, $w = 16\frac{2}{3}$, $h = 9\frac{1}{6}$

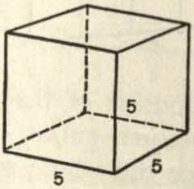
7. How many cubic inches are there in a box 18 inches by 20 inches by 24 inches?

8. Find the number of cubic feet of air in a room 18 feet by 14 feet by $9\frac{1}{2}$ feet high.

9. How many square feet of surface are there in the ceiling and walls of the room of Exercise 8?

10. Show that the volume of a cube with edge e is e^3. Write a formula for the volume of a cube with edge x.

11. Each edge of this cube is 5 inches long. What is the area of each of its six faces? What is the area of all six faces? What is the volume of the cube?

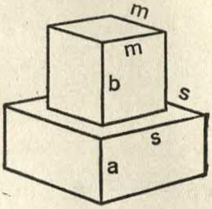


12. Write a formula for finding the area of all six faces of a cube, letting the length of an edge equal e.

B

13. How many bushels of wheat are needed to fill a granary 4 feet by 8 feet by 7 feet? (1 bushel = $1\frac{1}{4}$ cubic feet, approximately.)

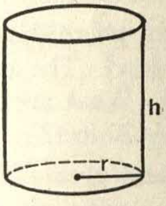
14. Find the volume of the lower rectangular solid in this figure. Find the volume of the upper solid. Write a formula for the volume of the combined solids.



15. Compare the volumes of two cubes whose edges are 3 feet and 6 feet respectively.

16. Compare the total surfaces of two cubes whose edges are 2 feet and 3 feet respectively.

17. The ordinary vegetable can is an example of a right circular cylinder. Its volume is given by the formula $V = \pi r^2 h$, where r is the radius of the base and h is the height. Find the volume of a right circular cylinder if the radius of the base is 10 inches and the height is 16 inches.



18. $S = 2\pi rh$ is the formula for finding the area of the curved surface of a right circular cylinder. The area of the curved surface is called the *lateral area*. Find the lateral area of a tin can if the radius of the base is 2 inches and the height is 4 inches.

19. The total area of a right circular cylinder is found by adding the areas of the two ends to the lateral area. Find the total area of a drum 16 inches in diameter and 18 inches high.

20. Find the volume and lateral area of the following right circular cylinders, when

a. $r = 6$ in., $h = 8$ in.

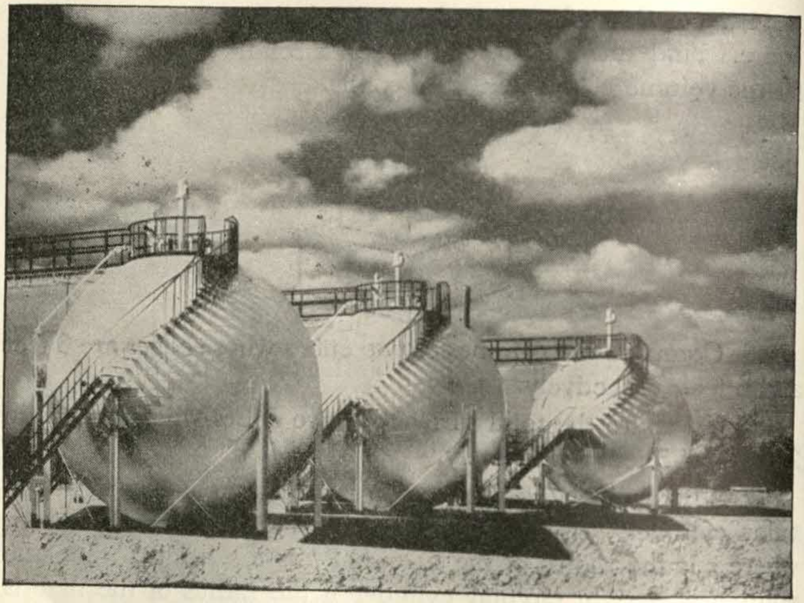
b. $r = 4\frac{1}{2}$ ft., $h = 10$ ft.

c. $r = 5\frac{1}{4}$ ft., $h = 12\frac{1}{8}$ ft.

d. $r = 2.3$ in., $h = 8.6$ in.

e. $r = 3.25$ in., $h = 7.5$ in.

f. $r = 12.67$ in., $h = 7.32$ in.



Shell Oil Company

Spherical Tanks Used to Store Gasoline

21. The volume of each sphere shown in the photograph is given by the formula $V = \frac{4}{3}\pi r^3$. The area of the surface is found by the formula $S = 4\pi r^2$. What are the volume and area of one sphere if its diameter is 20 feet?

22. Find the volume and area of a soft ball 2 inches in diameter.

23. Find the volumes and areas of the spheres whose radii are

a. 6 in.
b. 10 in.
c. $3\frac{1}{2}$ in.
d. $4\frac{1}{4}$ in.
e. $6\frac{1}{3}$ ft.
f. 5.25 in.
g. 16.4 ft.
h. 10.5 ft.

24. How many gallons of oil can be stored in a spherical tank having a radius of 15 feet? (1 gallon = 231 cubic inches.)

Making Formulas from Rules [A]

In making formulas you should remember that a formula contains two equal expressions connected by an equals sign. For example, $A = bh$ is a formula, but bh is not a formula.

EXERCISES [A]

1. The difference between the selling price (S) and cost (C) of an article is called the margin (M). Which of the following formulas express this fact?

a. $S = C + M$
b. $S = CM$
c. $S = M - C$
d. $M = C + S$
e. $M = S \div C$
f. $M = S - C$
g. $C = M + S$
h. $C = M - S$
i. $C = S - M$

2. The margin is often called the markup. It is really the gross profit. The actual profit or gain (G) is what is left when the expenses (E) are taken out of the margin (M). Which of the following formulas express this fact?

a. $G = M + E$
b. $G = E - M$
c. $G = M - E$
d. $M = G + E$
e. $M = G \div E$
f. $M = G - E$
g. $E = G - M$
h. $E = M - G$
i. $E = M + G$

EXERCISES

A

Write formulas for the following rules, using the letters suggested:

1. The selling price (s) is found by adding the cost (c), the desired profit or gain (g), and the expenses (e).

2. The number of gallons (g) in a container is found by dividing the number of cubic inches (V) in it by 231.

3. The reading on a Fahrenheit thermometer (F) is found by taking $\frac{9}{5}$ of the reading on the Centigrade thermometer and adding 32 to the result.

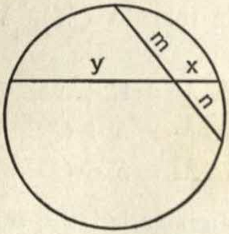
4. The perimeter (P) of a square is equal to 4 times one of its sides (s).

5. Percentage (p) is found by multiplying the base (b) by the rate (r).

6. The average (a) of two numbers (x and y) is equal to the sum of the numbers divided by 2.

7. The volume (V) of a circular cone is equal to the product of one third of the base (b) and the altitude (h).

8. In the circle below, the product of the two line segments m and n is equal to the product of the two line segments x and y.



9. The perimeter (P) of a rectangle is equal to the sum of twice the width (w) and twice the length (l).

10. The perimeter (P) of a triangle is equal to the sum of the three sides (a, b, and c).

11. The distance (d) passed over by any moving object is equal to the product of the rate (r) and time (t).

12. The sum of the interior angles (A, B, and C) of a triangle is 180°.

13. The number of seats (n) in a room is equal to the number of rows (r) multiplied by the number of seats in a row (s).

14. The diagonal (d) of a square equals the length of a side (s) times 1.414.

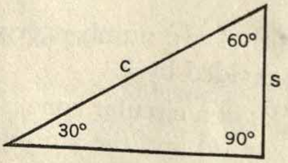
15. The yearly rent (R) of a house is the rent per month (r) multiplied by 12.

16. The area (A) of a circle is equal to π times the square of its radius (r).

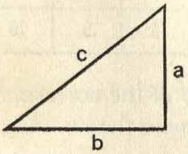
B

Write formulas for the following rules or expressions, using suitable letters to represent each of the numbers.

17. The side opposite a 30° angle in a right triangle is equal to one half the hypotenuse.



18. The reading on the centigrade thermometer can be found by subtracting 32 from the reading on the Fahrenheit thermometer and taking $\frac{5}{9}$ of the result.



19. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

20. The fare charged by a bus company is 15 cents for the first mile and 5 cents for each additional mile. Let n represent the number of additional miles.

21. A power company charges 7 cents a kilowatt hour for the first 20 kilowatt hours, and 4 cents for each additional kilowatt hour.

22. What formula will express the total cost (c) of n articles at b cents each?

23. Write a formula for changing the number of feet into the number of inches.

24. The distance away, in miles, of a lightning flash is found by dividing the number of seconds between the flash and its accompanying thunderclap by 5.

25. The temperature in degrees Fahrenheit can be told by counting the number of chirps made by a cricket in 15 seconds and adding 37.

Making Formulas from Tables [A]

In making formulas from tables, we first study how the corresponding numbers change and then write a formula expressing this relation.

Example 1. Write a formula expressing the relation between x and y in this table:

x	1	2	3	4	5	6	7	8	9
y	6	7	8	9	10	11	12	13	14

Solution. From a study of the corresponding values of x and y we see that y is always 5 more than x. Then the formula is $y = x + 5$.

Example 2. Write a formula expressing the relation between a and b in this table:

a	1	2	3	4	5	6	7	8
b	4	7	10	13	16	19	22	25

Solution. From a careful study of this table we see that b is always 1 more than 3 times a. Then the formula is $b = 3a + 1$.

EXERCISES [A]

In each exercise write the formula that expresses the relation between the numbers, and then complete the table.

1.

Number of gallons of gasoline (n)	1	2	3	4	5	6	7
Cost in cents (c)	21	42	63	84	?	?	?

2.

Interest in dollars (i)	1	2	3	4	5	6	7
Total amount in dollars (A)	46	47	48	49	?	?	?

3. Changing pounds to ounces.

Number of pounds (p)	1	2	3	4	5	6	7
Number of ounces (z)	16	32	48	?	?	?	?

4.

x	1	2	3	4	5	6	7	8	9
y	3	5	7	9	?	?	?	?	?

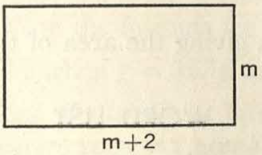
5.

x	1	2	3	4	5	6	7	8	9
y	1	3	5	7	?	?	?	?	?

Simplifying Formulas [A]

To be most useful a formula should be written in its simplest form.

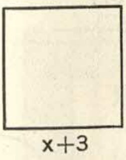
Example 1. Write a formula for the perimeter of this rectangle.



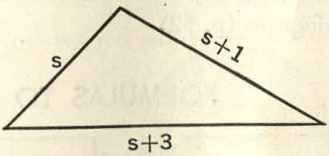
Solution. Since the perimeter of the rectangle is equal to the sum of the four sides, we have $P = m + m + m + 2 + m + 2$. Simplifying the formula, $P = 4\,m + 4$.

EXERCISES [A]

1. Each side of this square is $x + 3$. Write the formula for its perimeter in the simplest form.



Ex. 1



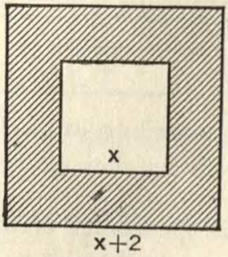
Ex. 2

2. Write the formula for the perimeter of this triangle in its simplest form.

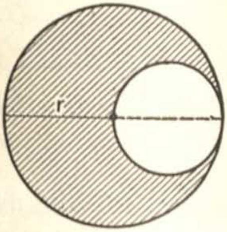
3. Write the formula for the perimeter of a rectangle if the width is x and the length $x + 4$.

4. The sides of a triangle are b, $b - 1$, and $b + 3$. Write the formula for its perimeter.

5. Two squares have sides of $x + 2$ and x respectively. Write a formula giving the sum of their perimeters.



Ex. 5



Ex. 6

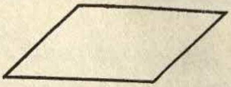
6. Write a formula giving the area of the shaded part of the larger circle.

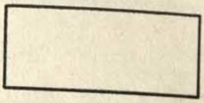
WORD LIST

Do you know the meaning of each of the following words? Can you spell each one correctly?

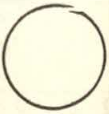
acute angle (p. 29)
centigrade (p. 61)
circle (p. 54)
circumference (p. 54)
complementary angles (p. 30)
diameter (p. 54)
exponent (p. 12)
Fahrenheit (p. 61)
parallelogram (p. 52)
parentheses (p. 11)
radius (p. 54)
rectangle (p. 5)
rectangular solid (p. 57)
sphere (p. 60)
square (p. 6)
supplementary angles (p. 30)
triangle (p. 31)
volume (p. 57)

FORMULAS TO REMEMBER



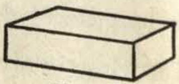


$A = bh$



$c = \pi d$
$c = 2\,\pi r$
$A = \pi r^2$

$A = \frac{1}{2}\,bh$



$V = lwh$

CHAPTER REVIEW [A]

1. What is a formula?

2. What is the formula for finding simple interest?

3. Name the three steps used in solving a problem by a formula.

4. $A = p + prt$. Find A when $p = \$350$, $r = 4\%$, and the time is 4 years 3 months.

5. The formula for finding the distance d when the rate r and the time t are known is $d = rt$. Write the formula for finding d when $r = 30$. Write the formula for finding d when $t = 5$.

6. $s = \frac{1}{2} gt^2$. Find s when $g = 32.16$ and $t = 3$.

7. Find the number of acres in a farm 180 rods long and 80 rods wide. (160 square rods = 1 acre.)

8. Write a formula for finding the area of a circle.

9. Write a formula for finding the volume of a rectangular solid.

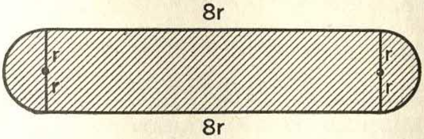


Fig. 1

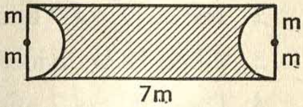


Fig. 2

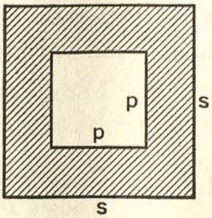


Fig. 3

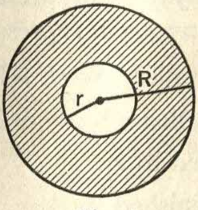


Fig. 4

10. Write a formula for the shaded area of each of the four figures above.

11. A bicycle wheel is 28 inches in diameter. What is its circumference?

12. A schoolroom is 30 feet long, 24 feet wide, and 12 feet high. If 30 people are in the room, find the average number of cubic feet of air for each person.

13. Write a formula for finding the cost of n articles at p cents each.

14. Write a formula for finding the average of the three numbers x, y, and z.

15. Write a formula expressing the relation of x and y in the table below. Then complete the table.

x	9	14	17	20	?	?	?	?	?	?	?
y	7	12	15	18	20	25	30	40	50	100	200

GENERAL REVIEW [A]

Complete the following statements by supplying the proper words:

1. The --?-- of 6 and 2 is 8.

2. The --?-- of 9 and 3 is 6.

3. When 10 is divided by 5, the --?-- is 2.

4. The --?-- of 7 and 3 is 21.

5. If x represents the number of feet in the length of a room, how many x will represent the number of inches in its length? How many x will represent the number of yards in its length?

6. Simplify $p = l + l + w + w$.

7. Indicate by symbols:

a. x added to y.

b. c diminished by 4.

c. y increased by 2.

d. The product of m and n.

e. a divided by c.

f. The sum of $4a$ and $3b$.

g. The sum of k and 6, divided by the product of 3 and k.

8. Indicate "h multiplied by five" three different ways.

9. What is the coefficient of y in the expression $3y^2 + 2y - 1$?

10. What is the second term in the expression $4a - 2b + 3c$?

11. Why do we use parentheses in algebra?

Tell what is needed to complete the following statements:

12. ab is a short way of writing a --?-- b.

13. $2a$ is a short way of writing --?--.

14. r^3 is a short way of writing --?--.

15. If $x = 3$, find the value of x^2; of $2x^2$; of x^3.

16. Simplify $9 + 6 \div 3$.

17. Simplify $\frac{1}{2}x - \frac{1}{3}x + x$.

18. Simplify $(9 + 6) \div 5$.

19. Find the value of $\frac{2a - b}{4}$ if $a = 8$ and $b = 4$.

20. Solve $c + 3c = 12$.

21. Solve $7m - 4m = 21$.

22. Write formulas for finding

a. The area of a rectangle.

b. The area of a triangle.

c. The area of a circle.

d. The volume of a rectangular solid.

23. Find the area of a square having a side m.

24. If a man earns $8 a day, write a formula expressing his wages (w) for n days.

25. The sum of two numbers is 144 and one of them is 5 times as large as the other. How large is each number?

26. Separate 90 into two parts such that one of them will be one fourth as large as the other.

27. The perimeter of a triangle is 210 inches. Find the lengths of its sides if the third side is one and one half times as long as each of the others.

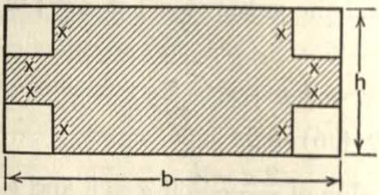
28. $F = \frac{9}{5}C + 32$. Find F when $C = 80$.

29. $C = \frac{5}{9}(F - 32)$. Find C when $F = 98$.

CHAPTER TEST [A]

Write formulas for

1. The simple interest i.
2. The circumference c of a circle whose diameter is d.
3. The area A of a circle whose radius is r.
4. The area of the shaded portion of this rectangle.



State in words

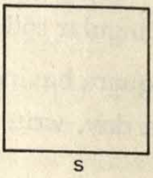
5. $r = \frac{1}{2}d$

6. $d = \frac{c}{\pi}$

7. $i = 12f$

8. $V = lwh$

The following sentences are based upon the formulas $p = 4s$ and $A = s^2$. Copy and complete each sentence.



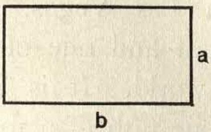
9. The value of p depends upon the value of --?--.
10. If s increases, p --?--.
11. If s is multiplied by 3, then p is multiplied by --?--.
12. If s is decreased, A is --?--.
13. If s is multiplied by 3, then A is multiplied by --?--.
14. $x = 2y - 3$. What is the value of x when $y = 5$?
15. $F = \frac{wa}{g}$. Find F when $w = 100$, $a = 8$, and $g = 32$.

CHAPTER TEST [B]

1. $F = \frac{9}{5}C + 32$. Find F when $C = 40$.

2. $C = \frac{5}{9}(F - 32)$. Find C if $F = 72$.

3. $p = 2a + 2b$. What change, if any, is made in p when b is decreased as much as a is increased?



4. Copy and complete the table for $p = 2n + 7$.

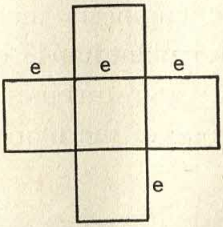
p	?	?	?	?	?	?
n	1	2	3	5	7	9

5. $V = lwh$. Complete: If l and w are doubled and h is trebled, then V is multiplied by --?--.

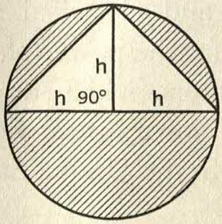
6. When a bomb is dropped from a plane a mile high, its approximate distance d in feet from the ground is given by the formula $d = 5280 - 16t^2$. Find how high the bomb is when t is 4.

Write formulas for

7. The area of the figure at the left below, consisting of adjoining squares having sides e.



Ex. 7



Ex. 8

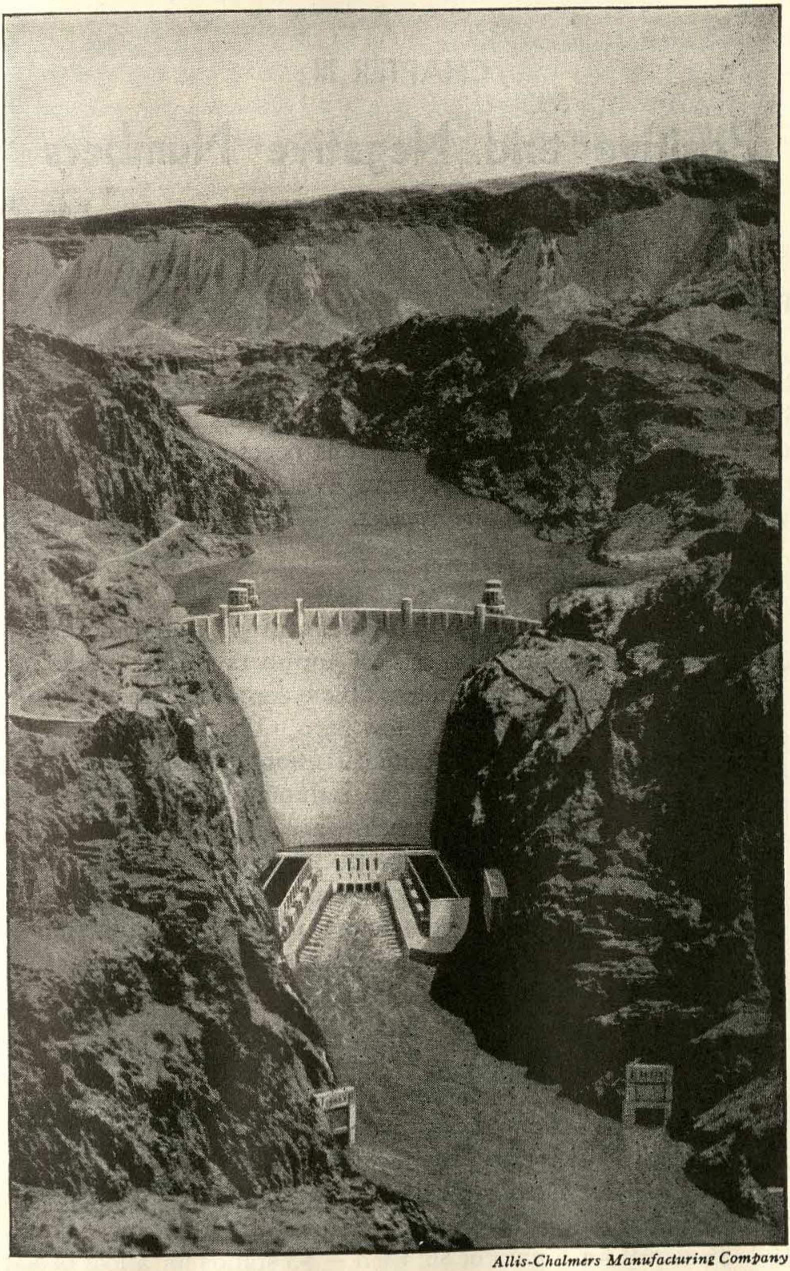
8. The area of the shaded portion of the circle in the figure at the right above.

HARNESSING THE FORCES OF NATURE

Hoover Dam is built across the Colorado River at a point about 25 miles from Las Vegas, Nevada. It is the highest and one of the largest dams in the world. It is 726 feet high above bedrock, and about one fifth of a mile long. It is in the shape of a horseshoe, with the bulge upstream. The thickness of the dam at its base is 650 feet, and its thickness at the top is 45 feet. More than one half billion barrels of cement were used in making this enormous block of concrete.

The dam has created a lake 115 miles long and at one place 45 miles wide. This vast amount of water is used to generate electricity, irrigate land, and supply cities.

Without mathematics this dam could not have been built. Engineers surveyed the river, its gorges, and the surrounding country. They knew the river's maximum and minimum flow of water, as well as its average yearly flow. Bridge or dam engineers made the plans for the dam, and electrical and hydraulic engineers made the plans for the power plant. This power plant has 17 electric generators, which are powered by water turbines. This dam, shown in the picture, is only one of the many large dams in the United States.



Allis-Chalmers Manufacturing Company

CHAPTER III

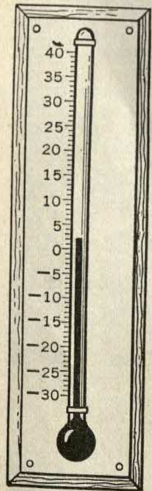
Positive and Negative Numbers

Positive Numbers [A]

The numbers that you used in arithmetic were whole numbers and fractions. No name was given to them. They were just called numbers. Now since we are going to use numbers which are the opposites of these, it is necessary to distinguish one kind of number from another. We shall call the numbers you have been using *positive numbers*.

Negative Numbers [A]

At the left of this page is a diagram of a thermometer. Notice that there are two sets of numbers on it — one set measuring distances above zero and one set measuring distances below zero. The two sets of numbers run in opposite directions from zero.



Now let us call all numbers above zero *positive* and the numbers below zero *negative*.

To show that a number is positive we shall place a *plus sign* (+) before it or else write the number without any sign before it; and to show that a number is negative we shall write the *minus sign* (−) before it. By this arrangement + 6, or 6, means "6 above zero" and + 10, or 10, means "10 above zero." Also, − 4 means "4 below zero" and − 7 means "7 below zero." This is one situation where positive and negative numbers can be used.

It is always convenient to use positive and negative numbers when we are dealing with numbers that are opposite in direction. Let us consider a few more cases. If + \$6 stands for a gain of \$6, then − \$3 stands for a loss of \$3;

if an increase of 3 pounds in one's weight is represented by + 3 pounds, then a decrease of 4 pounds in one's weight is represented by − 4 pounds; if going east 5 miles is represented by + 5 miles, then going west 10 miles is represented by − 10 miles.

Signed Numbers [A]

Signed numbers is the name applied to both positive and negative numbers. Can you suggest a reason why positive and negative numbers are called signed numbers?

The plus sign (+) is not always written before a number to show that it is positive, but the negative sign (−) is always written before a number to show that it is negative. So if a number has a plus sign, or no sign before it, it is positive; and if a number has a minus sign before it, it is negative. The following exercises are intended to give you a better understanding of signed numbers.

EXERCISES [A]

1. If + 6 stands for a gain of $6, what number will stand for a loss of $3?

2. If going east 4 miles is represented by + 4 miles, what does − 3 miles represent?

3. If − 3 feet means 3 feet below sea level, how can you represent 7 feet above sea level?

4. If we let + 80° stand for 80° east longitude, what does − 60° stand for?

5. If we let − 80° stand for 80° west longitude, what does + 60° stand for?

6. If positive numbers are used for distances to the right of zero, what will negative numbers stand for?

7. If + 8 inches means 8 inches above average, what does − 6 inches mean?

8. If 10 stories above the ground level are represented by + 10 stories, how would you represent 2 stories below ground level?

9. If 1870 A.D. is represented by + 1870, what does − 46 mean?

10. What does "6 in the hole" mean when you are keeping score in a game? Would you use + 6 or − 6 to stand for "6 in the hole"?

11. Which is colder, − 4° or − 8°?

12. Is an increase of − $10 actually an increase? Explain.

13. What is the difference between a gain of $3 and a loss of $5? What does (+ 3) − (− 5) mean?

14. Below are the grades in per cent made by sixteen pupils in an algebra test. The average of these grades is 80%. The first grade is 70%. This is 10% below average, so − 10 is written below the 70. The next grade is 80. The difference between it and 80 is zero, so 0 is written below the 80. Copy and complete the table.

Grades in %	70	80	90	60	100	80	55	65	100	95	80	75	70	80	85	95
Changes in %	−10	0	?	?	?	?	?	?	?	?	?	?	?	?	?	?

15. A man operating a grocery and meat market keeps a record of his monthly sales and compares them with the monthly sales of the previous year. He uses the plus sign to denote increase and the minus sign to denote decrease. Copy and complete the following table:

	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE
Sales last year	$8855.00	$8791.32	$7810.13	$7623.18	$7429.06	$7729.86
Sales this year	$8159.40	$8814.16	$8024.72	$8143.25	$7734.21	$7618.21
Changes in sales	−$695.60	+$22.84	?	?	?	?

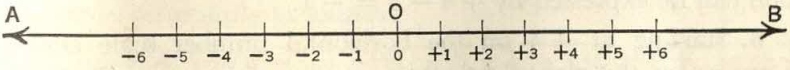
Double Use of the Plus and Minus Signs [A]

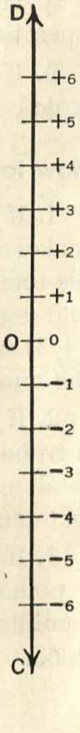
In arithmetic the plus sign (+) always indicates addition and the minus sign (−) always indicates subtraction. In algebra each of these signs has two uses. The plus sign may indicate addition or it may indicate that the number it precedes is positive; and the minus sign may indicate subtraction or it may indicate that the number it precedes is negative.

When the signs are used to denote addition and subtraction, they are called *signs of operation;* and when they are used to show that numbers are positive or negative, they are called *signs of quality*, or *signs of opposition.* In the expression − 8, the minus sign is a sign of quality; in the expression 9 − 8, the minus sign is a sign of operation; and in the expression (− 2) + (− 3) the plus sign is a sign of operation and the minus signs are signs of quality.

The *absolute value* of a signed number is the value of the number without regard to its sign. Thus the absolute value of + 7 is 7, of + 6 is 6, and of − 5 is 5.

Number Scales[A]





The horizontal line *AB* and the vertical line *CD* extend without limit in both directions. From any point *O* in each line equal distances are marked off in both directions. On *AB* the distances to the right of *O* are numbered + 1, + 2, + 3, etc. and the distances to the left of *O* are numbered − 1, − 2, − 3, etc. On *CD* the distances above *O* are numbered + 1, + 2, + 3, etc. and the distances below *O* are numbered − 1, − 2, − 3, etc. The starting point *O* in each line is the *origin.*

Any one of the division points of either line represents a positive or a negative integer (whole number), and any point between any two successive division points represents a positive or negative number that is not an integer. The positive numbers represent distances from *O* in one direction and the negative numbers represent distances from *O* in the opposite direction.

An arrangement of numbers on a line such as either of these is called a *number scale.*

ORAL EXERCISES [A]

Study either or both of the number scales on page 77 when answering the following questions:

1. Is + 3 greater or less than + 4?

2. Is − 2 greater or less than − 3?

3. Is zero greater or less than − 5?

4. Is zero greater or less than + 5?

5. A boy said 4 was the answer to a problem when the correct answer was − 4. How much too large was his answer?

6. How far apart on the number scale are 2 and − 4? − 3 and 2? 3 and − 1? 0 and 7? 2 and − 2?

7. Starting at + 4 on the horizontal number scale, count 5 spaces to the left. At what number did you stop? This operation can be expressed by $+4-5=-1$.

8. Starting at + 4 on the horizontal number scale count 5 spaces to the right. At what number did you stop? This operation can be represented by $+4+5=+9$.

9. If counting to the right 3 spaces on the horizontal number scale is represented by + 3, what does − 3 represent?

How to Add Positive and Negative Numbers [A]

1. If John, in playing a game in which one can go in the hole, makes a gain of 4 points and then makes another gain of 3 points, his total score is 7 points. We say that the sum of a gain of 3 and a gain of 4 is a gain of 7. This fact can be written $(+4)+(+3)=+7$. The plus signs within the parentheses mean that the numbers are positive.

2. If John has a loss of 6 points (goes in the hole 6 points) and then has a loss of 5 points (goes in the hole 5 points deeper), his total score is a loss of 11 points (11 points in the hole). This fact can be written $(-6)+(-5)=-11$.

3. If John makes a gain of 8 points and then makes a loss of 2 points, his score for the two times is a gain of 6 points, or 6 points out of the hole. This fact can be written $(+8)+(-2)=+6$.

4. If John makes a loss of 9 points (goes in the hole 9 points) and then makes a gain of 7 points, his total score for the two times is a loss of 2 points. He is still in the hole. This may be written $(-9) + (+7) = -2$. These four additions may be written as shown below:

$$\begin{array}{l} \text{gain } 4 \\ \text{gain } 3 \\ \hline \text{gain } 7 \end{array} \qquad \begin{array}{l} \text{loss } 6 \\ \text{loss } 5 \\ \hline \text{loss } 11 \end{array} \qquad \begin{array}{l} \text{gain } 8 \\ \text{loss } 2 \\ \hline \text{gain } 6 \end{array} \qquad \begin{array}{l} \text{loss } 9 \\ \text{gain } 7 \\ \hline \text{loss } 2 \end{array}$$

Using + and − signs, these additions can be written as follows:

$$\begin{array}{r} +4 \\ +3 \\ \hline +7 \end{array} \qquad \begin{array}{r} -6 \\ -5 \\ \hline -11 \end{array} \qquad \begin{array}{r} +8 \\ -2 \\ \hline +6 \end{array} \qquad \begin{array}{r} -9 \\ +7 \\ \hline -2 \end{array}$$

The additions above are vertical additions. They may be written horizontally as follows:

$$+4 + 3 = +7,\ -6 - 5 = -11,\ +8 - 2 = +6,\ -9 + 7 = -2.$$

ORAL EXERCISES [A]

Think of positive numbers as gains and of negative numbers as losses and add (or combine) the following:

	a	**b**	**c**	**d**	**e**	**f**	**g**
1.	5 $\underline{3}$	-8 $\underline{+4}$	-3 $\underline{-5}$	-4 $\underline{+6}$	-2 $\underline{-7}$	8 $\underline{6}$	-7 $\underline{+7}$
2.	-8 $\underline{+1}$	-6 $\underline{-5}$	9 $\underline{-7}$	-4 $\underline{6}$	-3 $\underline{+3}$	-2 $\underline{+8}$	-6 $\underline{+6}$
3.	$-5a$ $\underline{+4a}$	$7b$ $\underline{3b}$	$-7c$ $\underline{+3c}$	$7x$ $\underline{-3x}$	$-5y$ $\underline{6y}$	$-7m$ $\underline{-9m}$	$-8h$ $\underline{10h}$
4.	$5x^2$ $\underline{-8x^2}$	$-2r$ $\underline{+2r}$	-9 $\underline{0}$	$2s$ $\underline{s}$	$2k$ $\underline{-3k}$	$10m$ $\underline{-2m}$	$-8n$ $\underline{-4n}$
5.	$-3xy$ $\underline{-6xy}$	$5x^2$ $\underline{-x^2}$	$-16w$ $\underline{11w}$	$-11p$ $\underline{13p}$	$-10st$ $\underline{+10st}$	$14H$ $\underline{-5H}$	$-15T$ $\underline{-T}$

6. $(10) + (-3)$
7. $(-2) + (+7)$
8. $(-5) + (+5)$
9. $(-3) + (20)$
10. $(-3) + (-5)$
11. $+8 + (6)$
12. $(-9) + (-11)$
13. $(+3) + (-4)$
14. $-9 + (+9)$
15. $(-5) + (-25)$
16. $(-1) + (-1)$
17. $(-18) + (-10)$
18. $(-\frac{1}{2}) + (-\frac{1}{4})$
19. $(-\frac{2}{3}) + (\frac{5}{6})$
20. $(-.4) + (-.6)$

Rules for Adding Signed Numbers:

1. To add two numbers with like signs, use the same sign and find the sum of their absolute values.
2. To add two numbers with opposite signs, use the sign of the number having the greater absolute value and find the difference of their absolute values.

ORAL DRILL IN ADDITION [A]

To acquire speed and accuracy in the addition (or combining) of positive and negative numbers, practice daily the following exercises:

	a	b	c	d	e	f	g	h
1.	2 3 5	− 3 − 2 − 5	− 2 3	− 3 + 3	4 − 3	− 4 − 3	− 4 3	− 4 + 4
2.	7 6	− 7 5	− 7 6	6 3	− 6 − 3	− 3 6	− 2 − 6	− 2 + 2
3.	6 8	− 5 − 3	7 9	− 6 + 6	4 6	8 − 6	− 9 4	10 − 5
4.	5 − 5	− 5 − 5	− 8 − 9	− 5 9	− 10 6	11 − 5	10 − 1	− 10 + 10
5.	14 6	− 14 6	6 − 14	10 8	− 10 8	− 10 − 8	1 − 11	− 1 − 1
6.	4 7	− 4 − 7	7 3	− 7 − 3	7 5	− 5 − 7	− 7 − 1	− 7 − 8

	a	b	c	d	e	f	g	h
7.	−3	−10	21	−10	−2	8	−16	3
	−6	−11	−5	14	−13	−20	+16	2
8.	−1	14	2	−8	−8	40	−40	1
	−12	−14	−13	−8	+8	−1	−1	−40
9.	−20	−20	100	−75	−50	−20	41	−41
	3	−3	−50	−25	1	−30	5	1
10.	−18	16	−10	−100	−40	24	5	−20
	−10	−16	−3	+100	−50	−23	−20	−5
***11.**	$\frac{1}{4}$	$\frac{2}{3}$	$-\frac{1}{2}$	$\frac{2}{5}$	8	$-\frac{3}{5}$	$-\frac{7}{8}$	$3\frac{2}{3}$
	$\frac{3}{4}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{7}{5}$	$-4\frac{1}{2}$	$+\frac{3}{5}$	$-\frac{1}{8}$	$-1\frac{1}{3}$
12.	4.0	−3.0	−5.0	.00	−4.3	−6.3	4.1	−9.0
	−1.5	4.7	−1.0	−.25	−2.7	−2.7	−9.1	+.5

Adding Several Signed Numbers [A]

When two or more signed numbers are to be added, they may be combined in any order. As an illustration, the sum of 4, −3, −7, and 8 is found below in three ways.

Solution 1. $4 + (-3) = 1;\ 1 + (-7) = -6;\ -6 + 8 = 2$

Solution 2. $4 + (-7) = -3;\ (-3) + (-3) = -6;\ (-6) + 8 = 2$

Solution 3. $4 + 8 = 12;\ -3 - 7 = -10;\ +12 - 10 = 2$

Most people prefer Solution 3. They believe that it is easier first to add all the positive numbers, then add all the negative numbers, and finally add the two results.

EXERCISES [A]

Find the sum in each exercise.

1.	2.	3.	4.	5.	6.	7.
5	6	−9	4	−10	7	−4
−3	−7	+9	−6	6	−2	−3
−4	−8	7	7	+10	+5	−2

---

* Pupils needing remedial work in fractions will find explanations and exercises on pages 567–572.

8.	**9.**	**10.**	**11.**	**12.**	**13.**
$4a$	$-2b$	$-11m$	$10ab$	$7cd$	$4h$
$-7a$	$-5b$	$3m$	$12ab$	$-9cd$	$-9h$
$3a$	$6b$	$-6m$	$-10ab$	$-20cd$	$-3h$
$-6a$	$9b$	$-7m$	ab	$20cd$	$2h$

14. The numbers in the following diagram are arranged in 5 rows and 8 columns. Add the numbers in each row and in each column.

Add the row sums and the column sums to check your work.

	f	g	h	i	j	k	l	m	**Sums**
a	4	5	− 3	− 2	7	1	6	− 8	?
b	− 2	− 3	− 4	− 5	− 2	− 1	− 8	− 7	?
c	6	− 4	5	3	2	− 7	− 3	− 2	?
d	− 5	3	− 6	9	4	2	3	− 12	?
e	− 2	− 3	2	4	− 4	7	− 6	5	?
Sums	?	?	?	?	?	?	?	?	?

How to Indicate the Sum of Signed Numbers [A]

Parentheses are often used to distinguish the signs of opposition from the signs of operation. Thus in the expression $(+8) + (-6)$, the parentheses are used to show that positive 8 and negative 6 are to be added. Since adding -6 and $+8$ gives the same result as subtracting $+6$ from 8, the expression $(+8) + (-6)$ is equal to $(+8) - (+6)$. Then since the numbers within the parentheses in the last expression are positive, their signs and the parentheses may be removed and the expression becomes $8 - 6$. So if we wish to add $+8$ and -6, we may indicate the sum by $+8 - 6$.

In like manner, if we wish to indicate the sum of -3, $+7$, and -4, we can do so by writing $-3 + 7 - 4$. The written signs in the expression $-3 + 7 - 4$ are signs of operation, but they may be considered signs of opposition. From the above discussion the following rule may be inferred:

To show that two or more numbers are to be added, write the numbers one after another with their proper signs.

Example 1. If a man loses \$10 and then gains \$7, the sum of the two transactions is $-10+7=-3$.

Example 2. Indicate the sum of -6, 8, and -5.

Solution. The sign of 8 is + (understood). The indicated sum is $-6+8-5$. Then adding -6, $+8$, and -5, we get -3.

EXERCISES [A]

Using positive and negative numbers, rewrite the following and then combine the terms.

1. Earning \$7 and earning \$6.

2. Losing \$6 and gaining \$5.

3. Going up 30 feet and then going down 50 feet.

4. Gaining 10 cents and then losing 4 cents.

5. Going 6 steps backward and then going 10 steps forward.

6. Gaining 4 pounds, losing 6 pounds, and losing 2 pounds.

7. Making a profit of \$80, a loss of \$60, and a profit of \$20.

8. A rise of 8°, a drop of 12°, and a rise of 2°.

Remembering that $4-6+3$ is the same as $+4$ added to -6 added to $+3$, combine the following:

9. $4-6+3$

10. $8-5-10$

11. $7-9+6$

12. $4-2-2$

13. $9+6-4$

14. $3-8-5$

15. $-7-5+1$

16. $-8+5-3$

17. $-2-6-7$

18. $7+2+1$

19. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$

20. $.4+.3-.2$

21. What is the absolute value of -8? of $+6$? of 10? of -9?

22. In a game four boys made the following scores:

CHARLES	DICK	JESS	FRANK
3	2	− 5	− 2
− 5	6	3	− 4
7	− 3	− 6	− 5
− 6	− 5	8	6
2	7	3	4

Find the total score of each boy. Who made the highest score? the lowest score?

Simplifying Polynomials [A]

On page 14 you simplified polynomials by adding and subtracting positive numbers. You will now learn how to simplify polynomials such as $-4x + y - x - 3y$. In simplifying this polynomial it is necessary to combine negative numbers.

Example 1. Simplify $3a + 4a - 5a - 6a$.

Solution. $3a$ plus $4a = 7a$

$-5a$ plus $-6a = -11a$

$7a$ plus $-11a = -4a$

Then $3a + 4a - 5a - 6a = -4a$

EXERCISES [A]

Simplify by combining like terms:

1. $3x - 4x - x$
2. $a + 5a - 7a$
3. $4b - 6b + 3b$
4. $m + m - m$
5. $10r + 6r - 13r$
6. $-4x - 3x - 5x$
7. $5A - 2A + 3A$
8. $7h - 9h - h$
9. $6x - 4x + 9x$
10. $r - 4r - 6r$
11. $4x^2 - 3x^2 - 5x^2$
12. $2.5x + .5x - 6x$
13. $3.1y + 2.4y - 8y$
14. $3m^3 - 8m^3 - m^3$
15. $5bc - 8bc - bc$
16. $-xy - xy - xy$
17. $\frac{1}{2}x + \frac{5}{6}x - x$
18. $\frac{1}{3}a - \frac{5}{6}a + \frac{1}{12}a$
19. $-\frac{1}{2}b - \frac{1}{4}b - \frac{1}{3}b$
20. $\frac{1}{2}x + \frac{3}{4}x - \frac{1}{3}x$

Example 2. Simplify $4a + 3b + 5a - 8b$.

Solution. $4a$ and $5a$ are like terms and their sum is $9a$; $3b$ and $-8b$ are like terms and their sum is $-5b$. Then

$$4a + 3b + 5a - 8b = 9a - 5b.$$

EXERCISES [A]

Simplify:

1. $3a + 4b - a - 6b$
2. $-5x - 7y - 8x$
3. $x + y - x + 6y$
4. $5ab + 6 + 8ab - 10$
5. $-6x + 9 - 3x - 8$
6. $a + c + 7a - 4$
7. $m^2 + 2mn + n^2 - m^2 + 2mn + n^2$
8. $a^2 - 4ab - b^2 + 4ab + a^2 - b^2$

Indicate the sum of

9. a and $-b$
10. -3 and $+3$
11. $2x^2$ and $5x$
12. s and $-s^2$
13. x^2 and y^2
14. x^2, $-2x$, and -1
15. $4a$, $-b$, and $5c$
16. $-3r$, $2s$, and $-5t$
17. $2x^3$, $-5x^2$, and $-4x$
18. $-a$, b, and c.

Indicate the sum and simplify:

19. 4, 5, and -3
20. $2m$, 7, and $-5m$
21. $4a$, $-2a$, and h
22. $2x$, $3y$, and $-4x$
23. $-x^2$, $3x$, and $+x^2$
24. $2a$, $-b$, $-2a$, and b

Multiplication of Signed Numbers [A]

In algebra there are four types of products. These are a positive number times a positive number, a positive number times a negative number, a negative number times a positive number, and a negative number times a negative number.

Type I. *A Positive Number Times a Positive Number.* This is the type you learned in arithmetic.

If a man saves \$5 a week, he will be \$20 wealthier 4 weeks from now. The solution may be written

$$(+4) \times (+\$5) = +\$20.$$

Type II. *A Positive Number Times a Negative Number.* If a man loses \$5 a week, he will be \$20 poorer in 4 weeks.

If saving \$5 is represented by + \$5, then losing \$5 should be represented by − \$5; and if \$20 richer is represented by + \$20, then \$20 poorer should be represented by − \$20.

Then $(+4) \times (-\$5) = -\$20.$

Type III. *A Negative Number Times a Positive Number.* If a man has been saving \$5 a week, 4 weeks ago he was \$20 poorer than he is now.

If future time is represented by a positive number, then past time should be represented by a negative number.

Then $(-4) \times (+\$5) = -\$20.$

Type IV. *A Negative Number Times a Negative Number.* If a man has been losing \$5 a week, 4 weeks ago he was \$20 richer than he is now. Then $(-4) \times (-\$5) = +\$20.$

The examples above illustrate the following law of signs for multiplication.

> **The product of two numbers with *like* signs is positive.**
>
> **The product of two numbers with *unlike* signs is negative.**

EXERCISES [A]

Multiply, using the law of signs stated above:

	a	b	c	d	e	f	g	h
1.	3 5	2 -7	-6 $+3$	-4 -3	-5 $+1$	$+5$ -1	-2 -2	7 3
2.	-8 -4	7 -9	-6 -6	-7 $+1$	-8 1	-3 -3	$+3$ $+3$	-10 -1
3.	-4 -5	-5 4	-5 -10	6 -2	-5 7	-9 9	-9 1	-9 -1
***4.**	$\frac{1}{2}$ $\frac{1}{3}$	$-\frac{1}{4}$ $-\frac{1}{3}$	-8 $-\frac{1}{4}$	-9 $+\frac{1}{3}$	$-\frac{1}{5}$ $-\frac{1}{5}$	$\frac{2}{7}$ $-\frac{3}{7}$	$-\frac{1}{2}$ $-\frac{2}{4}$	$-\frac{3}{7}$ -1

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* For remedial work in fractions and decimals see pages 567–574.

5. $3(-5)$

6. $-6(4)$

7. $9(-1)$

8. $(-2)(10)$

9. $8(-10)$

10. $\frac{1}{2}$ of (-4)

11. $\frac{1}{3} \times (6)$

12. $-12(\frac{1}{4})$

13. $25(-\frac{1}{5})$

14. $(\frac{1}{2})(-\frac{1}{3})$

15. $-2(-8)$

16. $.3 \times -.4$

17. $.01 \times (100)$

18. $-2(.05)$

19. $-4(25)$

20. Add the numbers in each part of exercises 1, 2, 3, and 4.

21. Multiply 7, -3, 4, and -6 in turn by 1.

Complete: When a number is multiplied by 1, the product is the --?-- as the number itself.

22. Multiply 7, -3, 4, and -6 in turn by -1.

Complete: A number can be multiplied by -1 by --?-- its sign.

23. The expression $-4x$ means that the number x is to be multiplied by -4. What is the value of $-4x$ when $x = 3$? When $x = -3$?

24. Find the values of the following expressions when $a = 4$, $b = -5$, and $c = -6$.

a. $4b$

b. $-3a$

c. $4a$

d. $-4b$

e. $-3c$

f. $6b$

g. ab

h. ac

i. bc

j. $\frac{1}{2}a$

k. $-\frac{1}{5}b$

l. $.3c$

How to Find the Product of Three or More Numbers [A]

If three or more numbers are to be multiplied together, we can perform the multiplications in any order we choose. We may choose any one of the numbers and multiply it by any one of the remaining numbers. Then we multiply this product by any remaining number. We continue this process until all the numbers to be multiplied have been used. To illustrate, let us find the product of -4, $+3$, and -5.

$$(-4)(+3)(-5) = -4(-15) = 60$$
$$(-4)(+3)(-5) = (-12)(-5) = 60$$
$$(-4)(+3)(-5) = (+20)(+3) = 60$$

When finding the value of an expression such as $-4x^2$, we first do the multiplication indicated by the exponent.

Example. Find the value of $-7x^2$ when $x = -3$.

Solution. If $x = -3$, $x^2 = (-3)^2 = (-3)(-3) = 9$

Then, $-7x^2 = -7(9) = -63$

If a letter has no sign written before it, the $+$ sign is understood; and if it has no coefficient written before it, the coefficient 1 is understood. Thus x means $+1x$, x^2 means $+1x^2$, and $-cd$ means $-1cd$.

The Product of Any Number and Zero [A]

When you multiply zero and any other number together, the product is zero. Thus $3 \times 0 = 0$, $0 \times 10 = 0$, $0 \times (-8) = 0$, and $10{,}000 \times 0 = 0$.

It follows that a product is zero if any one of its factors is zero.

EXERCISES [A]

1. If $x = 2$, find the value of x^2; of $4x^2$; of $-x^2$.

2. If $a = -3$, find the value of a^2; of $5a^2$; of $-a^2$.

3. What does x^3 mean? What is the value of x^3 when $x = 4$? when $x = -4$? when $x = 0$?

4. If $m = -1$, what is the value of m^2? of $4m^2$? of $-8m^2$?

5. If $m = -2$, what is the value of m^3? of $5m^3$? of $-2m^3$?

6. If $a = 2$ and $b = -3$, what is the value of ab? of $3ab$? of $-ab$?

7. If $c = -6$, find the value of c^2; of c^3; of c^4; of $-c^3$; of $-c^4$.

If $a = 1$, $b = 2$, $c = 0$, and $d = -3$, find the value of

8. abc
9. ab^2
10. $-4ab$
11. $-3cd^2$
12. $-5c^2d$
13. $abcd$
14. $ac - a$
15. $ad - c^8$

Division of Signed Numbers [A]

You know from your study of arithmetic that if the product of two numbers is divided by either of them, the quotient is the other number. For example, if 15 (which is the product of

3 and 5) is divided by 3, the quotient is 5. This fact is true for any kind of number.

We can learn the law of signs for division by studying four examples.

1. Since $(+5)(+3) = +15$, then $(+15) \div (+3) = +5$.
2. Since $(-5)(+3) = -15$, then $(-15) \div (+3) = -5$.
3. Since $(+5)(-3) = -15$, then $(-15) \div (-3) = +5$.
4. Since $(-5)(-3) = +15$, then $(+15) \div (-3) = -5$.

The examples above illustrate the following law of signs for division:

> **1. The quotient of two numbers with like signs is positive.**
>
> **2. The quotient of two numbers with unlike signs is negative.**

EXERCISES

A

1. How does the law (rule) of signs for division compare with the law of signs for multiplication?

Divide as indicated:

	a	**b**	**c**	**d**	**e**	**f**	**g**
2.	$\frac{-12}{-3}$	$\frac{-10}{5}$	$\frac{-30}{5}$	$\frac{25}{-5}$	$\frac{8}{-1}$	$\frac{-6}{6}$	$\frac{0}{4}$
3.	$\frac{8}{-2}$	$\frac{-15}{5}$	$\frac{14}{-7}$	$\frac{-10}{-2}$	$\frac{12}{-2}$	$\frac{-4}{-4}$	$\frac{-18}{9}$
4.	$\frac{-15}{-1}$	$\frac{20}{-5}$	$\frac{-12}{3}$	$\frac{-8}{-4}$	$\frac{7}{1}$	$\frac{-7}{1}$	$\frac{0}{3}$
5.	$\frac{2.8}{-7}$	$\frac{-20}{-10}$	$\frac{-.04}{-2}$	$\frac{-3.0}{6}$	$\frac{100}{-10}$	$\frac{-80}{6}$	$\frac{-2.7}{.9}$
6.	$\frac{10}{-.2}$	$\frac{7}{3}$	$\frac{-.4}{2}$	$\frac{6\frac{2}{7}}{3\frac{1}{7}}$	$\frac{-60}{-30}$	$\frac{-99}{33}$	$\frac{80}{-16}$

7. $56 \div (-7)$

8. $-16 \div 8$

9. $(-20) \div (-4)$

10. $6\frac{2}{3} \div (-\frac{5}{6})$

11. $54 \div .06$

12. $-63 \div 7$

13. $40 \div 80$

14. $(-7\frac{1}{2}) \div (-\frac{3}{4})$

15. $(-5\frac{1}{3}) \div (2\frac{2}{9})$

16. $8.4 \div (-.4)$

17. $-50 \div 2.5$

18. $20 \div (-.05)$

19. Find the quotient if the dividend is -8 and the divisor is -1.

20. Find the quotient if the dividend is -27 and the divisor is -3.

21. The product of two numbers is -32. If one of the numbers is -16, what is the other?

22. If the multiplicand is -405 and the product is -2025, what is the multiplier?

23. If $a = -12$, find the value of

a. $\frac{a}{4}$

b. $\frac{3}{4}a$

c. $-\frac{1}{2}a$

d. $\frac{3a}{2}$

e. $-\frac{2}{3}a$

f. $-.5a$

g. $5a$

h. $-\frac{3}{2}a$

i. a^2

j. $a^2 \div 6$

k. $-a^2$

l. $-a^2 \div 72$

24. Simplify:

a. $\frac{2 \times 3}{2}$

b. $\frac{8 \times 9}{6}$

c. $\frac{8 \times -6}{4}$

d. $\frac{(-10)(-4)}{8}$

e. $\frac{9 \times 10}{-5 \times 3}$

f. $\frac{-4(100)}{50}$

B

25. If $x = 4$, $y = -6$, and $z = -8$, find the value of

a. xy

b. $\frac{xy}{z}$

c. $\frac{-xy}{-1}$

d. xz

e. $\frac{xz}{y}$

f. $-5xz$

g. $\frac{x^2}{y}$

h. $\frac{-yz}{x}$

i. $\frac{96}{xy}$

26. If $a = -6$, $b = 3$, and $c = -2$, find the value of

a. $\dfrac{a+b}{2}$ **c.** $\dfrac{b+c}{-1}$ **e.** $abc \div (-9)$

b. $\dfrac{a+c}{-4}$ **d.** $(ab) \div c$ **f.** $bc \div a$

27. $F = \frac{9}{5}C + 32$. Find the value of F when $C = 10$; when $C \doteq -10$; when $C = -30$.

C

28. $C_t = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}}$. In this formula C_1, C_2, and C_3 represent different values in the same way that the letters a, b, and c do. Find the value of C_t if $C_1 = 20$, $C_2 = 30$, and $C_3 = 40$.

Equations [A]

The equations which you have solved earlier have not required a knowledge of signed numbers. In solving the next group of equations it will be necessary to use addition and division of signed numbers.

Example 1. Solve $-3x = +21$.

Solution. $-3x = +21$

D_{-3} $x = -7$

Notice that to solve for x we divide both members by the coefficient of x.

Example 2. Solve $4x - 6x = -12 + 2$.

Solution. $4x - 6x = -12 + 2$

Simplifying, $-2x = -10$

D_{-2} $x = +5$

Check. Does $4(5) - 6(5) = -12 + 2$?

Does $20 - 30 = -10$?

Does $-10 = -10$? Yes.

EXERCISES [A]

Solve, and check as directed by your teacher:

1. $3x = +15$
2. $3x = -24$
3. $-4x = 12$
4. $5c = 0$
5. $-3x = 7$
6. $5b = 65$
7. $-x = -10$

   This means

   $-1x = -10$
8. $-c = 30$
9. $3y = 2.4$
10. $1.5h = 30$
11. $-1.2y = 6$
12. $0.03k = 30$
13. $4x - x = 18$
14. $3a - 5a = 30$
15. $x - 2x = 4$

    Do not forget that

    $-x$ means $-1x$
16. $7r - 11r = -8$
17. $9m - 2m = -4$
18. $-x - x = -6 - 6$
19. $-2.1k = 2.1 + 4.2$
20. $5h = -10.4 - 6.6$

REVIEW EXERCISES [A]

	a	b	c	d	e	f	g
1. Add:	-4 3	5 -3	-7 8	-2 10	9 -13	-10 $+10$	-3 -4
2. Multiply:	-9 4	-6 -3	-10 10	8 -5	-3 7	$.4$ $.3$	-8 $.5$

3. Simplify:

a. $(-4) + (-2)$

b. $6 - 8 + 7$

c. $-10 - 11$

d. $(-12)(-3)$

e. $-12 \div 3$

f. $(-10) \div (-10)$

g. $0 \div 6$

h. 7×0

i. $3 \div (-6)$

How to Multiply One Monomial by Another [A]

On page 12 it was said that an exponent is a small letter or figure placed above and to the right of a number to show how many times the number is used as a factor. Thus 6^2 is a short way of writing 6×6, m^3 is a short way of writing *mmm*, and

$(2x)^4$ is a short way of writing $(2x)(2x)(2x)(2x)$. We shall now learn how to multiply using monomials like $7m$ and $-3m^2$.

Example 1. Multiply $7a$ by 4.

Solution. $7a$ means 7 times a. Then 4 times $7a$ means 4 times 7 times a, or $4 \times 7 \times a$. This answer can be simplified because we can multiply 7 by 4, getting 28. The product of 28 and a can only be indicated, as $28a$.

Then $$7a \times 4 = 28a$$

Notice that we multiply only one of the two factors of $7a$ by 4.

Example 2. Multiply x^3 by x^2.

Solution. x^3 means xxx and x^2 means xx. Then $x^3 \cdot x^2 = xxx \cdot xx$. The short way of writing $xxxxx$ is x^5.

Then $$x^3 \cdot x^2 = x^5$$

How is the exponent 5 obtained?

Example 3. Multiply $4x^3$ by $-5x^4$.

Solution. $(4x^3)(-5x^4) = 4x^3(-5)x^4$. Since the multiplications may be performed in any order, this product may be written $4(-5)(x^3x^4)$. Since $4(-5) = -20$, and $x^3x^4 = x^7$, the product is $-20x^7$.

Then $$(4x^3)(-5x^4) = -20x^7$$

Example 4. Multiply $-6b$ by $+3a$.

Solution. $(-6b)(3a) = -6b \cdot 3a = -6 \cdot 3ab = -18ab$

EXERCISES [A]

1. Study the examples above and then tell how to complete the following law of exponents for multiplication:

The exponent of any letter in a product is found by __?__ the exponent of that letter in the multiplicand to the exponent of that letter in the multiplier.

2. The law of exponents for multiplication may be expressed as follows: $a^m \cdot a^n = a^{m+n}$. Explain.

Multiply:

3. $4x$ by 3
4. $3m$ by -2
5. $-5a$ by 4
6. $-6x$ by -5
7. $-3c$ by 4
8. $-4y$ by b
9. $-8c$ by $-d$
10. $4r$ by s
11. $6h$ by $-k$
12. $3p$ by -6
13. $2xy$ by -2
14. $3ab$ by -4
15. $-xy$ by z
16. $-2bc$ by -5
17. $4hk$ by g

Example 5. $4a \times -5a = -20a^2$.

EXERCISES [A]

Multiply:

1. $(-7x) \cdot (5x)$
2. $(3a) \cdot 4a$
3. $(6m) \cdot (-2m)$
4. $(-4c) \cdot (-5c)$
5. $(-x) \cdot (-x)$
6. $-2h(-3h)$
7. $-\frac{1}{2}k(\frac{1}{3}k)$
8. $\frac{2}{3}c(\frac{1}{2}c)$
9. $-\frac{2}{5}h(-\frac{1}{2}h)$
10. $\frac{3}{5}s(\frac{2}{3}s)$
11. $(.2x) \cdot (5x)$
12. $(1.3x) \cdot (10x)$
13. $(0.02p) \cdot (5p)$
14. $(1.2w) \cdot (6w)$
15. $(1.5c) \cdot (\frac{1}{3}c)$

Example 6. $(5a^2)(-3a^4) = -15a^6$.

In multiplying one monomial by another, first determine the sign of the product. Next find the numerical part of it. Then find the literal part.

EXERCISES [A]

Find the following products:

1. $(4a^2)(-7a)$
2. $(-5x^2)(3x^5)$
3. $8(-10m)$
4. $(3a^2)(2a^3)$
5. $x^4 \cdot x^6$
6. $-6b^2 \cdot 5b^4$
7. $(-5a^2)(-5b^4)$
8. $(-4x^3)(5a^3)$
9. $6b^4(-7b^3)$
10. $-14(2x)$
11. $-15x^3(\frac{2}{3}x)$
12. $(8m^2)(-4mn)$

13. $(-3c^2x)(-3c^2d)$

14. $(4h)(5h^2)$

15. $(.2x^4)(4x^5)$

16. $4^3 \cdot 4^2$

17. $10^2 \cdot 10^5$

18. $(1\frac{1}{4}m)(4m^7)$

How to Divide One Monomial by Another [A]

Study the three examples below and see if you can learn how to divide monomials without any help from your teacher.

Example 1. $a^7 \div a^2 = ?$

Solution. Since $a^2 \times a^5 = a^7$, then $a^7 \div a^2 = a^5$. Also,

$$\frac{a^7}{a^2} = \frac{\overset{1\,1}{\cancel{a}\cancel{a}}aaaaa}{\underset{1\,1}{\cancel{a}\cancel{a}}} = a^5$$

Which of these two explanations do you prefer?

Example 2. Divide $12x$ by $-4x$.

Solution. $$\frac{12x}{-4x} = \frac{\overset{-3}{\cancel{12}}\,\overset{1}{\cancel{x}}}{-\underset{1}{\cancel{4}}\,\underset{1}{\cancel{x}}} = -3$$

Example 3. Divide $-24a^3$ by $4a$.

Solution. $$\frac{-24a^3}{4a} = \frac{-\overset{-6}{\cancel{24}}\,\overset{1}{\cancel{a}}aa}{\underset{1}{\cancel{4}}\,\underset{1}{\cancel{a}}} = -6a^2$$

ORAL EXERCISES [A]

1. Study the three examples above and complete the following law of exponents for division:

> The exponent of any letter in a quotient is found by --?-- the exponent of that letter in the divisor from the exponent of that letter in the dividend.

2. When x^6 is divided by x^2, is the quotient x^3 or x^4? If you are not sure of your answer, think what the exponents mean as in example 1 above.

3. The law of exponents for division may be expressed by $a^m \div a^n = a^{m-n}$. Explain.

4. When x^9 is divided by x^3, how do you find the exponent of x in the quotient?

Divide as indicated:

5. $x^3 \div x$
6. $m^4 \div m$
7. $a^6 \div a$
8. $\frac{x^7}{x^3}$
9. $n^5 \div n$
10. $x^4 \div x^4$
11. $c^5 \div c$
12. $\frac{-m^{10}}{m^5}$
13. $-x^5 \div x^2$
14. $5^3 \div 5$
15. $10^4 \div 10$
16. $\frac{-8^4}{-8^4}$

WRITTEN EXERCISES [A]

Divide:

1. $15a^3 \div 3$
2. $8a \div 2$
3. $-6x \div 3$
4. $20m^2 \div (-4)$
5. $(4ab) \div (-1)$
6. $3x^2 \div (-.3)$
7. $-14x \div (-7)$
8. $-x^7 \div (-x^2)$
9. $(-4x^6) \div (2x^3)$
10. $16a^6 \div (4a)$
11. $(12p^3) \div (-6p)$
12. $(4x^2) \div (-4x^2)$
13. $(-8a^2) \div (-2a^2)$
14. $(6x^2y) \div (2y)$
15. $(-3m^4) \div (-1)$

Simplify the following fractions by dividing the numerators by the denominators:

16. $\frac{-28x}{4}$
17. $\frac{16m}{8}$
18. $\frac{-14m^3}{-2}$
19. $\frac{-24x}{-2x}$
20. $\frac{16m^5}{4m}$
21. $\frac{-30a}{-15}$
22. $\frac{8y^6}{y^6}$
23. $\frac{-2ab}{2a}$
24. $\frac{abc}{bc}$

Example. If $a = 2$ and $b = -3$, find the value of $\frac{2\,ab - 3\,b^2}{ab}$.

Solution. This fraction means that the numerator $2\,ab - 3\,b^2$ is to be divided by the denominator ab. We shall first evaluate the numerator. The value of the term $2\,ab = 2 \times 2 \times -3 = -12$. The value of $-3\,b^2$ is $-3(-3)^2$, which is -3×9, or -27. Then $-12 - 27 = -39$. The value of the denominator $ab = 2(-3) = -6$. Then $(-39) \div (-6) = 6\frac{1}{2}$.

The solution should be written as follows:

$$\frac{2\,ab - 3\,b^2}{ab}$$

$$= \frac{2(2)(-3) - 3(-3)^2}{2(-3)}$$

$$= \frac{-12 - 27}{-6} = \frac{-39}{-6} = 6\tfrac{1}{2}$$

EXERCISES

A

If $a = -3$ and $b = -4$, find the value of

1. $a^2 + b^2$
2. $b^2 + 6\,a$
3. $\frac{a^3}{b^2} + \frac{1}{9}$
4. $2\,a + b^2$
5. $-3\,b + ab$
6. $\frac{a}{b} + \frac{b}{a}$
7. $ab^2 + 4$
8. $ab + 12$
9. $\frac{ab + 6}{a^2 + b}$

10. If $x = -4$, what is the value of $-x$? of $-x^2$? of $-x^3$?

11. If $-x = -6$, what is the value of $+x$? of $-3\,x$? of $3\,x$?

12. If $a = 0$, what is the value of $2\,a^2 - 3\,a + 4$?

13. If $b = -3$, find the value of $b^3 + 3\,b^2$.

14. If $x = -4$, find the value of $x^2 + (x^3 - x)$.

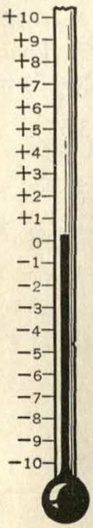
C

15. Repeat exercises 1–9 inclusive by letting $a = \frac{1}{2}$ and $b = \frac{1}{3}$.

16. Repeat exercises 1–9 inclusive letting $a = .3$ and $b = -.1$.

Subtraction of Signed Numbers [A]

Subtraction may be thought of as the process of finding what number must be added to a given number to equal another given number. Thus subtracting 2 from 5 is finding what number must be added to 2 so that the sum is 5.



If we wish to subtract 2 from 5 by the use of the vertical scale at the left, we count the number of spaces between the subtrahend + 2 and the minuend + 5. The distance is 3, and since the direction is upward, the difference is + 3.

To subtract − 3 from + 5, we count from − 3 to + 5. The distance is 8 and the direction is upward, so the difference is + 8.

To subtract + 6 from − 2, we count from + 6 to − 2. The distance is 8 and the direction is downward, so the difference is − 8.

To subtract − 3 from − 7, we count from − 3 to − 7. The distance is 4 and the direction is downward, so the difference is − 4.

These examples illustrate the following rule:

To subtract one number from another using the number scale, count the number of spaces from the subtrahend to the minuend, and to this number prefix a positive sign if the counting is upward and a negative sign if the counting is downward.

EXERCISES [A]

1. In arithmetic can 10 be subtracted from 8? What is the remainder when 10 is subtracted from 8 using the number scale?

2. Using a number scale, subtract:

	a	b	c	d	e
Minuend	4	− 3	5	0	3
Subtrahend	− 3	+ 1	− 6	5	− 7
Remainder	?	?	?	?	?

3. Using a number scale, subtract:

	a	b	c	d	e
Minuend	− 2	3	− 6	− 4	− 8
Subtrahend	7	6	7	− 5	− 8
Remainder	?	?	?	?	?

4. Add the following:

	a	b	c	d	e
Addend	− 2	3	− 6	− 4	− 8
Addend	− 7	− 6	− 7	+ 5	+ 8
Sum	?	?	?	?	?

5. Compare Exercise 4 with Exercise 3. Are the upper addends of Exercise 4 like the minuends of Exercise 3? How do the lower addends of Exercise 4 compare with the subtrahends of Exercise 3? How do the sums of Exercise 4 compare with the remainders of Exercise 3? How can you change a subtraction problem into an addition problem?

Tell how to complete the following rule:

To subtract a number, __?__ its opposite.

This rule is often stated as follows:

To subtract one number from another, change the sign of the subtrahend and add the result to the minuend.

Example 1. Subtract − 6 from + 10.

Solution. We change the sign of − 6 and get + 6. Then we add + 6 and + 10, obtaining + 16.

Example 2. − 10 − (− 3).

Solution. This means that − 3 is to be subtracted from − 10, so we change the sign of − 3, getting + 3. Then we combine − 10 and + 3, getting − 7. The solution can be written as follows:

$$\begin{aligned} &-10-(\underset{\downarrow}{-}3)\\ &=-10\quad+3\\ &=-7\end{aligned}$$

Example 3. From − 15
take + 9

Solution. When the problem is written in this form, we usually do the work mentally. We think this way: "Change the sign of + 9, getting − 9. Then − 15 and − 9 make − 24."

EXERCISES [A]

In each of the following exercises subtract the lower number from the number above:

	a	b	c	d	e	f	g
1.	8 4	− 8 − 5	− 6 − 4	5 − 6	8 10	− 20 25	18 − 2
2.	7 − 3	− 8 − 3	5 6	10 − 10	− 8 8	0 5	0 − 6
3.	− 5 − 6	− 6 − 5	9 − 4	− 4 9	3 2	2 3	7 − 8
4.	9 6	6 9	− 9 6	6 − 9	− 3 − 2	8 0	0 8
5.	− 4 − 10	− 10 − 4	− 13 −6	15 − 8	− 15 − 8	15 8	− 11 17

6. In exercises 1–5 above, find the sum in each case. +

7. In exercises 1–5, find the product in each case. ×

Subtract:

	a	b	c	d	e	f	g
8.	$4x$ $3x$	$6a$ $4a$	$3c$ $-5c$	$-7a$ $9a$	$-6x$ $2x$	$-8m$ $-3m$	$4r$ $-4r$
9.	$4x^2$ $-3x^2$	$5m^3$ m^3	s^2 $-4s^2$	xy $-xy$	$-ab$ $-ab$	$-18m$ $2m$	$5c$ $-c$
10.	$3ab$ $4ab$	$-5c^2$ $7c^2$	$-5h$ $14h$	$3x^2$ $2x^2$	$6y$ $5y$	$4y$ $5y$	0 rs

	a	**b**	**c**	**d**	**e**	**f**	**g**
11.	x	c^2	$\frac{3}{5}x$	$-\frac{2}{3}h$	$2\frac{1}{3}p$	$-7\frac{1}{2}k$	$3\frac{1}{3}y$
	$\frac{1}{2}x$	$-\frac{1}{2}c^2$	$\frac{1}{5}x$	$-\frac{5}{6}h$	$4\frac{1}{3}p$	$\frac{1}{2}k$	$4\frac{2}{3}y$
12.	$1.4h$	$5.0c$	$-4w$	$-m$	$-5.6c$	$2k$	$1.5r$
	$.4h$	$.4c$	$1.5w$	$.8m$	$-.6c$	$.5k$	$-.5r$

Do as indicated by the signs connecting the parentheses:

13. $(4)+(-6)$

14. $(4)-(-6)$

15. $(10)+(8)$

16. $(10)-(8)$

17. $(-6)+(8)$

18. $(-6)-(-8)$

19. $(-5)+(-7)$

20. $(-5)-(-7)$

21. $(5)+(-3)$

22. $(5)-(-3)$

23. $(-7)+(-7)$

24. $(-7)-(-7)$

25. The Battle of Thermopylae was fought in 480 B.C. and Carthage was destroyed in 146 B.C. How many years apart were these two events?

26. According to many scholars Christ was born in the year 4 B.C. and died in the year 29 A.D. How old was He at the time of his death?

27. In a football game the Bears had the ball on their opponents' 14-yard line. They made a gain of 7 yards on one play and then received a 5-yard penalty. Where was the ball placed after the penalty was made?

Reducing Fractions to Lowest Terms[A]

A fraction is in its lowest terms when the numerator and denominator do not have any common divisor except 1. A fraction is reduced to lowest terms by dividing both numerator and denominator by the largest number that is an exact divisor of each of them.

Example 1. Reduce $\frac{12}{15}$ to lowest terms.

Solution. The largest divisor of 12 and 15 is 3.

Then $$\frac{12}{15}=\frac{12\div 3}{15\div 3}=\frac{4}{5}$$

Example 2. Reduce $\frac{5x}{-15x^2}$ to lowest terms.

Solution. The greatest common divisor of $5x$ and $-15x^2$ is $5x$ or $-5x$.

Then $$\frac{5x}{-15x^2} = \frac{5x \div (5x)}{-15x^2 \div (5x)} = \frac{1}{-3x}$$

If both terms of the fraction are divided by $-5x$, the fraction reduces to $\frac{-1}{3x}$. Which answer do you prefer?

ORAL EXERCISES [A]

Reduce to lowest terms:

1. $\frac{4}{8}$
2. $\frac{7}{21}$
3. $\frac{8}{20}$
4. $\frac{30}{42}$
5. $\frac{25}{63}$
6. $\frac{24}{33}$
7. $\frac{4a}{5a}$
8. $\frac{3x}{5x}$
9. $\frac{2x^2}{3x^2}$
10. $\frac{x^4}{x^2}$
11. $\frac{a}{a^3}$ 9
12. $\frac{x^4}{x^3}$
13. $\frac{-6a^3}{12a}$
14. $\frac{a^2b}{ab}$
15. $\frac{xy^2}{x^2y}$
16. $\frac{8a^2}{12a}$
17. $\frac{x^5}{x^8}$
18. $\frac{5a^2}{-10a}$

Multiplication and Division of Fractions [A]

Study the following examples and observe that fractions are multiplied and divided in algebra as they are in arithmetic.

Example 1. Simplify $\frac{2}{5} \times \frac{5}{6}$.

Solution. $$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} = \frac{1}{3}$$

To multiply one fraction by another, we multiply the numerators together for the numerator of the product and multiply the denominators together for the denominator of the product. Then we reduce the product to lowest terms. The above solution is a simplification of this procedure.

Example 2. Multiply $\frac{a}{b}$ by 6.

Solution.

$$\frac{a}{b}\cdot\frac{6}{1}=\frac{6a}{b}$$

Example 3. Find the product of $\frac{3x^2}{4}$ and $\frac{6}{x}$.

Solution.

$$\frac{\overset{3x}{\cancel{3x^2}}}{\underset{2}{\cancel{4}}}\cdot\frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{x}}}=\frac{9x}{2}$$

Example 4. Divide $\frac{4a^3}{21}$ by $\frac{2a}{7}$.

Solution. The divisor $\frac{2a}{7}$ must be inverted. Then we proceed as in multiplication.

$$\frac{4a^3}{21}\div\frac{2a}{7}=\frac{\overset{2a^2}{\cancel{4a^3}}}{\underset{3}{\cancel{21}}}\times\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2a}}}=\frac{2a^2}{3}$$

EXERCISES [A]

Perform the indicated operations:

1. $\frac{2}{3}\times\frac{6}{7}$

2. $6\div\frac{1}{3}$

3. $\frac{7m}{5}\times 3$

4. $\frac{a}{b}\times\frac{c}{d}$

5. $\frac{-c}{d}\times\frac{3}{4}$

6. $\frac{x}{3}\div\frac{x}{2}$

7. $\frac{m}{n}\times\frac{4}{5}$

8. $\frac{c}{5}\div\frac{c}{3}$

9. $10\div\frac{1}{5}$

10. $b^2\div\frac{b}{a}$

11. $b^2\times\frac{b}{a}$

12. $\frac{2y}{3}\times\frac{6}{y}$

13. $\frac{4x^2}{1}\cdot\frac{1}{x}$

14. $\frac{-ab}{3}\div\frac{b}{4}$

15. $15\div\frac{2}{3}$

16. $\frac{15}{a} \div \frac{2}{3\,a}$

17. $\frac{4\,xy}{1} \div \frac{2\,x^2}{3}$

18. $6 \div \frac{1}{9}$

19. $\frac{1}{10} \div \frac{a}{50}$

20. $\frac{c}{d} \div \frac{a}{b}$

21. $\frac{-8\,a^3}{15\,x} \div \frac{-4\,a}{3\,x}$

Combining Fractions [A]

Fractions which have the same denominator may be added (or subtracted) by placing the sum (or difference) of their numerators over their common denominator.

Example 1. $\frac{2}{3} + \frac{5}{3} = \frac{7}{3}$ **Example 2.** $\frac{a}{b} - \frac{3}{b} = \frac{a-3}{b}$

EXERCISES [A]

Combine the fractions as indicated, giving the answers at sight.

1. $\frac{2}{3} + \frac{1}{3}$

2. $\frac{4}{3} - \frac{2}{3}$

3. $\frac{3}{7} + \frac{1}{7}$

4. $\frac{3}{7} - \frac{1}{7}$

5. $\frac{4}{3} + \frac{2}{3}$

6. $\frac{a}{6} + \frac{3\,a}{6}$

7. $\frac{x}{4} + \frac{3\,x}{4}$

8. $\frac{a}{3} + \frac{b}{3}$

9. $\frac{a}{c} + \frac{b}{c}$

10. $\frac{2}{a} + \frac{3}{a}$

11. $\frac{1}{c} + \frac{1}{c}$

12. $\frac{3}{x} - \frac{4}{x}$

13. $\frac{4}{xy} - \frac{3}{xy}$

14. $\frac{-4}{m} - \frac{3}{m}$

15. $\frac{5\,x}{8} + \frac{3\,x}{8}$

When fractions which are to be added or subtracted do *not* have the same denominator, they must be changed to equal fractions which do have the same denominator.

Example 3. $\frac{2\,x}{3} + \frac{x}{6} = \frac{4\,x}{6} + \frac{x}{6} = \frac{5\,x}{6}$

Notice that both terms of the first fraction are multiplied by 2.

EXERCISES [A]

Combine:

1. $\frac{2}{3}+\frac{5}{6}$

2. $\frac{x}{2}+\frac{3x}{4}$

3. $\frac{5c}{8}-\frac{c}{4}$

4. $\frac{m}{5}+\frac{m}{10}$

5. $\frac{x}{3}-\frac{x}{9}$

6. $\frac{a}{4}-\frac{3a}{2}$

7. $\frac{m}{4}+\frac{m}{6}$

8. $\frac{h}{5}-\frac{h}{2}$

9. $\frac{x}{10}-\frac{3x}{5}$

10. $-\frac{1}{3}-\frac{1}{2}$

11. $\frac{m}{3}+\frac{m}{4}$

12. $\frac{a}{2}-\frac{a}{3}$

13. $\frac{a}{4}-\frac{a}{3}$

14. $\frac{1}{2c}-\frac{1}{3c}$

15. $\frac{2}{a}-\frac{5}{2a}$

WORD LIST

At this time you should be able to use and spell each of the following words correctly.

coefficient (p. 10)
dividend (p. 11)
factor (p. 10)
monomial (p. 13)
parentheses (p. 11)
polynomial (p. 13)
positive (p. 74)
quotient (p. 88)

CHAPTER REVIEW [A]

Complete the following statements:

1. Subtracting $+6$ from a number gives the same result as --?--.

2. Subtracting -3 from a number gives the same result as --?--.

	a	b	c	d	e	f	g
3. Add:	7	-9	-5	-9	10	12	7
	6	-3	$+6$	8	-6	10	-4
4. From	10	6	-8	6	5	3	0
take	6	10	4	-4	3	-5	8

5. Add:

	a	b	c	d	e	f	g
	$-2x$	$-6x$	x^2	w	T	10	$-18c$
	$4x$	$-3x$	$-x^2$	$-4w$	$-T$	-10	$-6c$

Simplify the following:

6. $8-6+3$

7. $10-4-12$

8. $-4-5-1$

9. $x^2-2x+6-x^2-x-1$

10. $2a+5b-c-a-7b$

11. $3m^2-10m-1+7-m$

12. $(-4)(3)$

13. $(-3)9$

14. $(-6)(-6)$

15. $7(-10)$

16. x^2x^3

17. $(-3x)(4x)$

18. $(5w)(3w)$

19. $x^3 \cdot x^4$

20. $(-3x)(4x^2)$

21. $(-4c)(5c)$

22. $10(-4x)$

23. $4c^2(-3c^3)$

24. $12 \div (-3)$

25. $(-18) \div (-9)$

26. $42 \div 3$

27. $-30 \div 10$

28. $40 \div (-1)$

29. $(-x^9) \div (-x)$

30. $4xy \div 4$

31. $(6x^2y) \div (2y)$

32. $(xyz) \div z$

33. $(20x^2) \div (5x)$

34. $(-18x^3) \div (6x)$

35. $40x \div (8x)$

36. $10+(-4)$

37. $10-(-4)$

38. $2c-4c$

39. $3h-(-5h)$

40. $-5x+(-2x)$

41. $-5x-(-2x)$

42. $-9-6$

43. $x-4x$

44. $10-18$

45. $-3c+4c$

46. $0-(-8)$

47. $90+(-90)$

GENERAL REVIEW [A]

1. Write the following using the symbols of algebra:

a. The sum of x and y.

b. Twice x, decreased by three times y.

c. The sum of 4, -6, and m.

d. The square of h decreased by the cube of m.

e. The larger of two numbers when the smaller is 10 and the difference between them is d.

10+d.

2. Write the following using exponents:

a. xx **d.** $hh + h$ **g.** $(xx)(xxx)$

b. xxx **e.** $xx + xxx$ **h.** $2\,yy - yyy$

c. mmm **f.** $2\,cc + 3\,ccc$ **i.** $10 \times 10 \times 10$

3. Write a formula for

a. The area of a parallelogram whose base is b and altitude h.

b. The perimeter of a rectangle whose base is b and altitude a.

c. The circumference of a circle whose diameter is d.

d. The volume of a rectangular solid whose dimensions are a, b, and c.

4. If $x = 2$, $y = -4$, and $z = 3$, evaluate:

a. $3\,x + 4\,z$ **c.** $xy - z$ -11 **e.** $2\,x^2 - 10$ - 2

b. $xz - 4$ **d.** $x^3 + y^3$ - 56 **f.** $y^2 - z^2$ 7

5. A taxi charges 20 cents a mile plus a flat rate of 15 cents. Write a formula for the cost c (in cents) of riding m miles.

6. $c = 36 + 3(n - 10)$ is a formula for finding the cost (c) in cents of sending a telegram of ten or more words between two cities, where n is the number of words.

a. Find the cost of sending 10 words.

b. Find the cost of sending 20 words.

c. What is the minimum charge?

7. Write formulas expressing the relation between x and y as shown in the tables. Then copy and complete the tables.

a.

When $x =$	0	1	2	3	4	5
Then $y =$	10	9	8	7?	6?	5?

b.

When $x =$	0	1	2	3	8	?20
Then $y =$	0	5	10	15	40	100

8. $F = \frac{9}{5}C + 32$. Find F if $C = 20$; if $C = 100$.

9. $C = \frac{5}{9}(F - 32)$. Find C when $F = 32$; when $F = 212$; when $F = 0$.

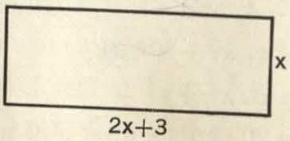
10. Write a formula for finding the cost c of n articles at y cents each.

11. Write a formula for the statement, "The amount is found by multiplying the principal, the rate, and the time together and adding this product to the principal."

12. What operation cannot always be performed by using positive numbers only, but can always be performed by using both positive and negative numbers?

13. Complete: In the expression $5x^2$, the 5 is called the --?-- of x^2 and the 2 is called the --?-- of x.

14. Write the formula for the perimeter p of this rectangle. What is the perimeter of this rectangle when $x = 1$?



15. Solve the following equations:

a. $7x + 3x = 30$ 3

b. $8p + 3p - p = 50$ 5

c. $4x + 6x = 20 - 80$ -6

d. $7c + c = 82 + 80$ 20¼

e. $10m - 14m = 14 + 13$ -6¾

f. $2c - .4c = 18 - 12$ 3.75

g. $7.2x - 3.5x = 6 + 14$ 5.405

h. $.04p = 100$ 2500

16. A rectangular field is enclosed by a fence 3960 rods in length. If the length is 5 times the width, how wide is the field?

17. The sum of three numbers is 1280. The first is 5 times the second and the third is 4 times the second. Find the numbers.

18. Two angles are complementary and one is 5 times the other. Find the angles.

19.

When t =	0	1	2	3	4	5	6	7
s =	0	16	64	?	?	?	?	?

When t is doubled, how does s change?
Complete the formula $s =$ --?-- t^2.
Complete the table.

20. Write the formula for the area (A) of Fig. 1, which consists of a square and halves of four circles.

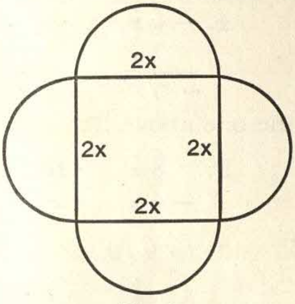


Fig. 1

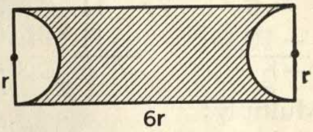


Fig. 2

21. Write the formula for the area of the shaded surface of Fig. 2.

22. Dale Smith joined the Pals Club on January 10. On the day he joined the club he paid an initiation fee of 50 cents. The monthly dues of the club are 10 cents and they are payable on the first day of each month.

a. If Dale is in good standing, how much money has he paid to the club by July 4 of the same year?

b. Write a formula for finding the amount of money he has paid to the club at the end of m months.

Simplify:

23. $\frac{-24x^2}{-4x}$ $6x$

24. $\frac{12y^3}{y^3}$ 12

25. $\frac{8x^2}{-2x}$ $-4x$

26. $\frac{2x^3}{5x^3}$ $\frac{2}{5}$

27. $\frac{a}{a^2}$ $\frac{1}{a}$

28. $\frac{ab}{a^2b^2}$ $\frac{1}{ab}$

29. $\frac{16a^2}{24a^5}$ $\frac{2}{3a^3}$

30. $\frac{17xy}{51x^2y^2}$ $\frac{1}{3xy}$

Perform the indicated operations:

31. $5 \times \frac{a^3}{15}$ $\frac{a3}{3}$

32. $\frac{8}{x^3} \cdot 2x$ $\frac{16}{x^2}$

33. $\frac{x}{5} \div \frac{x}{3}$ $\frac{3}{5}$

34. $10a^2 \div \frac{2a}{7}$ $35a$

35. $\frac{5}{c} \div \frac{3}{c}$ $\frac{5}{3}$

36. $\frac{1}{x} \cdot 4x^2$ $4x$

CHAPTER TEST [A]

Find the sum in each of the following:

1. $\begin{array}{r} 7 \\ -3 \\ \hline +4 \end{array}$ **2.** $\begin{array}{r} -3 \\ -6 \\ \hline -9 \end{array}$ **3.** $\begin{array}{r} -8 \\ +4 \\ \hline -4 \end{array}$ **4.** $\begin{array}{r} -6x \\ 2x \\ \hline -4x \end{array}$ **5.** $\begin{array}{r} 7c \\ -c \\ \hline +6c \end{array}$

Subtract each lower number from the one above it:

6. $\begin{array}{r} -8 \\ -10 \\ \hline +2 \end{array}$ **7.** $\begin{array}{r} -11 \\ 11 \\ \hline -22 \end{array}$ **8.** $\begin{array}{r} 3c \\ 5c \\ \hline -2c \end{array}$ **9.** $\begin{array}{r} 8x \\ -6x \\ \hline +14x \end{array}$ **10.** $\begin{array}{r} 9.1y \\ 4.6y \\ \hline 4.5y \end{array}$

Multiply:

11. $\begin{array}{r} -9 \\ 4 \\ \hline -36 \end{array}$ **12.** $\begin{array}{r} 4y \\ -5 \\ \hline -20y \end{array}$ **13.** $\begin{array}{r} -6x \\ -7x \\ \hline +42x \end{array}$ **14.** $\begin{array}{r} 10p \\ -6 \\ \hline -60p \end{array}$ **15.** $\begin{array}{r} 0 \\ 4 \\ \hline 0 \end{array}$

Find the quotients:

16. $-8 \div 4$ **17.** $(-8x) \div (-2)$ **18.** $18y^3 \div (-9y)$

Simplify by collecting like terms:

19. $5x - 6 - 4x$

20. $4a + 6a - 12a$

21. $5h - k + 7k - 3k$

22. $9x^2 - 4x - 6x^2 - 3x$
$+3x^2 \quad -7x$

Which of the statements in exercises 23–25 are true?

23. $3x$ and 3 are similar terms.

24. $-2x$ and $2x$ are similar terms.

25. The product of $\frac{2x}{3}$ and $\frac{x}{2}$ is $\frac{3x}{5}$.

Solve:

26. $9x + 3x = 30$

27. $-y - y = -4$

28. $3a - 5a = 16$

29. $4c + 10c = -7$

30. $1.2x = 6$

31. $\frac{3}{4}k = 12$

If $a = -4$ and $b = 2$, find the value of

32. $7a + 4b =$
$-28 + 8 = 2$

33. $ab + 5 =$
$-8 + 5$

CHAPTER TEST [B]

Perform the indicated operations:

✓ **1.** $(+4) - (1.5)$

2. $18 \div (-6)$

3. $(-15) + (+19)$

4. $(-10) - (+20)$

5. $x^2 \cdot x^3$

6. $20a^4 \div (-4a)$

7. $(7x^2)(-2x)$

8. $x^8 \div x^2$

9. $(a)(-b)(-c)$ $(-a)(b)(-c)$

10. $10p + (4p) - (6p)$

Simplify by collecting like terms:

11. $4x^2 - 10xy + y^2 - 7xy - x^2 - 9y^2$

12. $4a - 3b + 4c - 7b - 5c$

13. Reduce the fraction $\frac{4xy}{2x}$ to lowest terms.

14. Multiply: $\frac{2m^2n}{3x} \times \frac{x}{4m^3}$

15. Add: $\frac{x}{3} + \frac{y}{3}$

16. Subtract: $\frac{5}{m} - \frac{3}{2m}$ $\frac{7}{2m}$

Solve:

17. $-8x + 2x = 15 - 6$

18. $2.1c - 1.4c = 2.8$

19. $3b - 4b = 19$

20. $8p - 4p = -6 + 2$

21. What is the absolute value of -3?

22. What must be added to -9 to give $+4$?

23. The sum of two numbers is -11. If one of them is -5, what is the other?

24. The product of two numbers is $5x^4$ and one of the numbers is x^3. Find the other number.

25. The difference between two numbers is $3c$. What is the larger number if the smaller is $-4c$?

A

$4c$ $x - (-4c) = 3c$

$x + 4c = 3c$

MATHEMATICS USED IN THE MACHINE SHOP

Some machine operators do not use much mathematics in their work. However, a good machinist not only knows how to operate the machine but understands the principles by which the machine works. He knows how to read his handbook; he understands graphs; he can use the formulas and can develop some of them. When he is given a job, he can make the necessary calculations, can set his machine, and do the work without the foreman's assistance.

The mathematics needed in the machine shop is arithmetic, algebra, plane and solid geometry, and elementary trigonometry.

Without mathematics the opportunity for advancement is limited. Many machinists, realizing this fact, study mathematics in vocational night schools and university extension classes.

Wright Aeronautical Corporation

The Shapes of Gears Are Determined by Equations

CHAPTER IV

Equations and Problems

In the solution of equations in this chapter we shall use three fundamental principles called *axioms*. An *axiom* is a statement about numbers accepted as true without proof. A good understanding of these axioms is necessary in order to solve equations quickly and accurately. The three axioms that you will learn are used not only in algebra but in all mathematics.

Using the Division Axiom [A]

You have already solved equations such as $3x = 15$, the solution of which is $x = 5$. In solving this equation you probably think, "$1x$ is $\frac{1}{3}$ of $3x$. Then $1x$ is $\frac{1}{3}$ of 15." You can also solve it by thinking, "I must divide $3x$ by 3 to obtain $1x$. Then if I divide the left member of the equation by 3, I must also divide the right member by 3." By solving the equation in the latter manner you are making use of the *division axiom*. The division axiom is stated as follows:

> **If both members of an equation are divided by the same number, except zero, the members are still equal.**

Example 1. Solve $-7x = 28$.

Solution. Solving an equation for x is finding the value of $+1x$. To obtain $+1x$ or x in the left member, we must divide by -7. If we divide the left member of the equation by -7, we must also divide the right member by -7. Dividing both members of the equation by -7, we obtain $x = -4$.

Example 2. Solve $-5x + x = 2 - 18$.

Solution. Simplifying each member, we have

$$-4x = -16$$

*D_{-4} $\qquad x = 4$

Check. Does $\quad -5(4) + 4 = 2 - 18$?

Does $\quad -20 + 4 = 2 - 18$? Yes.

ORAL EXERCISES [A]

Solve the following equations mentally, first telling what number you are dividing both members of the equation by, and then stating what the letter x equals.

1. $-5x = 60$
2. $4x = -20$
3. $-2x = -12$
4. $7x = 21$
5. $3x = -18$
6. $-3x = -18$
7. $-3x = 18$
8. $3x = 18$
9. $4x = 14$
10. $-11x = 22$
11. $5x = -12.5$
12. $-2x = 7$
13. $1.5x = 3$
14. $5x = 3$
15. $6x = 2$

Example 3. Solve $1.2x = .36$.

Solution. In the equation $12x = 36$ you easily recognize that 36 divided by 12 is 3; but .36 divided by 1.2 is not so easy. Unless you are absolutely sure of the correct quotient you should actually perform the division, either to the right of your solution or on another piece of paper. Thus

$$1.2x = .36$$

$D_{1.2}$ $\qquad x = .3 \qquad \begin{array}{r} .3 \\ 1.2_{\wedge}\overline{\smash{)}\,.3_{\wedge}6} \\ \underline{3\,6} \end{array}$

WRITTEN EXERCISES [A]

Solve and check:

1. $-3x = 24$
2. $-x = -10$
3. $-x = 10$
4. $7x = 35$
5. $-4x = 9$
6. $x - 3x = -10$
7. $2m - 5m = 42$
8. $x - 2x = 8$
9. $c + 3c = -8$

---

* D_{-4} means "dividing each member of the equation by -4."

10. $2.1x = 10.5$
11. $5c + 4c = 100 - 10$
12. $4x = 2$
13. $.5x = 20$
14. $.25y = 100$
15. $m + .4m = -70$

Using the Addition Axiom[A]

We shall now learn how another axiom can be used to solve equations. Let us examine the equation $3x - 5 = 7$. This equation states that if 5 is subtracted from three times a number, the remainder is 7. We know that 3 times the number is 5 greater than 7, or 12. If 3 times the number is 12, the number is 4. The *addition axiom* which follows enables us to solve the equation more easily:

> **If the same number is added to both members of an equation, the members are still equal.**

Example 1. Solve $3x - 5 = 7$.

Solution. We wish only terms containing x in the left member. We do not wish -5 in the left member. Since $+5$ added to -5 equals 0, we shall add $+5$ to the left member, giving $3x$ as the result. Since we are adding 5 to the left member, we must also add it to the right member. The solution should be written as follows:

$3x - 5 = 7$
*A$_{+5}$ $3x - 5 + 5 = 7 + 5$
Simplifying $3x = 12$
D$_3$ $x = 4$

Check. Does $3(4) - 5 = 7$?
Does $12 - 5 = 7$? Yes.

Example 2. Solve $4x - 8 = -20$.

Solution. $4x - 8 = -20$
A$_{+8}$ $4x - 8 + 8 = -20 + 8$
Simplifying $4x = -12$
D$_4$ $x = -3$

---

* A$_{+5}$ means "adding + 5 to each member of the equation."

Check. Does $4(-3) - 8 = -20$?
Does $-12 - 8 = -20$? Yes.

EXERCISES [A]

Solve the following equations, using the method shown in Examples 1 and 2:

1. $x - 5 = 8$
2. $2x - 5 = 7$
3. $3c - 9 = 12$
4. $5x - 5 = -25$
5. $-3x - 4 = 8$
6. $-4m - 13 = -5$
7. $8y - 20 = 0$
8. $3x - 4 = -10$
9. $4r - 24 = 0$
10. $-h - 1 = 5$
11. $-2r - 19 = 23$
12. $b - .5 = .9$
13. $s - .4 = 2.6$
14. $n - 3.2 = 5$
15. $-e - 10 = 3.6$
16. $7.2x - 5 = 9.4$
17. $-6.1x - 11 = 7.3$
18. $3.1x - 62 = 0$
19. $3.14x - 4 = 2.28$
20. $-9x - 11 = -25$

Example 3. Solve $7x + 2 = 16$.

Solution. In this equation we must eliminate the $+2$ in the left member. To do this we must add -2. Since we add -2 to the left member, we must also add -2 to the right member. You should write your solution as follows:

$7x + 2 = 16$
A_{-2} $7x + 2 - 2 = 16 - 2$
Simplifying $7x = 14$
D_7 $x = 2$

Check. Does $7(2) + 2 = 16$?
Does $14 + 2 = 16$? Yes.

EXERCISES [A]

1. $x + 3 = 7$
2. $c + 1 = 8$
3. $2y + 3 = 21$
4. $3m + 5 = -7$

A-1

5. $-2x + 1 = -10$
6. $-3c + 8 = 23$
7. $-9h + 1 = -17$
8. $6k + 24 = 0$
9. $x + 1 = 0$
10. $-x + 9 = 0$

.39

11. $c + 2 = 2.7$
12. $n + .25 = 3$
13. $-7x + 11 = -66$
14. $-9x + 15 = 24$
15. $2n + 1.75 = 4.45$
16. $-7n - 7 = 0$

17. Explain why adding a negative number to each member of an equation is sometimes called "using the subtraction axiom"?

18. What is meant by the root of an equation?

Example 4. Solve $7x = 16 + 3x$.

Solution. We wish the terms containing x to be in the left member. To eliminate the $+3x$ in the right member, we must add $-3x$ to each member of the equation.

$$7x = 16 + 3x$$

A_{-3x} $\qquad 7x - 3x = 16 + 3x - 3x$

Simplifying $\qquad 4x = 16$

D_4 $\qquad x = 4$

Check. Does $7(4) = 16 + 3(4)$?

Does $28 = 16 + 12$? Yes.

Example 5. Solve $-8y = 15 - 3y$.

Solution. To eliminate the $-3y$ in the right member of the equation, we must add $+3y$ to both members of the equation.

$$-8y = 15 - 3y$$

A_{+3y} $\qquad -8y + 3y = 15 - 3y + 3y$

Simplifying $\qquad -5y = 15$

D_{-5} $\qquad y = -3$

Check. Does $-8(-3) = 15 - 3(-3)$?

Does $+24 = 15 + 9$? Yes.

EXERCISES [A]

1. $3x = x + 4$
2. $6m = m + 20$
3. $2x = -7x + 18$
4. $3r = 5r - 8$

5. $-6c = 30 - 4c$

6. $2p - 9 = 12$

7. $5h + 13 = -27$

8. $4s = -25 - 6s$

9. $5x = 2 + x$

10. $15 + 5y = 23y$

11. $9 = 2x - 5$

12. $0 = 6 - 2x$

Using the Multiplication Axiom [A]

Equations which contain fractions are usually more easily solved by changing them into equations which do not contain fractions. An equation containing fractions can be changed into another equation free from fractions by multiplying each member by a number that is exactly divisible by each denominator. The smallest number that is exactly divisible by each denominator is called the *lowest common denominator* (abbreviated L.C.D.).

Example 1. Solve $\frac{1}{3}x = \frac{1}{5}$.

Solution. The L.C.D. is 15.

$$\tfrac{1}{3}x = \tfrac{1}{5}$$

*M_{15} $\qquad 15(\tfrac{1}{3}x) = 15(\tfrac{1}{5})$

$$5x = 3$$

D_5 $\qquad x = \tfrac{3}{5}$

Check. Does $\qquad \tfrac{1}{3}(\tfrac{3}{5}) = \tfrac{1}{5}$? Yes.

Example 2. Solve $\dfrac{3x}{7} = -2$.

Solution. Since 7 is the only denominator, it is the L.C.D. The denominator of -2 is 1.

$$\frac{3x}{7} = -2$$

M_7 $\qquad 7\left(\dfrac{3x}{7}\right) = 7(-2)$

$$3x = -14$$

D_3 $\qquad x = -4\tfrac{2}{3}$

Check. Does $\qquad \dfrac{3(-4\frac{2}{3})}{7} = -2$?

Does $\qquad -\dfrac{14}{7} = -2$? Yes.

---

*M_{15} means "multiplying each member by 15."

Example 3. Solve $.02\,x = 50$.

Solution. The denominator of the term containing x is 100, since $.02\,x$ is the same as $\frac{2}{100}\,x$. The denominator of 50 is 1. Then the L.C.D. is 100.

$$.02\,x = 50$$

$$\text{M}_{100} \qquad 100(.02\,x) = 100(50)$$

$$2\,x = 5000$$

$$x = 2500$$

Example 4. Solve $.04\,x = 51.2$.

Solution. The L.C.D. is 100.

$$\text{M}_{100} \qquad 100(.04\,x) = 100(51.2)$$

$$4\,x = 5120$$

$$\text{D}_4 \qquad x = 1280$$

In the four examples above we have used the following *multiplication axiom:*

> **If both members of an equation are multiplied by the same number, the members are still equal.**

EXERCISES [A]

Solve and check the following equations, using the multiplication axiom:

1. $\frac{x}{5} = 3$
2. $\frac{1}{3}m = 6$
3. $\frac{2}{7}x = 14$
4. $\frac{c}{8} = \frac{1}{4}$
5. $.2\,x = 7$
6. $.3\,x = 2.1$
7. $\frac{x}{3} = 45$
8. $-\frac{1}{2}x = \frac{1}{6}$
9. $\frac{3}{7}m = -21$
10. $\frac{s}{10} = \frac{1}{15}$
11. $.03\,k = 6$
12. $.06\,k = 12.6$
13. $\frac{y}{4} = 3$
14. $\frac{1}{6}n = 1$
15. $\frac{1}{3}x = 0$
16. $\frac{4}{5}n = \frac{5}{4}$
17. $\frac{1}{2}c = -\frac{15}{8}$
18. $.2\,r = 6.12$

A Short Method of Applying the Addition Axiom [A]

In the solution of equations the division axiom or the multiplication axiom needs only to be used once. However, if an equation contains more than 3 terms in its members, the addition axiom would need to be used more than once. To make the solution of equations containing several terms easier, we shall now learn a short method of applying the addition axiom. Compare the two solutions in Examples 1 and 2, and answer the questions that follow them.

Example 1. Solve $2x + 3 = 7$.

Solution A		**Solution B**	
$2x + 3 = 7$	(1)	$2x + 3 = 7$	(1)
$2x + 3 - 3 = 7 - 3$	(2)	$2x = 7 - 3$	(2)
$2x = 4$	(3)	$2x = 4$	(3)
$x = 2$	(4)	$x = 2$	(4)

1. Which of the two forms of solution have you been using?

2. How do the two solutions compare as to the number of steps?

3. Which equations in the two solutions are alike and which are unlike?

4. In Solution A what number is added to each member of equation (1) to give equation (2)?

5. In Solution B what number is added to each member of equation (1) to give equation (2)?

6. Complete: In Solution B, equation (2) is obtained from equation (1) by omitting --?-- in the left member and writing --?-- in the right member.

Example 2. Solve $2x - 3 = 7$.

Solution A		**Solution B**	
$2x - 3 = 7$	(1)	$2x - 3 = 7$	(1)
$2x - 3 + 3 = 7 + 3$	(2)	$2x = 7 + 3$	(2)
$2x = 10$	(3)	$2x = 10$	(3)
$x = 5$	(4)	$x = 5$	(4)

Apply the six questions in Example 1, above, to Example 2.

These two examples illustrate the following principle:

> **Any term may be omitted from either member of an equation if its opposite is written in the other member.**

This short method of applying the addition axiom by omitting a term in one member of an equation and writing the term with its sign changed in the other member is called *transposition.*

Example 3. Solve $2x + 15 = 5x - 6$.

Solution. We wish to have all terms containing x in the left member and all other terms in the right member. The term $+ 2x$ is in the left member, so we write it in the left member with the same sign. The $+ 5x$ is in the right member, so we write it in the left member as $- 5x$. So far the solution appears as follows:

$$2x - 5x =$$

The $+ 15$ is in the left member, so we write it in the right member as $- 15$. We leave the $- 6$ in the right member with the same sign. The first step in our solution appears thus:

$$2x - 5x = -15 - 6$$

Then, completing the solution as usual, we have,

Simplifying $\quad -3x = -21$

D_{-3} $\quad x = 7$

Check. Does $\quad 2(7) + 15 = 5(7) - 6$?

Does $\quad 14 + 15 = 35 - 6$? Yes.

Example 4. Solve $7x - 18 = 5x + 4 - 2x$.

Solution. We leave the $+ 7x$ in the left member with the same sign. We then write the $+ 5x$ in the left member as $- 5x$, and write the $- 2x$ in the left member as $+ 2x$. This makes the left member $7x - 5x + 2x$. Then we write the $- 18$ in the right member as $+ 18$ and keep the $+ 4$ in the right member with the same sign. The solution appears as follows:

$$7x - 18 = 5x + 4 - 2x$$

Transposing $7x - 5x + 2x = +18 + 4$

Simplifying $4x = 22$

D_4 $x = 5\frac{1}{2}$

Check. Does $7(5\frac{1}{2}) - 18 = 5(5\frac{1}{2}) + 4 - 2(5\frac{1}{2})$?

Does $38\frac{1}{2} - 18 = 27\frac{1}{2} + 4 - 11$?

Does $20\frac{1}{2} = 20\frac{1}{2}$? Yes.

EXERCISES [A]

Solve and check:

1. $x + 3 = 15$

2. $x - 5 = 0$

3. $3m - 5 = 16$

4. $2x = 20 - 8x$

5. $3c = 8 - 4c$

6. $3m - 9 = 37$

7. $4r = 8r - 20$

8. $5x - 7x - 10 = 0$

9. $16 = 10 - 3x$

10. $11y + 12y = 46$

11. $6 - 3x + 12 = 0$

12. $7m = 3m - 20$

13. $6x - 15 = 84 - 3x$

14. $5x + 15 = 40 + x$

15. $8 - 5y = y - 4$

16. $7c + 28 = 5c + 6$

17. $9k + 2 = 8 + 5k$

18. $2x + 15 - 10 = -3x$

19. $10h - 2 = -7 + 5h$

20. $4c + 13 = -2c + 50$

21. $0 = 8x - 24$

22. $11a - 8 - 2a - 64 = 0$

23. $8y - 2 - 5y - 8 = 0$

24. $14 = 6x - 2 + 2x$

25. $7x - 3x + 18 = 36 - 2x$

26. $9y - 13 + 6y - 7 = 2y$

27. $8c + 300 - 3c = 2c + 1200$

28. $5m + 10 - m = 12 - m + 3$

29. $5r + 12 - 3r = -2r - 13 - r$

30. $8x + 15 - 6x - 6 = 45$

31. $10y + 13y - 5y - 6 = 7y + 6 + 12y - 5$

32. $11c - 7 - 6c - 5 = 11c - 8c$

33. $8x - 11 + 4x - 17 - 5x = 0$

34. $4h + 17 - 7h + 16 - 6h + 3 = 0$

Example 5. Solve $\frac{2}{3}x - 5 = \frac{3}{4}x - \frac{9}{2}$.

Solution. When an equation contains either common or decimal fractions, we first obtain an equation free of fractions by multiplying both members of the equation by the lowest common denominator.

$$\frac{2}{3}x - 5 = \frac{3}{4}x - \frac{9}{2}$$

M_{12} $$8x - 60 = 9x - 54$$

Transposing $$8x - 9x = 60 - 54$$

Simplifying $$-x = 6$$

D_{-1} $$x = -6$$

Check. Does $\frac{2}{3}(-6) - 5 = \frac{3}{4}(-6) - \frac{9}{2}$?

Does $-4 - 5 = -\frac{9}{2} - \frac{9}{2}$? Yes.

EXERCISES [A]

1. $\frac{2}{3}x - 2 = 10$

2. $\frac{3}{5}y - 1 = \frac{3}{4}$

3. $\dfrac{4x}{3} - 1 = \dfrac{1}{6}$

4. $\dfrac{7a}{8} + 1 = \dfrac{1}{2}$

5. $.4x + 5 = 9$

6. $p + .05p = 31.5$

7. $8.2r + 6 = 3.5r + 3.6$

8. $.09c + 11 = 10.8$

9. $\frac{3}{5}x + 3 = \frac{2}{5}x + 5$

10. $9 + \frac{1}{10}x = \frac{1}{5}x - 6$

11. $\dfrac{y}{3} + \dfrac{y}{2} = 10$

12. $\dfrac{m}{5} + \dfrac{m}{3} = 24$

13. $.4x + .5x - 3.2 = 0$

14. $.03x - 2.8 + .04x = 0$

15. $1.8y = 7.5 - 2.17y$

16. $4.3m - .68 - .9m = 0$

Example 6. If $3x - 2y = 18$ and $y = 3$, find the value of x.

Solution. $$3x - 2y = 18$$

If $y = 3$, then $$3x - 2(3) = 18$$

$$3x - 6 = 18$$

$$3x = 18 + 6$$

$$3x = 24$$

$$x = 8$$

EXERCISES

A

1. If $2x - y = 9$ and $x = 5$, find the value of y.
2. If $4x - 3y = 7$ and $x = 1$, find the value of y.
3. If $a + b = 8$ and $a = -8$, what is the value of b?
4. If $3x + 2y = 13$ and $y = 0$, find the value of x.
5. Solve $5x - 3y = 25$ for x when $y = -5$.
6. Solve $x - 2y = 6$ for x when $y = 0$.

B

Solve:

7. $.6x - 12 = 1.6x$
8. $.09c - 11 = 10.8$
9. $4.3m + .68 = .9m$
10. $p + .12p = 560$
11. $r + .06 = 5625$
12. $4.48c - 8 - 2.1c = 39.6$
13. $3.1r + 9.8 = 6.3r + 3.8r$
14. $3.5k + 14 = 2.1k + 60.2$
15. $c - .09c + 200 = 455.2$
16. $8x + 8.45 = -7.55$

Solving Problems by Formulas [A]

The formula $A = bh$ can be used to find the area of a rectangle or parallelogram when the base and height are known, to find the height when the area and base are known, and to find the base when the area and height are known.

Example 1. The area of a floor is 210 square feet and the base is 12 feet. What is the height?

Solution 1. $A = bh$. Substituting 210 for A and 12 for b in the formula, we have,

$$210 = 12h$$

$$-12h = -210$$

D_{-12} $$h = 17.5$$

The height is 17.5 feet.

Solution 2. Since we are to find h, we may turn the formula around.

Then $$12h = 210$$

D_{12} $$h = 17.5$$

The height is 17.5 feet.

The formula $i = prt$ states how to find the interest when the principal, rate, and time are known. It can also be used to find the principal, rate, or time.

Example 2. What principal will produce \$45 interest in 3 years at 5%?

Solution.

$$
\begin{aligned}
& & i &= prt \\
& & 45 &= p \times \tfrac{5}{100} \times 3 \\
M_{100} & & 4500 &= 15\,p \\
& & -15\,p &= -4500 \\
& & p &= 300.
\end{aligned}
$$

The principal is \$300.

To Solve a Problem by a Formula:

1. Write the formula.
2. Substitute the values for the known letters.
3. Solve the resulting equation for the remaining letter.

EXERCISES

A

Using the formula $A = lw$ or $lw = A$, find the value l when

1. $A = 320, w = 16$

2. $A = 576, w = 24$

3. $A = 324, w = 12$

4. $A = 242.54, w = 13.4$

5. $A = 453.75, w = 12.5$

6. $A = 42.78, w = 4.6$

Using the same formula, find w when

7. $A = 136, l = 8$

8. $A = 225, l = 15$

9. $A = 70.98, l = 15.1$

10. $A = 19.44, l = 5.4$

Using the formula $A = \frac{1}{2}\,bh$, find the value of the unknown letter:

11. $b = 8, h = 22$

12. $A = 324, b = 36$

13. $h = 14, A = 182$

14. $A = 750, h = 6$

15. $A = 1008, h = 56$

16. $b = 3\frac{1}{3}, A = 11\frac{1}{4}$

Using the formula $V = lwh$, find the value of the unknown letter:

17. $l = 18,\ w = 8,\ h = 6$

18. $V = 60,\ l = 15,\ w = 4$

19. $V = 567,\ h = 7,\ w = 9$

20. $V = 175,\ l = 5,\ h = 7$

21. $V = 147.56,\ w = 7,\ h = 3.4$

22. $l = 10\frac{1}{2},\ w = 2\frac{1}{4},\ h = 3\frac{1}{2}$

Using the formula $i = prt$, find the value of the unknown:

23. $p = \$500,\ r = 6\%,\ t = 3$ yr.

24. $i = \$90,\ r = 5\%,\ t = 3$ yr.

25. $i = \$150,\ r = 4\%,\ t = 5$ yr.

26. $p = \$350,\ r = 3\%,\ i = \$21$

27. $i = \$63.75,\ r = 2\frac{1}{2}\%,\ t = 6$ yr.

28. $p = 452,\ r = 4\frac{1}{2}\%,\ i = \$50.85$

29. $i = \$60,\ p = \$600,\ t = 5$ yr.

30. $i = \$51,\ p = \$850,\ t = 2$ yr.

Solve each of the following formulas and equations:

31. $2x + 3y = 13$ when $x = 2$.

32. $3y = 13 - 2x$ when $x = 6$.

33. $3x - y = 7$ when $x = 5$.

34. $2x - 3y = 12$ when $x = 0$.

35. $4x - 5y = 4$ when $x = -4$.

36. $V = abc$ when $V = 2730$, $a = 13$, and $b = 14$.

37. $A = p + prt$ when $p = 150$, $r = 4\%$, and $t = 3\frac{1}{2}$.

38. $S = \dfrac{v^2}{2g}$ when $v = 40$ and $g = 32.2$.

B

Solve:

39. $T = t + 273$ when $t = -80$.

40. $T = t + 273$ when $t = 140$.

41. $F = \frac{9}{5}C + 32$ when $C = -10$.

42. $F = \frac{9}{5}C + 32$ when $F = 72$.

43. $A = \dfrac{h(b + b')}{2}$ if $b = 8$, $b' = 18$, and $h = 14$.

44. $V = \pi r^2 h$ is a formula for finding the volume of a cylinder. Evaluate the formula when $r = 6$, $h = 14$, and $\pi = 3\frac{1}{7}$.

45. $K = \frac{1}{2} mv^2$ is a formula used in physics. Evaluate it when $K = 306.3125$ and $v = 6.5$.

C

46. Solve $C = \frac{5}{9}(F - 32)$ for F when $C = 100$.

47. Solve $C = \frac{5}{9}(F - 32)$ for F when $C = -273$.

48. Evaluate $V = \frac{4}{3} \pi r^3$ when $r = 2$ and $\pi = 3.14$.

49. In the formula $A = p + prt$, A is the amount, p is the principal, r is the rate, and t is the time. Find the value of p when $A = \$306.25$, $r = 5\%$, and $t = 4\frac{1}{2}$ years.

Making Tables from Equations [A]

We shall now continue making tables from equations, as you did in Chapter II.

EXERCISES [A]

Copy and complete the following tables:

1. $x = 3y$

x	?	?	?	?	?	?	?	?	?
y	0	1	2	3	4	− 1	− 2	− 3	− 4

a. As the values of y increase, what can you say about the values of x?

b. How does x change when the values of y decrease?

2. $x + y = 10$

x	?	?	?	?	0	2	4	6
y	0	2	4	6	?	?	?	?

Do x and y increase together?

3. $x - 2y = 10$

x	?	?	?	?	?	?	?	?
y	− 4	− 2	0	2	4	6	8	10

When y increases does x increase or decrease?

4. $xy = 16$

x	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
y	?	?	?	?	?	?	?	?

When x decreases, how does y change?

5. When two numbers, x and y, change together and their quotient $\left(\frac{x}{y}\right)$ always remains the same, the two numbers *vary directly;* when two numbers change together and their product (xy) always remains the same, the numbers *vary inversely.*

a. How do x and y vary in the equation $x = 3y$?

b. How do m and n vary in the equation $mn = 16$?

c. How do x and y vary in the equation $x - 2y = 0$?

Making Equations from Tables [B]

In the last set of exercises you made tables which contained sets of values satisfying equations. We shall now reverse the process and make the equations whose solutions are given in the tables.

Example. Write the equation which expresses the relation between x and y as shown in the following table:

x	1	3	5	7	9	11	13	15
y	0	1	2	3	4	5	6	7

Solution. By inspection, $x = 1$ when $y = 0$. This fact tells us that x equals 1, plus some term containing y. From the table we know that x is increased by 2 when y is increased by 1. In other words, x increases twice as fast as y. From these two facts the equation $x = 1 + 2y$ is suggested. Then we check the equation $x = 1 + 2y$ with each set of values of x and y given in the table. On doing so we find that $x = 1 + 2y$ is the required equation.

EXERCISES

B

Write equations which express the relation between the letters in the following tables, and complete the tables:

1.

y	0	2	4	6	8	10	12	14
x	0	1	2	3	4	?	?	?

2.

y	0	1	2	3	4	5	6	?
x	0	3	6	9	12	15	?	21

3.

m	7	8	9	10	11	?	17	21
n	0	1	2	3	4	5	?	?

4.

h	2	5	8	11	14	17	?	26
k	0	1	2	3	4	5	6	?

5.

a	10	9	8	7	6	5	?	?
b	0	1	2	3	4	5	7	12

SUGGESTIONS. What is the value of a when $b = 0$? Does a increase or decrease when b increases? Does a decrease as fast as b increases?

6.

x	18	15	12	9	6	?	?	?
y	0	1	2	3	4	5	6	7

7.

p	infinitely large	36	18	12	9	?	?
v	0	1	2	3	4	6	9

SUGGESTION. What is the product of p and v?

8.

s	0	4	16	36	64	100	?	?
t	0	1	2	3	4	5	6	7

SUGGESTIONS. Note that when either *s* or *t* is zero, the other is zero also. As *t* is increased by one, is *s* increased by the same amount each time? Divide the different values of *s* by their greatest common divisor. What relation do these quotients bear to the corresponding values of *t*?

C

9.

n	3	5	10	11	20	25	30	35
m	15	21	36	39	66	?	?	?

SUGGESTIONS. Observe that as *n* changes from 3 to 5, *m* changes from 15 to 21; that is, an increase of 2 in the value of *n* causes an increase of 6 in the value of *m*. Again, observe that as *n* changes from 5 to 10, *m* changes from 21 to 36; that is, an increase of 5 in the value of *n* causes an increase of 15 in the value of *m*. In both cases *m* seems to increase 3 times as fast as *n*. If this is true, then a decrease of 3 in the value of *n* will cause a decrease of 9 in the value of *m*; that is, if *n* decreases from 3 to 0, *m* decreases from 15 to 6. Therefore the formula is $m = 6 + 3n$. Check this formula with the other sets of values and, if true, complete the table.

10.

d	3	5	8	9	10	12	20	24
c	14	22	34	38	?	?	?	?

11.

y	2	3	4	?	7	?	10	12
x	3	?	13	18	28	33	43	?

12. Some pupils using a protractor measured the angles of polygons having 3, 4, 5, 6, and 7 sides. Then they made the following table:

Number of sides (n)	3	4	5	6	7
Sum of the angles (S)	180°	360°	540°	720°	900°

Write the formula for finding the sum of the angles of any polygon in terms of the number of sides.

Algebraic Representation[A]

In order to solve problems algebraically, it is necessary to express number relations by the use of symbols.

EXERCISES[A]

1. The sum of two numbers is 30. If x stands for one of the numbers, represent the other in terms of x.

2. If m represents one number, what will represent a number 4 times as large? What will represent a number 4 smaller than m? What will represent a number 4 larger than m? What will represent a number $\frac{1}{4}$ as large as m?

3. Indicate: the sum of 5 and 7; the sum of x and y; the product of x and y.

4. The difference of two numbers is 8. If x stands for the smaller number, what will represent the larger?

5. What is the cost of x pounds of sugar at 8 cents a pound? of $2x$ pounds of raisins at 27 cents a pound?

6. How many cents are there in 3 dimes? in x dimes? in $2x$ dimes?

7. How many inches are there in 5 feet? in y feet? in $3c$ feet?

8. How many ounces are there in 4 pounds? in x pounds? in $5p$ pounds?

9. How many oranges at 5 cents each can be bought for 20 cents? for $10x$ cents? for $30y$ cents? for $5xy$ cents?

10. The product of two numbers is $8x$ and one of them is 4. What is the other number?

11. The sum of two numbers is 10 and one of them is x. What is the other number?

12. The quotient of two numbers is $3c$ and the divisor is $4c$. What is the dividend?

13. The difference of two numbers is 6 and the smaller is $3x$. What is the larger?

14. Find 6% of 1020; 4% of 80; 100% of 32; 200% of 35; 4% of x; 10% of y.

15. Find x% of 80; y% of 25; r% of s; p% of m.

16. A rope is x yards long. If 6 yards are cut off, how many yards are left?

17. What is the perimeter of a rectangle x feet wide and $2x$ feet long? What is the area of the rectangle?

18. What is the area of a rectangle $6a^2$ feet long and $3a$ feet wide? What is its perimeter?

19. The length of a rectangle is 6 feet more than its width. If x represents the width, express the length in terms of x.

20. A train travels 60 miles an hour. How far will it travel in 3 hours? in 4 hours? in x hours? in y^2 hours?

21. If an automobile travels 30 miles an hour, how long will it take to go 90 miles? 500 miles? x miles? $120x$ miles?

22. Find the cost of 72 cookies at 10 cents a dozen; of x cookies at y cents a dozen; of $12b$ cookies at 8 cents a dozen.

23. How many pints are there in 8 quarts? in 12 quarts? in k quarts? in $2c$ quarts?

24. If n stands for one side of an equilateral triangle, what will represent its perimeter?

25. If 2 numbers are in the ratio of 2 : 3 and $2x$ represents the smaller number, then $3x$ represents the larger. If two numbers are in the ratio of 3 : 5 and $3x$ stands for the smaller number, what will represent the larger?

26. A boy is x years old. How old will he be in 6 years? in 10 years? in y years?

27. A girl is y years old. How old was she 5 years ago? 8 years ago? x years ago?

28. Letting x stand for the unknown number, write in algebraic language

a. A number increased by 7 equals 28.

b. Twice a number plus the number equals 81.

c. One third of a number, less 8, equals 13.

d. Seven times a number equals the number increased by 30.

Problems

Before you attempt to solve any of these problems, you should review the directions for solving problems on page 23.

EXERCISES

A

1. The sum of a certain number and 6 is 19. Find the number.

2. A certain number is equal to 25 diminished by 13. What is the number?

3. 42 is equal to a number increased by 14. What is the number?

4. The sum of a number and -5 is 4. Find the number.

5. Four times a certain number is equal to 96 increased by the number. Find the number.

6. If 15 is subtracted from 4 times a certain number, the remainder is 17. Find the number.

7. If 6 is subtracted from a certain number, the difference is 36. What is the number?

8. If 20 is added to 3 times a certain number, the sum is 101. Find the number.

9. The sum of two numbers is 60 and one is 3 times the other. Find the numbers.

10. The sum of two numbers is 63 and one of them is 27 larger than the other. Find the numbers.

Let $n =$ the smaller number

Then $n + 27 =$ the larger number

and $n + n + 27 =$ --?--. Complete the equation and solve.

11. The sum of two numbers is 44 and one of the numbers is 18 larger than the other. What are the numbers?

12. Four times a certain number, decreased by 5, equals 25 decreased by 6 times the number. Find the number.

13. A man paid \$2605 for two automobiles, paying \$255 more for one than the other. What was the cost of each?

14. One number is 3 more than twice the other. What are the numbers if their sum is 18?

15. A man has 360 feet of fencing and wishes to enclose a rectangular lot whose length is 20 feet more than 3 times the width. What are the dimensions of the lot?

16. The number of boys in an algebra class is one less than 3 times the number of girls. If there are 39 pupils in the class, how many of them are girls?

B

17. The sum of two consecutive integral numbers is 37. What are the numbers?

SUGGESTION. 1, 2, 3, 4, 5 etc., are integers.

Let $x =$ the smaller number

Then $x + 1 =$ the larger number

18. The sum of three consecutive integral numbers is 117. What are the numbers?

19. The sum of three consecutive integers is 345. What are the integers?

20. The sum of three consecutive integers is 21 more than the smallest integer. Find the three integers.

21. The sum of two consecutive odd integers is 92. What are they?

SUGGESTION. The odd integers are 1, 3, 5, 7, etc.

Let $x =$ the smaller integer

Then $x + 2 =$ the larger integer

22. Find three consecutive odd integers whose sum is 51.

23. The sum of three consecutive even integers is 60. Find the integers.

24. The sum of three consecutive even integers is 50 more than the smallest one. Find the integers.

PERCENTAGE PROBLEMS[A]

A per cent of a given number is called the *percentage*. The number of which the per cent is taken is the *base*. The per cent is the *rate*. The expression "per cent" (abbreviated %) means hundredths. Thus 6% means .06 or $\frac{6}{100}$. When solving percentage problems you should always keep in mind that a common way of expressing a fraction whose denominator is 100 is as a number of per cent.

When we say "14 is 25% of 56" we are comparing the two numbers 14 and 56, and we mean that the first number is $\frac{25}{100}$ of the second. In this example, 14 is the *percentage*, 56 is the *base*, and $\frac{25}{100}$ or .25 or 25% is the *rate*. Since the word "*of*" *means times*, we have

$$14 = .25 \times 56$$

In general terms,

$$\text{percentage} = \text{rate} \times \text{base}$$

or

$$P = rb$$

This formula is called the percentage formula and is usually written in the form $P = br$.

Percentage problems are of three kinds, and most people who try to do them by arithmetic become confused and do not know what operations are to be performed with the numbers. With the use of algebra the three kinds of percentage problems are easy. All we need to do is use the formula $P = br$, substitute the known values, and then find the value of the remaining letter.

The three kinds of percentage problems are as follows:

1. Finding a per cent of a number. For example:

What is 3% of 120?

2. Finding what per cent one number is of another. For example:

26 is what per cent of 65?

3. Finding a number when a per cent of it is known. For example:

169 is 65% of what number?

We shall now solve each of the above examples.

Example 1. What is 3% of 120?

Solution. Here the number 120 is called the base, 3% or .03 is the rate, and the number to be found is the percentage.

$$P = br$$
$$P = 120 \times .03$$
$$P = 3.6$$

Then 3.6 is 3% of 120.

Example 2. 26 is what per cent of 65?

Solution. Here the percentage is 26, the base is 65, and the number to be found is the rate.

$$P = br$$
$$26 = 65\,r$$
$$-65\,r = -26$$

D_{-65} $$r = .4$$

.4 expressed as hundredths is .40 or $\frac{40}{100}$. Therefore the rate is 40%.

Example 3. 169 is 65% of what number?

Solution. Here 169 is the percentage, .65 is the rate, and the number to be found is the base.

$$P = br$$
$$169 = b \times .65$$

or $$169 = .65\,b$$

M_{100} $$16900 = 65\,b$$
$$-65\,b = -16900$$

D_{-65} $$b = 260$$

The number is 260.

EXERCISES

A

1. Find the following numbers:

a. 4% of \$180

b. 3% of \$250

c. 7% of \$900

d. 16% of \$225

e. 40% of 75

f. 300% of 18

g. 250% of 50

h. 200% of 70

i. 500% of 1

j. 18% of 18

k. 25% of 100

l. 700% of 7

2. Find the following per cents or rates:

a. 8 is what % of 16?
b. 24 is what % of 40?
c. 18 is what % of 72?
d. 72 is what % of 18?
e. 16 is what % of 80?
f. What % of 60 is 48?
g. What % of 6 is 12?
h. What % of 100 is 33?
i. What % of 250 is 50?
j. What % of 6 is 4800?

3. Find the following:

a. 15 is 40% of what number?
b. 40% of a certain number is 28. What is the number?
c. 125% of a number is 40. Find the number.
d. 84 is 400% of what number?
e. 8% of a number is 1640. Find the number.
f. 60 is 15% of what number?
g. 11 is 11% of what number?
h. 40% of a number is 4. What is the number?

4. Find $3\frac{1}{2}\%$ of 720. ($3\frac{1}{2}\% = .035 = \frac{7}{200}$.)

5. Find $37\frac{1}{2}\%$ of 124.

SUGGESTION. Write $37\frac{1}{2}\%$ as $\frac{37\frac{1}{2}}{100}$ and reduce to $\frac{3}{8}$.

6. What is $12\frac{1}{2}\%$ of 96?

7. What is $66\frac{2}{3}\%$ of 450?

8. Find $8\frac{1}{3}\%$ of 48.

9. What is $a\%$ of b?

10. What per cent of 90 is 20?

11. $292.50 is what % of $900?

12. What % of 850 is 63.75?

13. 32% of a number is 9.6. What is the number?

14. 42% of a number is 106.26. Find the number.

15. 43.75 is 125% of another number. Find the second number.

16. $12\frac{1}{2}\%$ of a man's monthly income is spent for rent. Find his income each month if his rent is \$48 a month.

17. A store had a 40% discount sale. What was the former price of a davenport which sold at the sale for \$117?

B

18. What is $\frac{1}{2}\%$ of 250?

SUGGESTION. $\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{200}$. Also, $\frac{1}{2}\% = .005$.

19. Find $\frac{1}{8}\%$ of 96.

20. What is .1% of 2000?

21. What is $\frac{3}{4}\%$ of 444?

22. $\frac{1}{2}$ is what % of $\frac{3}{4}$?

23. .02 is what % of .08?

24. A man received \$18 interest on an investment of \$400. What rate did he receive?

25. A suit of clothing that was marked to sell for \$60 was sold for \$52. The actual selling price was what per cent of the marked price?

26. A chair was marked to sell for \$55. If a discount of \$10 was allowed for cash, the cash discount was what per cent of the marked price?

27. A farmer took clover seed containing 1.5% plantain seed to be "cleaned," or have the plantain seed removed. If 9.6 pounds of plantain seed were removed, how much seed did the farmer have originally?

28. How much money must a man invest at 4% a year to produce an annual income of \$600?

29. How many \$1000 bonds paying 5% are needed to produce an income of \$600 a year?

C

Example. 10% of a number increased by 24% of the same number equals 142.8. What is the number?

Solution. Let $x = \text{the number}$

Then $.10\,x + .24\,x = 142.8$

$.34\,x = 142.8$

M_{100} $34\,x = 14280$

D_{34} $x = 420$

The number is 420.

30. 8% of a number plus 12% of the number is 62. Find the number.

31. 14% of a number plus 12% of the number, less 8% of the number, is 380. Find the number.

32. A number decreased by 10% of itself equals 405. What is the number?

33. A number decreased by 8% of itself is 322. Find the number.

34. A number increased by 16% of itself equals 185.6. What is the number?

35. A merchant sold goods for $2592 and made a gain of 8% of the cost. What was the cost of the goods?

36. How cheap can a grocer afford to sell berries that cost 12 cents a quart if he must make a profit of 20% based on the selling price?

37. A baseball team with a standing of .816 has won 84 games. How many games has it lost?

38. A man spent 30% of his annual salary for rent and 20% of the remainder for food. If he spent $500 each year for food, what was his annual salary?

39. An electric refrigerator sold for $105.30 after discounts of 10% and 10% were given. What was the marked price?

SUGGESTION. If p represents the marked price, the first discount is $.10\,p$ and the second discount is $.09\,p$. Why?

40. A radio was sold for $69.50 after discounts of 10% and 5% were allowed. What was the marked price of the radio?

WORD LIST

Can you spell each of the following words? Do you know the meaning of each one?

addition axiom (p. 116)
binomial (p. 13)
coefficient (p. 10)
cylinder (p. 59)
diameter (p. 54)
equation (p. 21)
evaluating formulas (p. 125)
equilateral (p. 31)
exponent (p. 12)
integer (p. 135)
integral (p. 135)
member of an equation (p. 21)
monomial (p. 13)
multiplication axiom (p. 119)
negative number (p. 74)
percentage (p. 136)
perimeter (p. 6)
polynomial (p. 13)
terms (p. 13)
transposing (p. 122)

CHAPTER REVIEW[A]

Solve the following equations and check:

1. $2x - 3 = 15$

2. $3x + 6 = -9$

3. $-4y + 1 = 0$

4. $c - 1 = 0$

5. $-3x - 5 = 9x$

6. $3.14c = 9.42$

7. $\frac{1}{3}n = 5$

8. $\frac{p}{2} = 3$

9. $y - 48 = 2.6y$

10. $-x + 1 = x - 3$

11. $5c - 6 = c + 2$

12. $3h - 1 = 5h - 7$

13. $3p - 10 = 7p - 150$

14. $2x + 7 = 3 + x$

15. $2c + 10 = c - 2$

16. $\frac{2p}{7} + 4 = \frac{p}{3}$

17. $\frac{5}{6}y - 1 = \frac{1}{4}y + \frac{1}{2}y$

18. $x - 8 = -6 - 2x + 1$

19. If $x - 2y = 7$, find y when $x = 3$.

20. If $2x - y = 5$, find y when $x = 0$.

21. If $7x + 2y = 11$, find y when $x = -1$.

22. If $5x + 3y = 12$, find y when $x = -3$.

Write, using algebraic language:

23. The sum of x and -6.

24. 10 subtracted from y.

25. a increased by twice b.

26. The product of m and n.

27. 4 less than c.

28. 2 less x.

Let x stand for the numbers in exercises 29–32.

29. Twice a number plus three times the number equals 30.

30. A number decreased by 8 equals 22.

31. 8 increased by twice a number equals 40.

32. One fourth of a number is 16.

33. How many cents are there in 4 nickels? in b nickels? in $3x$ nickels?

34. How many cents are there in 2 quarters? in y quarters? in $4x$ quarters?

35. What is the cost of $4x$ books at $2x$ cents each?

36. How many feet are there in $24x$ inches? in $6c$ inches?

37. If b bananas cost c cents, what is the cost of n bananas? cn/b

38. Separate 106 into 2 parts so that one part will be 16 more than the other.

39. Separate 64 into 2 parts such that one part will be $\frac{7}{9}$ of the other.

40. Write the rules expressed by the following formulas:

a. $P = br$

b. $i = prt$

c. $V = lwh$

d. $A = bh$

e. $A = \frac{1}{2}bh$

f. $C = 2\pi r$

41. 15 is what per cent of 75?

42. 42 is $37\frac{1}{2}\%$ of what number?

43. 16 is what per cent of 320?

44. 32 is 8% of what number?

45. How much money must be invested at 4% to earn \$100 a year?

46. $V = lwh$. Find w, when $V = 1080$, $l = 12$, $h = 9$.

47. $i = prt$. Find p, when $i = 43.20$, $r = .04$, $t = 3$.

48. $A = \frac{1}{2}bh$. Find h when $A = 38$ and $b = 10$.

49. $C = 2\pi r$. Find r when $C = 18.84$ and $\pi = 3.14$.

50. $x - 2y = 2$. Copy and complete the table.

x	− 4	− 2	0	2	4	6	8
y	?	?	?	?	?	?	?

GENERAL REVIEW

A

On your paper write one word, number, or letter, and only one, for each blank to make the following ten statements true:

1. In algebra figures and --?-- are used to represent numbers.

2. xy means x --?-- y.

3. The figure 3 in the expression $4x^3$ is called an --?--.

4. The sign = means --?--.

5. The --?-- sign is omitted in the expression ab.

6. There are three --?-- in the expression $2x^2 - 5x + 1$.

7. The coefficient of x is --?--.

8. $2m$ is a short way of writing $m +$ --?--.

9. The coefficient of b^3 in the expression $7b^3$ is --?--.

10. In the expression $x^3 + 4x - 2x^3 + 8$, x^3 and 8 are --?-- terms.

11. If n stands for a certain number, tell what each of the following expressions means:

a. $n + 2$ **b.** $2n$ **c.** n^2 **d.** $\frac{n}{3}$

12. In what two operations in algebra do we change signs of terms?

13. Why should one make a sketch when it is possible to do so in solving a verbal problem?

14. If $2y - x = 5$, find y when $x = -7$.

If $x = 2$ and $y = -3$, find the values of:

15. $3xy$	**18.** x^2	**21.** $(xy)^2$
16. $-xy$	**19.** $2y^3$	**22.** $xy - 10$
17. $10 + y$	**20.** $-y^3$	**23.** $10 - xy$

24. Divide:

a	b	c	d	e	f
$\frac{x^2}{x}$	$\frac{y^3}{y}$	$\frac{4xy}{y}$	$\frac{8c}{-4}$	$\frac{ab}{-a}$	$\frac{-9k}{k}$

25. Give an example of each of the following:

a. Monomial.

b. Binomial.

c. Trinomial.

d. Binomial with like terms.

e. Binomial with unlike terms.

f. Monomial with exponent 4.

26. Find the area of a triangle whose base is 10.3 and whose altitude is 5.2.

27. Find the radius of a circular 1-mile race track. (Use $\pi = 3.14$.)

28. Find the missing values of x and y which satisfy the equation $3x - y = 6$.

x	?	?	?	− 3	− 1	0	?	?
y	0	− 1	3	?	?	?	− 3	− 12

29. The sum of two numbers is 119 and their difference is 9. Find the numbers.

30. A farmer divided his farm of 257 acres among his three sons so that the eldest received 13 more acres than the youngest, and the third son received 6 acres less than the eldest. How many acres did each son receive?

31. If a sum of money is increased by \$21, the result is the same as when 5 times the sum is subtracted from \$177. What is the original sum of money?

32. What principal will produce \$28.80 interest in 3 years, at 4% per year?

B

33. Combine: $x^3 - 3x^2y - 2x^2y + 3xy^2 - 5y^3 - 4xy^2 + 5x^2y - 6y^3 - 7x^3$.

Find the numerical value of each expression when $a = \frac{3}{4}$, $b = \frac{1}{2}$, $c = \frac{2}{3}$, and $x = 3$.

34. $\dfrac{a-b}{a+b} + \dfrac{a+b}{a-b}$

35. $\dfrac{12a - 10b + 12c}{4a - 8b + 6c}$

36. $\left(\dfrac{1}{b} - \dfrac{1}{a}\right)\left(\dfrac{1}{c} + \dfrac{1}{x}\right)$

37. $\dfrac{abc}{x} - c^3$

38. The sum of three numbers is 96. The first is $\frac{1}{5}$ of the second and $\frac{1}{2}$ of the third. What are the numbers?

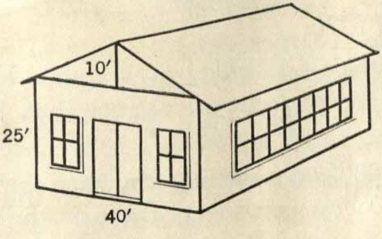
39. Distribute $174 among three people so that the first may receive 5 times as much as the second, and $42 more than the third.

40. Divide 155 marbles among three boys so that one will receive 5 times as many as the second, and $\frac{1}{5}$ as many as the third.

41. $F = .004\,Av^2$ is a formula for finding the force of a wind that blows against any flat surface. F is the number of pounds in the force, A is the number of square feet in the surface, and v is the velocity of the wind in miles an hour.

If a wind blows at 40 miles an hour directly against a flat sail with an area of 120 square feet, what is the force of the wind against the sail?

42. How much force would be exerted against this building by a wind blowing 50 miles an hour directly against the end? (See Exercise 41.)



CHAPTER TEST [A]

Part I. Equations (60%)

Solve:

1. $-3c = 21$
2. $4y = 12$
3. $0.5c = -15$
4. $7y - 3 = -17$
5. $6x + 5 = 23$
6. $8y - 8 = 0$
7. $4y = 2 + 2y$
8. $\frac{2}{3}c = 24$
9. $0.03h = 0.12$
10. $3x + 4 = x - 10$

11. Show that 3 is a root of $7x - 5 = x + 13$.

12. $x + 2y = 10$. Find the value of x when $y = 4$.

13. $A = lw$. Find l when $A = 144$ and $w = 9$.

14. $i = prt$. Find i if $p = \$600$, $r = 4\%$, and $t = 2\frac{1}{2}$ years.

15. Copy and complete the following table for $x = 4y - 2$:

x	?	?	?	?	?	?
y	0	1	2	3	−1	−2

Part II. Problems (40%)

1. The sum of two numbers is 18 and one of them is x. What is the other one?

2. Find $x\%$ of \$150.

3. The difference between two numbers is 5. If x represents the smaller number, what will represent the larger?

4. A number increased by 32 equals 75. Find the number.

5. If 7 is subtracted from twice a certain number, the remainder is 23. Find the number.

6. Find 4% of \$270.

7. The attendance on Friday at Fremont High School was 782. What was the per cent attendance that day if the enrollment was 850?

8. June made 80% on a spelling test when she spelled 48 words correctly. How many words were there on the test?

CHAPTER TEST[B]

Part I. Equations (60%)

Solve:

1. $7c + 7 = 35$

2. $-8c = 32$

3. $0.4x = -15$

4. $-2x + 6 = 0$

5. $7y - 9 = y + 7$

6. $11x - 5 = x - 32$

7. $0 = 8x + 40$

8. $10 = 6x - 4 + 2x$

9. $-7p = -2p - 20$

10. $\frac{x}{5} + \frac{x}{3} = -64$

11. $7c - 4d = 15$. Find d if $c = 3$.

12. $i = prt$. Find r if $p = \$350$, $i = \$31.50$, and $t = 2$ years.

13. $T = t + 273$. Find T when $t = -100$.

14. Show that -2 is not a root of the equation $4x + 5 = x + 1$.

15. $3x - 2y = 15$. Find the value of y when $x = -3$.

Part II. Problems (40%)

1. The sum of two numbers is 26 and one of them is n. What is the other?

2. The larger of two numbers is 13. Find the smaller if their difference is d.

3. There are 30 boys and girls in Miss Kendrick's algebra class and there are two-thirds as many boys as girls. How many girls are there in the class?

4. The sum of three consecutive integers is 105. Find them.

5. A family spends \$30 a month for rent, which is 12% of its monthly income. What is its income per month?

6. Dale wishes to make a picture frame using 65 inches of picture moulding. In order that the frame may have the right shape, the width of the frame will be 0.62 of the length. How long shall each of the four sides of the frame be?

7. An article priced at \$18 is sold at a discount of 20%. Find the selling price after the discount is allowed.

ARCHITECTURAL DRAFTING

When we admire a beautiful building such as a church, a large public building, a residence, a skyscraper, or a factory building, we usually do not realize the vast amount of planning, calculating, drawing, and detailed work that had to be done before the actual construction of the building was begun.

The architect supervises the drawing of the plans and the construction of the building. He must have a broad knowledge of construction, heating, lighting, acoustics, interior decorating, etc. He is assisted in preparing the plans by draftsmen, and the details of construction are worked out by the structural engineer, the heating engineer, the electrician, and others.

You may begin the study of mechanical drawing without a knowledge of algebra and geometry. If you wish to make a further study of drafting and take up architectural drafting or machine drafting, you must have a knowledge of algebra, geometry, and trigonometry. To become an architect you must have a broad education. Vitruvius, the great Roman architect, said of an architect,

> *Let him be skillful with a pencil, instructed in mathematics, history, literature, and philosophy, understand music, have some knowledge of medicine, know the opinion of jurists, and be acquainted with astronomy and the theory of the heavens.*

A Student in an Architectural Drafting Class

CHAPTER V

Operations with Polynomials

Adding Polynomials [A]

In this chapter you will learn how to add and subtract polynomials and how to multiply and divide them by monomials.

Polynomials may be added by writing them one after another and then combining like terms; but an easier method of adding is to write them so that like terms are in columns, and then add the columns. We shall use the latter method.

Example 1. Add 2 yards + 1 foot + 3 inches and 4 yards + 1 foot + 5 inches.

Solution.

$$\begin{array}{l} 2 \text{ yd.} + 1 \text{ ft.} + 3 \text{ in.} \\ \underline{4 \text{ yd.} + 1 \text{ ft.} + 5 \text{ in.}} \\ 6 \text{ yd.} + 2 \text{ ft.} + 8 \text{ in.} \end{array}$$

The sum is 6 yards + 2 feet + 8 inches.

Additions may be checked by adding the columns in reverse order. For example, if the columns are first added from top to bottom, the solutions may be checked by adding the columns from bottom to top.

A more common method of checking additions is that of substitution, which will be used in checking the following example.

Example 2. Add: $4a + 3b - 4c$; $5a + 7c - 8b$; and $-6a - b + 9c$. Check by letting $a = 1$, $b = 2$, and $c = 3$.

Solution **Check**

$$\begin{array}{rcl} 4a + 3b - 4c & = & 4 + 6 - 12 = -2 \\ 5a - 8b + 7c & = & 5 - 16 + 21 = 10 \\ \underline{-6a - b + 9c} & = & -6 - 2 + 27 = 19 \\ 3a - 6b + 12c & = & 3 - 12 + 36 = 27 \end{array} \qquad \left.\begin{array}{r} -2 \\ 10 \\ 19 \end{array}\right\} 27$$

For $a = 1$, $b = 2$, and $c = 3$, the value of the first polynomial is -2, the value of the second is 10, and the value of the third is 19. The sum of -2, 10, and 19 is 27. For these values of the letters, $3a - 6b + 12c$ equals 27. Therefore the solution checks.

The fact that a solution checks is not a conclusive proof that the solution is correct, as you can see from the following incorrect addition that checks for $x = 5$ and $y = 3$.

Check

$$\begin{array}{rcl} x + 3y & = & \left.\begin{array}{r} 5 + 9 = 14 \\ 10 - 3 = 7 \end{array}\right\} 21 \\ \underline{2x - y} & & \\ 6x - 3y & = & 30 - 9 = 21 \end{array}$$

Example 3. Find the sum of $3x^3 - 2y^3$, $x^2y + 2xy^2 + y^3$, and $-4x^3 - 7x^2y + 5xy^2 - 6y^3$. Check by letting $x = 2$ and $y = 3$.

Solution **Check**

$$\begin{array}{rrrrcl} 3x^3 & +0 & +0 & -2y^3 & = & \left.\begin{array}{r} 24 + 0 + 0 - 54 = -30 \\ 0 + 12 + 36 + 27 = 75 \\ -32 - 84 + 90 - 162 = -188 \end{array}\right\} -143 \\ & x^2y & +2xy^2 & +y^3 & = & \\ \underline{-4x^3} & \underline{-7x^2y} & \underline{+5xy^2} & \underline{-6y^3} & = & \\ -x^3 & -6x^2y & +7xy^2 & -7y^3 & = & -8 - 72 + 126 - 189 = -143 \end{array}$$

You can save time and have a more orderly solution if you arrange the polynomials in descending (or ascending) powers of one of the letters. In the Solution of Example 3 the terms are arranged in descending powers of x. Notice that the first column contains the third power of x, the second column contains the second power of x, the third column contains the first power of x, and the fourth column contains no x factor. It so happens that the polynomials were arranged in ascending powers of y. Can you give a reason why the zeros were written in the first polynomial?

The solutions of these three examples illustrate the following rule:

> **To add two or more polynomials, write them with like terms in the same column and add the columns.**

EXERCISES

A

Add the following polynomials, checking as many solutions as your teacher indicates.

1. $\begin{array}{r} 6 \text{ ft.} + 7 \text{ in.} \\ 3 \text{ ft.} + 5 \text{ in.} \\ \hline \end{array}$

2. $\begin{array}{r} 3 \text{ lb.} + 7 \text{ oz.} \\ 7 \text{ lb.} - 4 \text{ oz.} \\ \hline \end{array}$

3. $\begin{array}{rrr} 3x & +2y & -z \\ 5x & -4y & \\ \hline \end{array}$

4. $\begin{array}{rrr} -2a & +4b & -6c \\ 3a & -9b & +5c \\ \hline \end{array}$

5. $\begin{array}{rrr} 4c & +5b & -a \\ -3c & -5b & +7a \\ \hline \end{array}$

6. $\begin{array}{rr} 2m & +6 \\ -m & +10 \\ \hline \end{array}$

7. $\begin{array}{rrr} -3k^2 & +4k & -9 \\ 2k^2 & -5k & +3 \\ -k^2 & -10k & +1 \\ \hline \end{array}$

8. $\begin{array}{rrr} x^2 & -4x & +3 \\ -3x^2 & +5x & -2 \\ x^2 & -2x & -5 \\ \hline \end{array}$

9. $\begin{array}{rrr} 2a & -3b & +7c \\ -4a & -9b & -8c \\ -7a & -b & +c \\ \hline \end{array}$

10. $\begin{array}{rrr} -m & -n & +p \\ m & -n & -p \\ m & +2n & -p \\ \hline \end{array}$

11. $\begin{array}{rrr} x^2 & -xy & +y^2 \\ -3x^2 & -7xy & -4y^2 \\ 2x^2 & -3xy & -8y^2 \\ \hline \end{array}$

12. $\begin{array}{rrr} 3m^2 & -m & -6 \\ m^2 & & -4 \\ & 4m & -1 \\ \hline \end{array}$

13. $\begin{array}{rrr} 4ax & -3ab & -6 \\ -2ax & -5ab & -10 \\ ax & +8ab & -1 \\ \hline \end{array}$

14. $\begin{array}{rrr} x^4 & -ax^3 & \\ & -2ax^3 & -bx^2 \\ & ax^3 & +4bx^2 \\ \hline \end{array}$

15. $\begin{array}{rrr} x^3 & -3x^2 & -2 \\ -4x^3 & -x^2 & +1 \\ x^3 & & -10 \\ \hline \end{array}$

Add:

16. $2x + 4y - 12$, $8x - 5y - 2$, and $-9x - 8y - 1$.

17. $7a - 9b + c$, $-5a + c$, and $3a + 8b - 2c$.

18. $-5x^2 - 3x + 2$, $-3x^2 + 6$, and $-4x^2 - 7x$.

19. $x - y - z$, $2x - 3y - 7z$, and $2z - 5x + y$.

20. $x^2 - 6 + 7x$, $7x - 4 + x^2$, and $2x^2 - x - 1$.

21. $a^2 - b^2 + ab$, $b^2 - a^2 - ab$, and $4ab + b^2$.

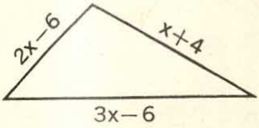
22. $3xy - 7z$, $5xy - 4z$, $-7xy + z$, and $-xy - 10z$.

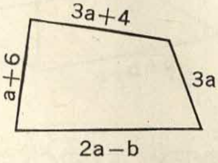
23. $5a + b + 4c$, $-3a - 6b$, and $4b - 6c + 5a$.

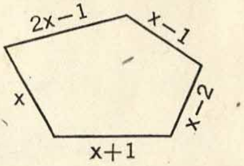
24. $2x^2 - 3xy + y^2$, $-8xy - 9y^2 - x^2$, and $-3x^2 - 4y^2$.

25. The length of a rectangle is $3a + 1$ and the width is $a - 6$. Find its perimeter.

26. The width of a rectangle is $x - 4$ and its length is $x^2 - x$. Find its perimeter.







27. Find the perimeter of the triangle above.

28. Find the perimeter of the quadrilateral (the figure with 4 sides).

29. Find the perimeter of the pentagon (the figure with 5 sides).

B

Add:

30. $13x + 4y - 12$, $18x - 5y - 2$, and $-9x - 8y - 4$.

31. $4x^4 - 7x^3 + 6x^2 - x + 1$ and $-x^4 + x^3 - 5x^2 + 6x - 9$.

32. $2a + 3b - 4c + 8d - 1$ and $-6a - 5b - 9d + 5$.

33. $-8x^3 + 5x^2 - 9x - 15$ and $-7x^3 + 5 + 6x - 3x^2 + 10$.

34. $x^4 - x^3y + x^2y^2$, $x^3y - x^2y^2 + xy^3$, and $x^2y^2 - xy^3 + y^4$.

35. $\frac{1}{3}x + \frac{1}{4}y - \frac{1}{5}z$ and $\frac{1}{2}x - \frac{3}{4}y + \frac{4}{5}z$.

36. $\frac{1}{2}a + \frac{1}{3}b - \frac{1}{4}c$ and $\frac{1}{3}a - \frac{1}{2}b - \frac{3}{4}c$.

37. $\frac{2}{3}a^2 - \frac{1}{4}a - 3$, $\frac{1}{2}a^2 - a + \frac{2}{3}$, and $\frac{3}{4}a - \frac{1}{6}a^2$.

38. $2.3x^2 - 3.2x - 4$, $x^2 + .7$, and $4.1x - 5.6 - 4.7x^2$.

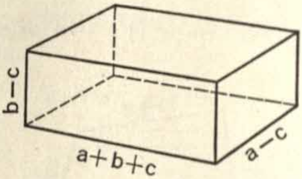
39. $.3x + .2y - .6z$ and $.3x - 1.4y + 15.6z$.

40. $3.15x - 1.60y + .70z$ and $1.25x - 3.40y - 1.00z$.

41. $(x^2 - 2xy + y^2) + (3x^2 - 8xy - y^2)$

42. $(a - b + c) + (2a + b - 3c) + (a + b - c)$

43. Find the sum of the lengths of the 12 edges of this rectangular solid.



REVIEW EXERCISES [A]

Simplify:

1. $9x - 3x + 5x$

2. $7m - 9m - m$

3. $-4 - 6 + 1$

4. $4xy - 8xy + 6xy$

5. $3m - 4m - m$

6. $2a^2 + 3a^2 - 5a^2$

7. $\frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}x$

8. $-4.3y - 2.7y$

9. $\frac{1}{3}c - \frac{1}{6}c - \frac{5}{6}c$

10. $h - \frac{1}{2}h - \frac{1}{3}h$

11. $x^2 - 4xy + y^2 - 2x^2 - y^2 - 10xy$

12. $abc - c^2 - 6abc - 10c^2$

Subtract:

	a	**b**	**c**	**d**	**e**	**f**	**g**
13.	-8 $+3$	10 -2	-6 -5	-1 -7	14 10	15 20	$+10$ -10
14.	$7h$ $-h$	$10c$ $3c$	$-4x$ $5x$	$3b$ $-5b$	0 $7x$	c $-5c$	$-9m$ m
15.	x $\frac{1}{2}x$	$3y$ $.5y$	$.1x$ $.4x$	$.55p$ $-.15p$	$-4.1y$ $1.1y$	$2.3h$ $2.1h$	$-c$ $.01c$

Multiply:

	a	**b**	**c**	**d**	**e**	**f**	**g**
16.	4 2	-9 2	-20 -4	25 -1	-14 2	-3 9	0 7

	a	**b**	**c**	**d**	**e**	**f**	**g**
17.	$\begin{array}{r} x^2 \\ \underline{x^2} \end{array}$	$\begin{array}{r} x \\ \underline{x} \end{array}$	$\begin{array}{r} x^3 \\ \underline{x} \end{array}$	$\begin{array}{r} x^4 \\ \underline{x} \end{array}$	$\begin{array}{r} a^2 \\ \underline{a^7} \end{array}$	$\begin{array}{r} y^6 \\ \underline{y} \end{array}$	$\begin{array}{r} m^4 \\ \underline{m^3} \end{array}$
18.	$\begin{array}{r} 2x \\ \underline{5} \end{array}$	$\begin{array}{r} 3a \\ \underline{-4} \end{array}$	$\begin{array}{r} 5c \\ \underline{c} \end{array}$	$\begin{array}{r} 10h \\ \underline{2h} \end{array}$	$\begin{array}{r} a^2b \\ \underline{a} \end{array}$	$\begin{array}{r} 3ac^2 \\ \underline{4c} \end{array}$	$\begin{array}{r} -4b \\ \underline{3c} \end{array}$

Divide:

	a	**b**	**c**	**d**	**e**	**f**
19.	$-2\overline{)10}$	$-4\overline{)-20}$	$6\overline{)-18}$	$7\overline{)-14}$	$-3\overline{)-30}$	$-7\overline{)42}$
20.	$\frac{x^5}{x}$	$\frac{x^7}{x^2}$	$\frac{-6a}{2}$	$\frac{-20c}{c}$	$\frac{12y}{-3y}$	$\frac{8m^5}{2m}$

Subtracting Polynomials [A]

The method of subtracting one polynomial from another will be illustrated by the solution of two examples. At this time review the rule for subtracting one number from another on page 98.

Example 1. Subtract $x^2 - 4x + 3$ from $4x^2 + 2x + 1$, and check by letting $x = 1$.

Solution **Check**

$$\begin{array}{r} \left.\begin{array}{rcl} 4x^2 + 2x + 1 &=& 4 + 2 + 1 = 7 \\ \underline{x^2 - 4x + 3} &=& 1 - 4 + 3 = 0 \end{array}\right] = 7 \\ 3x^2 + 6x - 2 = 3 + 6 - 2 = 7 \qquad \end{array}$$

Example 2. From $x^3 + 4x$ take $5x^3 + x^2 - 5x + 7$, and check by letting $x = 2$.

Solution **Check**

$$\begin{array}{r} \left.\begin{array}{rcl} x^3 + 0 + 4x + 0 &=& 8 + 0 + 8 + 0 = 16 \\ \underline{5x^3 + x^2 - 5x + 7} &=& 40 + 4 - 10 + 7 = 41 \end{array}\right] = -25 \\ -4x^3 - x^2 + 9x - 7 = -32 - 4 + 18 - 7 = -25 \qquad \end{array}$$

The solution of a subtraction problem may be checked by adding the remainder to the subtrahend. This sum should equal the minuend.

EXERCISES

A

Subtract and check:

1. $\begin{array}{r} 3x+6 \\ x-1 \\ \hline \end{array}$

2. $\begin{array}{r} a+b+c \\ a-b+c \\ \hline \end{array}$

3. $\begin{array}{r} 4x-3y+z \\ 2x-7y-3z \\ \hline \end{array}$

4. $\begin{array}{r} 2a-3b+4c \\ a-b+6c \\ \hline \end{array}$

5. $\begin{array}{r} 2r-4s+t \\ 5r+s-t \\ \hline \end{array}$

6. $\begin{array}{r} x^2-xy+2y^2 \\ x^2+xy-4y^2 \\ \hline \end{array}$

7. $\begin{array}{r} 2a-b \\ a+7b-6 \\ \hline \end{array}$

8. $\begin{array}{r} 100-4x \\ 10-6x \\ \hline \end{array}$

9. $\begin{array}{r} x^2-3x \\ 4x^2-x+3 \\ \hline \end{array}$

10. Take $x-1$ from 20.

11. Subtract $2k+3m-6n$ from $4k+6$.

12. Subtract $-4a+5b-6$ from 0.

13. From $a-b+c$ take $2a-5b-3c$.

14. The minuend is $2a+5b+4c$ and the subtrahend is $7a-b+6c$. Find the remainder.

15. $a+2b-3c$ is how much greater than $-a-b-5c$?

16. Take $2x^3-4x^2+6x-7$ from x^3-3x^2+4x-5.

17. From 0 subtract $y-x$.

18. x^2-6x+7 is how much larger than x^2+6x-7?

19. How much greater than $2a-b$ is $a+3b+2$?

20. How much less than $x-4y+3z$ is $3x-6y-z$?

21. The perimeter of a triangle is $7x-10$. One of the sides is $x-6$ and another is $2x+1$. Find the third side.

22. From the sum of $a+2b-1$ and $3a-b+6$ subtract $7a-2b-10$.

23. Subtract $\frac{3}{4}a-\frac{1}{4}b+\frac{1}{5}c$ from $\frac{1}{2}a-\frac{1}{3}b+\frac{2}{5}c$.

24. The perimeter of a rectangle is $10x-80$. If one side is $2x-7$, find the sum of the remaining sides.

B

25. Subtract $5a^2-3a+6$ from $7a^2$.

26. Take $2x^3-3x^2y+7xy^2$ from $x^3-2x^2y-y^3$.

27. By how much does $2a - b$ exceed $4a + b - c$?

28. Take $x^2 - 3xy + 21$ from the sum of $x^3 - 21$ and $2x^2 - 4xy$.

29. From $\frac{1}{10}a - \frac{1}{5}b - \frac{1}{2}c$ take $\frac{2}{5}a + \frac{1}{4}b - \frac{1}{3}c$.

30. Take $.2m + .35n + p$ from $.3m - .45n - p$.

31. From $16a^3 - 7$ take $3a^3 + 2a^2 - 5a + 4$.

32. From $\frac{1}{5}a - \frac{1}{3}b + \frac{1}{6}c$ take $\frac{2}{15}a - \frac{1}{5}b - \frac{1}{8}c$.

33. The length of a rectangle is $x^2 - 4x + 9$ and the length exceeds the width by $7x - 2$. Write a formula for the perimeter of the rectangle.

34. The side of a square is $2x$. The width of a rectangle is $3x - 1$ and its length is $x^2 + 2x + 1$. How much larger than the perimeter of the square is the perimeter of the rectangle?

Multiplying Monomials[A]

As a preparation for the problems which you will soon be asked to solve, you need to be able to multiply one monomial by another with speed and accuracy. It is for this reason that the following review is given.

ORAL REVIEW EXERCISES[A]

Multiply:

1. $3x$ by 6
2. $10x^2$ by 4
3. $8m$ by -1
4. $-12y^2$ by 6
5. $-8m$ by -1
6. $4r$ by r
7. x by x
8. abc by d
9. xy by -4
10. $-a$ by a
11. πr by r
12. $2xy$ by -2
13. $-4x^3$ by 9
14. x^2y by y^3
15. 2π by r
16. $4x$ by $3x$
17. $-4a$ by $5a$
18. $-7h$ by $-4h$
19. h by $-h$
20. $-x^2$ by x^2
21. $4x$ by $\frac{1}{2}x$
22. $.2m$ by $.3$
23. $(4xy)(xy)$
24. $(-3ab)(4ac)$
25. $(\frac{4}{7}p)(14p)$
26. $(.1r)(-.1r)$
27. $(\frac{1}{2}y)(\frac{1}{5}y^2)$
28. $(-.4w)(-5w)$
29. $(6a^2y)(-3ay^2)$
30. $(ch)(c^2h)$

31. State the law of signs for multiplication.

32. State the law of exponents for multiplication.

Powers of Monomials [A]

You are again reminded that when you are working with exponents you should always keep in mind what the exponents mean. For example, x^2 means $(x)(x)$; m^4 means $(m)(m)(m)(m)$; and $(2\,ab^2)^3$ means $(2\,ab^2)(2\,ab^2)(2\,ab^2)$.

Study the following solutions:

Example 1. Find the square of x^3.

Solution. $(x^3)^2 = (x^3)(x^3) = x^6$

Example 2. Simplify $(x^7)^2$.

Solution. $(x^7)^2 = (x^7)(x^7) = x^{14}$

Example 3. Find the indicated power in $(-5\,x^4)^3$.

Solution. $(-5\,x^4)^3 = (-5\,x^4)(-5\,x^4)(-5\,x^4) = -125\,x^{12}$

After you have had sufficient practice, you will be able to omit the middle expressions in the solutions.

EXERCISES [A]

Find the indicated powers:

1. $(x^2)^2$
2. $(x^3)^2$
3. $(a^4)^2$
4. $(b^5)^2$
5. $(c^6)^2$

6. $(x^2)^3$
7. $(x^3)^3$
8. $(a^4)^3$
9. $(x^5)^3$
10. $(m^6)^3$

11. $(-3\,a)^2$
12. $(4\,x^3)^2$
13. $(-5\,a^3)^2$
14. $(-6\,x)^3$
15. $(xy)^2$

16. Choose some small negative number. Find the 1st, 2d, 3d, 4th, 5th, 6th, 7th, and 8th powers of it. What is the sign of each odd power? What is the sign of each even power? What sign has any power of a positive number?

Simplify:

17. x^2x^3
18. $(x^2)^3$
19. a^4a^3

20. $(a^4)^3$
21. a^2a^5
22. $(a^5)^2$

23. $(\frac{1}{2}\,x)^2$
24. $(.5\,c)^2$
25. $(a^2b^3)^2$

26. $(2\,x)^3$
27. $(-m^2n)^3$
28. $(-4\,c^2)^3$

29. $(2\,a)(3\,a)(4)$

30. $(-3)(4\,x)(-a)$

31. $(5\,c)(-c)(b)$

32. $-8(ax)(x)$

33. $(-4)(-3)(cd)$

34. $+a+a+a+a$

35. $a\cdot a\cdot a\cdot a$

36. $aaa+aaa$

Multiplying a Polynomial by a Monomial[A]

Suppose that an iron casting weighs 4 lb. 3 oz. and we wish to find what six of these castings weigh. The long way of finding the total weight of the six castings is to add the six weights as shown at the right of the page.

As you know, the short way of finding the combined weight is to multiply the weight of one casting by 6. Notice that *each term* of the multiplicand is multiplied by 6, and that there are just as many terms in the product as there are in the multiplicand.

$$\begin{array}{rr}
4\text{ lb.} + & 3\text{ oz.}\\
4\text{ lb.} + & 3\text{ oz.}\\
4\text{ lb.} + & 3\text{ oz.}\\
4\text{ lb.} + & 3\text{ oz.}\\
4\text{ lb.} + & 3\text{ oz.}\\
4\text{ lb.} + & 3\text{ oz.}\\
\hline
24\text{ lb.} + & 18\text{ oz.}\\
=25\text{ lb.} + & 2\text{ oz.}
\end{array}$$

$$\begin{array}{rr}
4\text{ lb.} + & 3\text{ oz.}\\
 & 6\\
\hline
24\text{ lb.} + & 18\text{ oz.}\\
=25\text{ lb.} + & 2\text{ oz.}
\end{array}$$

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial.

Example 1. Multiply $a-b+6$ by 4.

Solution 1.

$$\begin{array}{r}
a-b+6\\
4\\
\hline
4\,a-4\,b+24
\end{array}$$

Solution 2.

$$4(a-b+6)=4\,a-4\,b+24$$

Pupils should acquire the habit of working from the *left to the right*. Do you know why?

Example 2. $-2x(3x^2 - 4x + 1) = ?$

Solution 1.

$$\begin{array}{r} 3x^2 - 4x + 1 \\ -2x \\ \hline -6x^3 + 8x^2 - 2x \end{array}$$

Solution 2.

$$\begin{array}{rl} & -2x(3x^2 - 4x + 1) \\ = & -6x^3 + 8x^2 - 2x \end{array}$$

EXERCISES

A

Multiply:

1. $\begin{array}{r} a + 5 \\ 4 \\ \hline \end{array}$

2. $\begin{array}{r} a^2 - 6 \\ 4 \\ \hline \end{array}$

3. $\begin{array}{r} -3b + c \\ -6 \\ \hline \end{array}$

4. $\begin{array}{r} -3x + 4y \\ -10 \\ \hline \end{array}$

5. $\begin{array}{r} 4x^2 - 5y \\ +a \\ \hline \end{array}$

6. $\begin{array}{r} m + n - p \\ -c \\ \hline \end{array}$

7. $\begin{array}{r} a - b + 5c \\ -3 \\ \hline \end{array}$

8. $\begin{array}{r} x^2 - 4x + 3 \\ +x \\ \hline \end{array}$

9. $\begin{array}{r} c^2 - 4cd + 3 \\ -4 \\ \hline \end{array}$

10. $\begin{array}{r} 4a^2 + 7a - 10 \\ 2a \\ \hline \end{array}$

11. $\begin{array}{r} 7m^2 - m + 3 \\ 3m \\ \hline \end{array}$

12. $\begin{array}{r} 3x^2 - 8x + 1 \\ 4x^2 \\ \hline \end{array}$

13. $3x(2x^2 - 3x - 1)$

14. $a(a^2 - a + 3)$

15. $-5(6a + b - 7)$

16. $\pi r(r + l)$

17. $-7(-c^2 + 2c + 3)$

18. $x(x^2 - 4x + 5)$

19. $2x(x^2 + x + 1)$

20. $x(x^2 - xy + y^2)$

21. $-1(x^2 + 3x + 6)$

22. $-1(a + b - c)$

23. Find the area of a rectangle if its base is $4a - 6$ and its altitude is $2a$.

24. Copy and complete this table of areas of rectangles:

Base	$x + 6$	$m - 6$	$8x - 3y$	$a^2 - 3a + 4$
Altitude	x	$2m$	$4xy$	$+5a$
Area	?	?	?	?

25. Find the cost of $7c - d$ articles at $+2c$ cents each.

26. Find the interest on x dollars, for y years, at p per cent. $\left(p \text{ per cent means } \frac{p}{100}.\right)$

27. Find the area of a triangle whose base is $x + 8$ and altitude $2x$.

B

Multiply:

28. $-2ab^2(2a^2 - 7ab - ab^2)$

29. $5cd(c^2d - cd^2 + 3cd)$

30. $\frac{3}{4}(12a^2 - 16a + 8)$

31. $(7x^2 - 4xy + 16y^2)4y$

32. $(-2x^2 - x + 3)(-1)$

33. $-5w(-1 + w + 3w^2)$

34. $12(\frac{2}{3}a - \frac{1}{6}b - \frac{1}{12}c)$

35. Show that $A = p(1 + rt)$ is the same as $A = p + prt$.

36. Change the formula $T = 2\pi r(r + l)$ so that it has no parentheses.

37. Remove the parentheses in the formula $A = \pi(R^2 - r^2)$. What does each term in the new formula represent?

Dividing a Polynomial by a Monomial[A]

When a polynomial is multiplied by a monomial, each of its terms is multiplied by the monomial. Since division is the inverse of multiplication, in dividing a polynomial by a monomial, *each term* of the dividend must be divided by the monomial.

Compare the multiplication and division below.

$$\begin{array}{r} 2\text{ hr.} + \;\;3\text{ min.} + \;\;5\text{ sec.}\\ 4\\ \hline 8\text{ hr.} + 12\text{ min.} + 20\text{ sec.}\end{array}$$

$$\begin{array}{r} 2\text{ hr.} + \;\;3\text{ min.} + \;\;5\text{ sec.}\\ 4\overline{)8\text{ hr.} + 12\text{ min.} + 20\text{ sec.}}\end{array}$$

What is the multiplicand? the multiplier? the product? the dividend? the quotient?

Remember that the quotient has the same number of terms as the dividend.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 1. $\dfrac{60 + 42 - 30}{2} = 30 + 21 - 15$

Example 2. $\dfrac{15\,x^3 - 40\,x^2 - 15}{-\,5} = -\,3\,x^3 + 8\,x^2 + 3$

Example 3. $(12\,x^3 - 8\,x^2 + 4\,x) \div (4\,x) = 3\,x^2 - 2\,x + 1$

Check. $4\,x(3\,x^2 - 2\,x + 1) = 12\,x^3 - 8\,x^2 + 4\,x$

Note. Do not make the error of omitting the + 1 in the quotient. Omitting the + 1 in the quotient changes the value of the answer.

EXERCISES

A

1. Explain why the quotient of $\dfrac{8\,x^2y}{2\,xy}$ is written $4\,x$ and not $4\,x\,1$.

2. Explain why the quotient of $\dfrac{8\,x^2 + 4\,x}{4\,x}$ is written $2\,x + 1$ and not $2\,x$.

Find the quotients:

3. $(4\,x - 8) \div 2$

4. $(10\,x^2 - 15) \div 5$

5. $(18\,r + 6\,s) \div 6$

6. $(x^2 + x) \div x$

7. $(20\,m^2 - 12) \div (-\,4)$

8. $(x^3 - x^2) \div x^2$

9. $(ab - ac) \div a$

10. $(\pi R - \pi r) \div \pi$

11. $(an - am) \div a$

12. $(x^4 - x^3) \div x^2$

13. $(24\,a^3 + 6\,a^2) \div 3$

14. $(14\,m^2 - 21\,m) \div (-\,7)$

15. $\dfrac{7\,c^3 - 14\,c^2 - c}{-\,c}$

16. $\dfrac{-\,4\,a^3 + 16\,a^2 - 10\,a}{-\,2}$

17. $\dfrac{24\,m^3 - 16\,m^2 - 8\,m}{8}$

18. $\dfrac{8\,x^5 - 10\,x^4 + x^3}{x^3}$

19. $\dfrac{a^{10} - a^9 + a^7}{a^3}$

20. $\dfrac{8\,a^3 - 4\,a^2 + 4\,a}{-\,4\,a}$

21. $(ax^2 - ax) \div (ax)$

22. $(14a - 7a^2) \div (7a)$

23. $(18x^2 - 6x) \div (-6x)$

24. $(b^3 - 8b^2) \div b^2$

25. $(-3x^2 - 6x) \div (2x)$

26. $(\pi r^2 - \pi rh) \div (\pi r)$

27. $(\pi {r_1}^2 - \pi {r_2}^2) \div \pi$

28. $(r^6 - r) \div r$

29. $(vt - \frac{1}{2}gt^2) \div t$

30. $(180n - 360) \div 180$

31. How many feet are there in $(24x - 36y)$ inches?

32. At $2y$ cents each how many pencils can be bought for $(8y^2 + 6y)$ cents?

33. How long will it take to travel am^2 miles if you travel at the rate of m miles a day?

34. If you save c cents a day, how many days are needed to save $(bc + c)$ cents?

B

Divide:

35. $(16x^4 - 20x^3 + 8x^2 + 4x) \div (4x)$

36. $(27a^3 - 24a^2 - 15a + 9) \div (-9)$

37. $(x - x^2 + 2x^3 - 3x^4 + x^5) \div (-1)$

38. $(x^3 - 6x^2 - 7x + 6) \div (-1)$

39. $(4x^5 - 32x^4 + 8x^3 - 4x^2) \div (-4x^2)$

40. $(8x^2y^3 + 4x^3y^4 - 2xy^3) \div (2xy^3)$

Simplifying Expressions Containing Parentheses [A]

You have learned that parentheses are used to show that all terms or factors within them are to be treated as a whole. For example, $4(2a - b)$ means that $2a - b$ is to be multiplied by 4, and $-x(x + 2)$ means that $x + 2$ is to be multiplied by $-x$.

Example 1. Remove parentheses in the expression

$$4x^2(x^2 - 3x + 3).$$

Solution. $4x^2(x^2 - 3x + 2) = 4x^4 - 12x^3 + 8x^2$

EXERCISES [A]

Find the indicated products:

1. $4(2x - 5)$
2. $3(a - b)$
3. $-2(c - m)$
4. $-5(x - 1)$
5. $6(-2 + x)$
6. $x(x - 1)$
7. $x^2(x - 1)$
8. $-4(a + b)$
9. $-3(1 - x)$
10. $-7(5 - 2c)$
11. $4(m - 7)$
12. $-8(c^2 - 5)$
13. $4(a^2 - x^2)$
14. $\pi(R + r)$
15. $p(1 + rt)$
16. $x^2(x + 4)$
17. $8(\frac{1}{2}a + \frac{1}{4}b)$
18. $6(.2x + y)$
19. $20(\frac{1}{5}x - 3)$
20. $-a^2(a - 5)$
21. $.2(5x - 10)$
22. $-.4(25c + 1)$
23. $18(\frac{1}{2}x - \frac{1}{3}y)$
24. $\frac{1}{2}x(4x - 6y)$

Example 2. Simplify $-4(a + b) - 3a$.

Solution. There are two terms in this expression. The first term is $-4(a + b)$ and the second is $-3a$.

Then

$$\begin{aligned} & -4(a + b) - 3a \\ &= -4a - 4b - 3a \\ &= -7a - 4b \end{aligned}$$

Example 3. Remove parentheses and simplify $4 - 5(c - 4)$.

Solution.

$$\begin{aligned} & 4 - 5(c - 4) \\ &= 4 - 5c + 20 \\ &= 24 - 5c \end{aligned}$$

EXERCISES [A]

Remove parentheses and simplify:

1. $4(2p - 3) - 4$
2. $2(m - 5) + 7$
3. $-3(2x - 1) - 4$
4. $-7(x - 4) - x$
5. $8(2 - y) - 4y$
6. $2 + 7(a - 5)$
7. $3c - 5(2c - 4)$
8. $5c - 4(c + 1)$
9. $7y - 4(y + 5)$
10. $8 - 3(-3y + 1)$

Example 4. Simplify $2(4x-3)-4(x-5)$.

Solution. This expression means that $4x-3$ is to be multiplied by 2; that $x-5$ is to be multiplied by -4; and that the two products are to be combined.

Then

$$\begin{aligned} &2(4x-3)-4(x-5)\\ &=8x-6-4x+20\\ &=4x+14 \end{aligned}$$

EXERCISES [A]

Simplify:

1. $4(2x-1)+3(x+1)$
2. $7(2c+1)-5(3c+1)$
3. $1(x+3)-2(2x-5)$
4. $a(a+1)-a(3a-1)$
5. $4(3c+2)+5(c-1)$
6. $3(m-n)-4(n-m)$
7. $-1(a+b)+1(a-b)$
8. $5(3+h)-6(h-2)$
9. $b(b-a)+a(b-1)$
10. $c(c-5)-5(c^2+2c)$

Example 5. Simplify $(a+5)+(3a-6)$.

Solution 1. This expression means that $3a-6$ is to be added to $a+5$. The solution is at the right.

$$\boxed{\begin{array}{r} a+5\\ 3a-6\\ \hline 4a-1 \end{array}}$$

Solution 2. Since the expressions in the parentheses are to be added, we write the polynomials one after the other *without* changing their signs.

Then

$$\begin{aligned} &(a+5)+(3a-6)\\ &=a+5+3a-6\\ &=4a-1 \end{aligned}$$

Solution 3. Since a number is not changed when it is multiplied by 1, we may proceed as in Example 4.

Then

$$\begin{aligned} &(a+5)+(3a-6)\\ &=1(a+5)+1(3a-6)\\ &=a+5+3a-6\\ &=4a-1 \end{aligned}$$

Example 6. Simplify $(y-5)-(3y-4)$.

Solution 1. This expression means that $3y-4$ is to be subtracted from $y-5$. In this solution the signs of the subtrahend are changed mentally before adding.

$$\boxed{\begin{array}{r} y-5 \\ \underline{3y-4} \\ -2y-1 \end{array}}$$

Solution 2. Since $3y-4$ is the quantity that is to be subtracted, $3y$ is changed to $-3y$ and -4 is changed to $+4$.

Then

$$\begin{aligned} &(y-5)-(\underset{\downarrow}{3y}\ \underset{\downarrow}{-\ 4}) \\ &= y-5 \quad -3y+4 \\ &= -2y-1 \end{aligned}$$

Solution 3. Placing 1 before the parentheses does not change the value of the expression.

Then

$$\begin{aligned} &(y-5)-(3y-4) \\ &= 1(y-5)-1(3y-4) \\ &= y-5-3y+4 \\ &= -2y-1 \end{aligned}$$

Rule 1. Parentheses preceded by a plus sign may be removed without changing the signs of the terms within. Parentheses preceded by a minus sign may be removed if the signs of the terms within are changed.

Another way of putting it is as follows:

Rule 2. If parentheses are preceded by a + sign or a − sign, write the monomial 1 before the parentheses and multiply as indicated.

EXERCISES

A

Simplify, following the method of either Rule 1 or Rule 2 above:

1. $x-(x-1)$

2. $c+(2c+1)$

3. $5a+(4-a)$

4. $3x-(2-4x)$

5. $10-(5+y)$

6. $8+(4+y)$

7. $x^2 - (x^2 - 6)$	**11.** $10 - (8 - m)$	**15.** $6t - (32 + 3)$
8. $2b + (-b + 1)$	**12.** $8 + (5 + b)$	**16.** $3p + (-p - 4)$
9. $-c - (c + 3)$	**13.** $4 - (x - 2)$	**17.** $5h + (2h - 1)$
10. $5y - (2y - 1)$	**14.** $2x - (8 - x)$	**18.** $R - (4r + 2)$

Remove parentheses and combine like terms:

19. $3x - (4 - 2x) + 5$	**25.** $2(x - 1) - (3x + 4)$
20. $3c - 5(c - 1) + 6$	**26.** $(5c + 2)3 - 10$
21. $(x + 6) - (x - 4)$	**27.** $x(x + 1) - (x^2 - 5x)$
22. $(4m - 3) - 5(m + 1)$	**28.** $4(m - 3) - 5(2m + 3)$
23. $2x - y - (x + y)$	**29.** $a^2 + b^2 + a(a + b)$
24. $x - (x - y) + 2y$	**30.** $a^2 + a(a + b)$

B

Simplify:

31. $(2a + 3b) + (a - 5b) - 2(2a - 7b)$

32. $x(x - 5) - 3x(x + 4) - (x^2 - 7x + 1)$

33. $ab(a^2 - ab + b^2) - 3a(a^2b + ab^2)$

34. $m^2(m - 1) - m(m^2 + 2m - 4)$

35. $4p - (2p^2 + 3p + 6) + (-4p^2 + 5p)$

Enclosing Terms in Parentheses [B]

It is sometimes convenient or necessary to enclose two or more terms of a polynomial in parentheses.

When terms are removed from parentheses preceded by a plus sign, the signs of the terms are not changed. Therefore when this operation is reversed, that is, when terms are placed within parentheses preceded by a plus sign, the signs of the terms are not changed.

When terms are removed from parentheses preceded by a minus sign, the signs of the terms are changed. Therefore when terms are enclosed in parentheses preceded by a minus sign, the signs of the terms must be changed.

Example 1. Enclose the last three terms of the polynomial $4x^3 - 2x^2 - 5x + 3$ in parentheses preceded by a plus sign.

Solution.

$$\begin{aligned} &4x^3 - 2x^2 - 5x + 3 \\ &= 4x^3 + (-2x^2 - 5x + 3) \end{aligned}$$

Example 2. Enclose the last two terms of $2a + 3b + 4c - d$ in parentheses preceded by a minus sign.

Solution.

$$\begin{aligned} &2a + 3b + 4c - d \\ &= 2a + 3b - (-4c + d) \end{aligned}$$

EXERCISES [B]

Enclose the first three terms of each polynomial in parentheses preceded by a plus sign, and the last three terms in parentheses preceded by a minus sign.

1. $m^2 - 2mn + 3 - 4y^2 - 3y + 2$

2. $c^2 - 3c + 1 - 7d^2 - 8d + 9$

3. $-4x^2 - 3x - 1 + h^2 + 5h - 8$

Enclose the terms containing powers of x in parentheses preceded by a plus sign, and the terms containing powers of y in parentheses preceded by a minus sign.

4. $2x^2 - y^2 - 3y + 4x - 5cx + 2y^2 - by$

5. $2x - 3y + bx + cy + 3x^2 - 8y^2$

6. $x^2 + y^2 + 3x - 5y + cx^3 - dy^3$

WORD LIST

You should be able to spell and use correctly each of the following words:

circumference (p. 54)
dividend (p. 11)
multiplier (p. 93)
monomial (p. 13)
parentheses (p. 167)
polynomial (p. 13)
product (p. 85)
quotient (p. 88)

CHAPTER REVIEW

A

Add:

1. $\begin{array}{r} 5x + 7y - 8z \\ 3x + y - 4z \\ \hline \end{array}$

2. $\begin{array}{r} 60 - 7x \\ 4x - y \\ \hline \end{array}$

3. $\begin{array}{r} 3x^2 - 2x + 7 \\ -x^2 + 2x - 5 \\ \hline \end{array}$

4. $\begin{array}{r} 3c + 5d + 3 \\ 11c + d - 9 \\ \hline \end{array}$

5. $\begin{array}{r} x^2 - 7xy + y^2 \\ -6x^2 - 8xy - 9y^2 \\ \hline \end{array}$

6. $\begin{array}{r} 2x^4 - x^3 + x^2 \\ +5x^4 + 4x^3 - x^2 \\ \hline \end{array}$

Subtract:

7. $\begin{array}{r} a - b + c \\ 2a - b - c \\ \hline \end{array}$

8. $\begin{array}{r} 4a - 3b + 6 \\ a - 4b - 1 \\ \hline \end{array}$

9. $\begin{array}{r} x^2 - 20 \\ x^3 + x^2 \\ \hline \end{array}$

10. $\begin{array}{r} 40 \\ 2x - 5 \\ \hline \end{array}$

11. $\begin{array}{r} 3a - 2b + 7c \\ -5a + b - c \\ \hline \end{array}$

12. $\begin{array}{r} x^3 - 1 \\ x^2 + x \\ \hline \end{array}$

Multiply:

13. $\begin{array}{r} x^2 - 4x + 3 \\ -4 \\ \hline \end{array}$

14. $\begin{array}{r} 2m^2 - m + 2 \\ 3 \\ \hline \end{array}$

15. $\begin{array}{r} a + b + c \\ -a \\ \hline \end{array}$

16. $\begin{array}{r} 2x^2 - 5x + 3 \\ -4 \\ \hline \end{array}$

17. $\begin{array}{r} m^2 - 4m + n^2 \\ mn \\ \hline \end{array}$

18. $\begin{array}{r} x^2 - 4xy + y^2 \\ -3xy \\ \hline \end{array}$

Divide:

19. $\dfrac{4a - 8b + 12c}{3}$

20. $\dfrac{5x^3 - 20x^2 - 5}{-5}$

21. $\dfrac{x^3 - 4x^2 + x}{x}$

22. $\dfrac{3x^3 - 6x^2 - 18x}{-3x}$

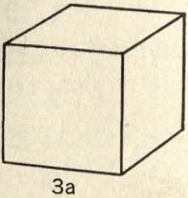
23. $-7\,\overline{\smash{)}21x^2 - 14x}$

24. $5c\,\overline{\smash{)}10abc - 5c^3}$

25. $(18c^3 - 12c^2 + 9c) \div (-3c) = ?$

26. Find the total area of the cube shown here.

27. Find the volume of the cube.



28. $(-3x)^4 = ?$

29. $(-4x^2)(-x)(-6x^4) = ?$

State the products:

30. $a^2b^3 \cdot a^3b$

31. $m^2n \cdot mn^4$

32. $(-ab)(-ab)$

33. $(-x^3y^2)(x^2y)$

34. $m \times n$

35. $7ab \times 0$

State the quotients:

36. $(8x^4) \div (-4x)$

37. $(30ab^3) \div (-15a)$

38. $(14t^3) \div (-7t^3)$

39. $(-12mn) \div (-6m)$

40. $(x^3 - 4x^2 + 5x - 8) \div (-1)$

41. $(4x^5 + 16x^4 - 20x^3 + 8x^2 - 4x) \div (4x)$

B

Simplify by removing parentheses and combining like terms:

42. $2a - 3(a + b)$

43. $(a - b + c)4 - 6$

44. $2x(x^2 - 4) - x^2(1 - 3x)$

45. $-3(3x + 1) - 4(2x - 3)$

Enclose the first two terms of each polynomial in parentheses preceded by a plus sign and the last two terms in parentheses preceded by a minus sign:

46. $4c + 1 - 3c^2 - 1$

47. $a + b - c + d$

48. $x^2 - 6x - y^2 - 9$

49. $a^2 - b^2 - c^2 + d^2$

C

Simplify:

50. $2(x-4)-3(2x-6)-(4x-28)$

51. $2c(c^2-3c+1)-3c(c^2+c-2)$

Enclose the last four terms in parentheses preceded by a minus sign:

52. $a^2-2ab+b^2-2bc+c^2-9$

53. $x^2-a^2-4ab-4b^2+16$

GENERAL REVIEW

A

1. The area of a rectangle is $4a-8$ and its base is 4. Find its height.

Solve:

2. $5x+4-12=32$

3. $3y+5y-7=4y$

4. $3x+14=11x-2$

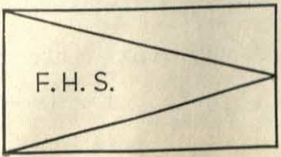
5. $x+.06x=424$

6. $c-.1c=81$

7. $\frac{3}{4}y=\frac{2}{3}$

8. Find by substitution which of the numbers 2, -2, 3, and -3 are roots of the equation $x^2-x=6$.

9. This triangular pennant is cut from a rectangular-shaped piece of cloth. What fraction of the cloth is wasted?



10. What must be added to $2a-4b$ to make the expression equal to zero?

11. $2x-3y=12$. What is the value of x when $y=4$? when $y=-4$? when $y=0$?

12. Subtract $+99$ from -99.

13. $i=prt$. Find p when $i=\$33.75$, $r=6\%$, and $t=2\frac{1}{2}$ years

14. Simplify $(2x-4)-10+(3x+6)$.

B

15. Simplify $(4x - 3xy) + 5y^2 - 2(x^2 - xy - y^2)$.

16. Simplify $x(xy^2 - y) - 6 - 4(x^2y^2 + 3)$.

Enclose the last two terms in parentheses preceded by a minus sign:

17. $x^2 - 4x + 3$

18. $c^2 + 5c - 3$

19. $p^2 - 3p + 2$

20. $8x^2 + 9x - 4$

21. Find three consecutive integers whose sum is 1353.

CHAPTER TEST[A]

Simplify:

1. $7c + (-3c)$

2. $2x - (-4x)$

3. $-x^3 \div x^2$

4. $(-3x^2)^2$

5. $-7p - 4p$

6. $-19 - (+19)$

7. $\dfrac{12x - 4}{4}$

8. $\dfrac{4y^2 - y}{-1}$

9. Add $3a^2 - 4 + 5a$ and $6a - a^2 - 10$.

10. Subtract $7x^2 - 9x + 7$ from $4x - x^2 + 6$.

11. Multiply $2m^2 - 6m + 3$ by $-3m$.

12. Divide $9x^3 - 6x^2 + 15$ by 3.

Remove parentheses and combine like terms:

13. $a - (a - b)$

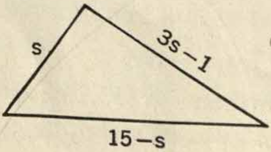
14. $2c + (d - 3c)$

15. $2(x - 1) - 3(x + 4)$

16. $5(y - 1) + 2(2y - 3)$

17. Find the cost of $2n$ articles at $(n + 1)$ cents each.

18. Find the perimeter of the triangle below.



add

19. The area of a rectangle is $2x^3 - 4x$ and the altitude is $2x$. Find the base.

20. In a schoolroom there are n rows and $n + 2$ pupils in each row. How many pupils are there in the n rows?

CHAPTER TEST[B]

Simplify:

1. $6c + (-3c)$

2. $(8xy)(6y^2)$

3. $(-2x)^3$

4. $xy(x - y)$

5. $\dfrac{12xy - x^2}{x}$

6. $-2a^2(a - ab)$

7. $(c^2 - 8c) \div c$

8. $\frac{1}{2}x(x^2 - 6x + 10)$

9. $7h - (-5h)$

10. $20 - (-20)$

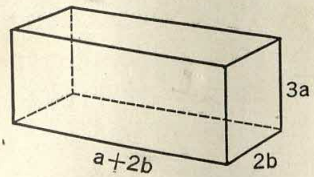
11. Add: $2a^2 - a + 6$; $a - 9$; and $5a - 10a^2$.

12. From $a^3 - 1$ subtract $2a^2 - 5a - 1$.

13. Multiply $x^2 - 2xy - y^2$ by $2xy$.

14. Find the area of a rectangle if its base is $m^2 - m - 1$ and its altitude is $2m$.

15. Find the area of the base of this rectangular solid.



16. Find the area of each end of the rectangular solid.

17. From the sum of $x^2 - x + 3$ and $2x^2 - 4x - 7$ subtract $4x^2 - 7x - 1$.

Remove parentheses and simplify:

18. $5(c - 3) - (c + 2)$

19. $p - (p + 3) - 3(2p + 1)$

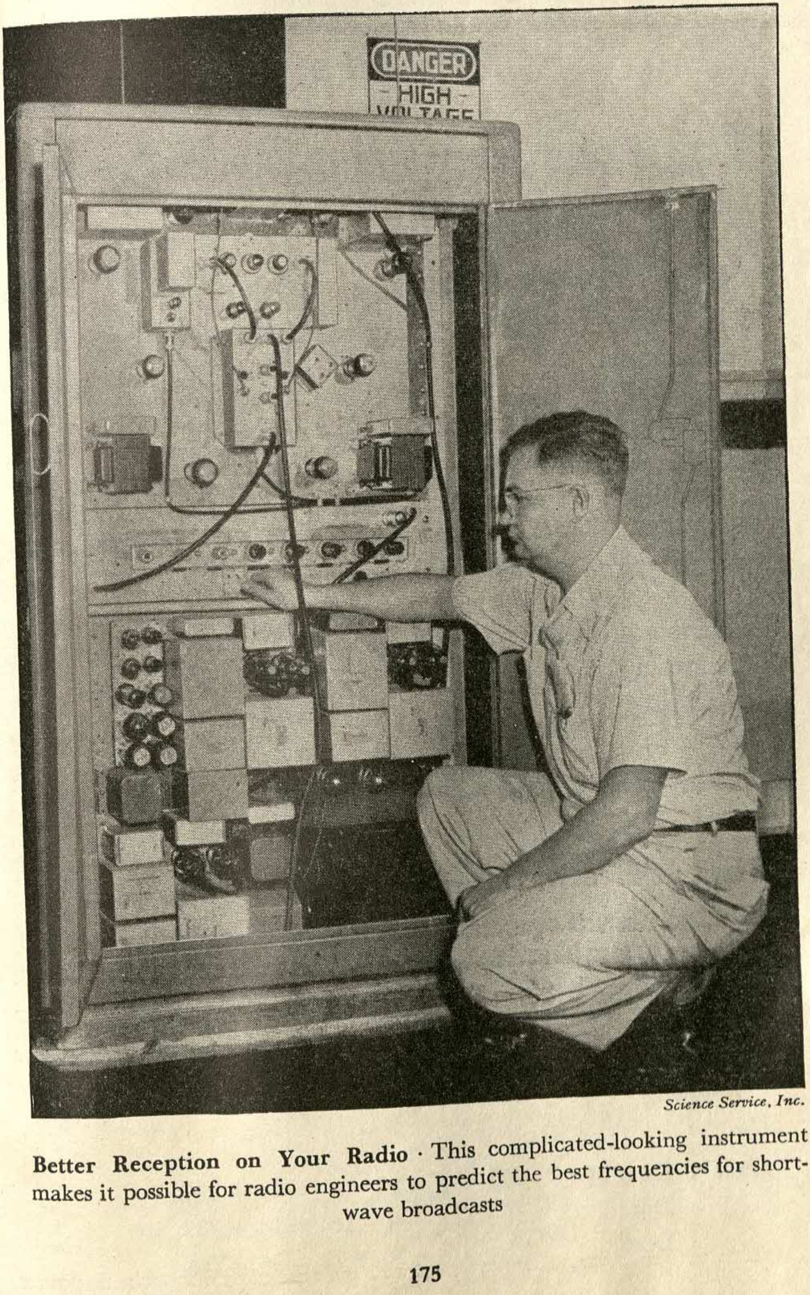
20. Insert the first two terms of $a - b + 3c - 5d$ in parentheses preceded by a plus sign and insert the last two terms in parentheses preceded by a minus sign.

MATHEMATICS AND RADIO

FIFTY to 250 miles above the earth's surface lies the ionosphere, a region of rarefied air in which many of the atoms have lost their electrons. Such atoms, called ions, are found in layers which block radio waves as effectively as a brick wall does a ball. Since radio waves travel in a straight line, and the earth is round, the waves would be lost in space if it were not for the layers of ions. As it is, the waves glance off these layers, and return to the earth, where they may be picked up on receiving sets. However, at times there are comparatively few ions in the ionosphere. Then most radio waves escape into space, and short-wave reception may be very poor.

Scientists have now developed an instrument which locates the layers of ions, and also makes possible predictions of where they will be weeks or months in the future. This information enables radio men to choose the best frequencies for their short-wave broadcasting. Once this was more or less a matter of guesswork; today the ideal frequencies may be determined with mathematical precision.

Mathematics has played a vital part in the development of radio, and is essential to an understanding of radio theory. In this, as in many other fields, mathematics has helped make possible scientific advance.



Science Service, Inc.

Better Reception on Your Radio · This complicated-looking instrument makes it possible for radio engineers to predict the best frequencies for short-wave broadcasts

CHAPTER VI

Equations Containing Parentheses

Removing Parentheses in Equations

In solving equations containing parentheses, we first remove the parentheses.

Example 1. Solve and check: $(6x - 5) - (6 - x) = 3$

Solution. Here $6 - x$ is to be subtracted from $6x - 5$. The minuend is $6x - 5$ and the subtrahend is $6 - x$. We can write the equation without the parentheses if we change the signs of the subtrahend. Our work should appear as follows:

$$(6x - 5) - (6 - x) = 3$$

Removing parentheses, $6x - 5 - 6 + x = 3$

Transposing $\quad 6x + x = 3 + 5 + 6$

Simplifying $\quad 7x = 14$

D_7 $\quad x = 2$

Check. Does $(12 - 5) - (6 - 2) = 3$?

Does $7 - 4 = 3$? Yes.

Example 2. Solve $3(x - 5) - 4(2x - 4) = -2x + 22$.

Solution. In this equation $(x - 5)$ is to be multiplied by $+3$ and $(2x - 4)$ is to be multiplied by -4. The work appears as follows:

$$3(x - 5) - 4(2x - 4) = -2x + 22$$

Multiplying $\quad 3x - 15 - 8x + 16 = -2x + 22$

Transposing $\quad 3x - 8x + 2x = 15 - 16 + 22$

Simplifying $\quad -3x = 21$

D_{-3} $\quad x = -7$

Check. Does $3(-7 - 5) - 4(-14 - 4) = 14 + 22$?

Does $3(-12) - 4(-18) = 14 + 22$?

Does $-36 + 72 = 36$? Yes.

EXERCISES

A

Solve, and check as directed by your teacher:

1. $2(x + 6) = 22$
2. $5(x - 1) = 6(x - 3)$
3. $6x - (3x - 4) = 14$
4. $2x - (13 - 2x) = 61$
5. $15 + 6x = 7(x - 2)$
6. $5(x - 4) - 4(x - 3) = 0$
7. $3x - 8 = 3(7 - x)$
8. $3 - (x + 29) = 12x$
9. $3c - 4(c + 2) = 5$
10. $(3x + 3) = (x + 4)$
11. $3y - (y + 4) = 0$
12. $4(x - 3) - 6(x + 1) = 0$
13. $9x - (3x - 18) = 36$
14. $8x - (2x + 5) = 0$
15. $9x - (5x - 2) = 8$
16. $2p - (4 - p) = 0$
17. $-(5y + 4) = -6y + 3$
18. $13x - (3 + 12x) = -3$
19. $9r = 5 - (10 - 4r)$
20. $10s + 3 + (-6s - 3) = 0$
21. $5(2p + 3) = 2(4p - 1)$
22. $5(2 - 3x) = 15 - (x + 7)$
23. $5 + (6m + 3) = 8m$
24. $7(x - 5) = 10 - (x + 1)$
25. $9x = 2x + 6 - (4 - 3x)$
26. $(r - 9) - (r + 6) = 4r$
27. $5c - (3c + 2) = 15$
28. $40 = 16 - 4(9 - 3k)$
29. $5x + 2(4x - 5) = 30$
30. $6x - (x - 7) = (x + 15)$

B

Solve:

31. $20h - (5h + 9) + (7 - 9h) = (12h - 5)$
32. $7(5y - 1) - 18y = 12y - (3 - y)$
33. $\frac{1}{3}(6x - 9) = \frac{1}{2}(8x - 4)$
34. $(.6y + .08) - 1.2y = -.8y - .22$
35. $(4x - 13) + (17 - 2x) - 3(x - 7) = 3(x - 5)$
36. $2(x + 1) - (x + 5) - (x - 4) - (3x - 4) = 0$
37. $6(x - 2) - 2(x - 5) - 2(x + 6) = 0$
38. $3(5 - n) - 4(2n - 7) = -11 - (10 + 3n)$
39. $5y - 4(y - 6) - 2(y + 6) + 12 = 0$
40. $3x - .2(x - 30) = x + .3(8x - 30)$
41. $24x^2 + 147 = 27 - 16x - 4x(1 - 6x)$
42. $5(m - 3) + 21(m - 2) = 11 - 4(17 - m)$

43. $x + 6.25 - 3(4x - 1) = 8(x + .25) - 3(x + .25)$

44. $190 - 21(y + 3) = 196 - 15(2y - 5)$

45. $2x - 5(.8x - 1.8) + 5(x + 4) = 41$

Conditional and Identical Equations [A]

The equation $3(x + 2) = 3x + 6$ is true for all values of x, since $3(x + 2)$ and $3x + 6$ are always equal. An equation like this is called an *identical equation* or an *identity.*

The equation $3x + 2 = 14$ is true when $x = 4$, and only when $x = 4$. No other value of x will satisfy the equation. In other words, the condition for $3x + 2$ to equal 14 is that x shall equal 4. An equation such as $3x + 2 = 14$ is called a *conditional equation.* Can you give a reason for its name?

The difference between a conditional equation and an identical equation is that the unknown letter may have any value in an identical equation, but can have only one or a limited number of values in a conditional equation. The equation $x^2 = 9$ is an example of a conditional equation in which x has more than one value. It is satisfied by either $+3$ or -3 but by no other values.

Example. Is the equation $3(x + 5) + 4 = 3x + 19$ conditional or identical?

Solution. $3(x + 5) + 4 = 3x + 19$

Removing parentheses $3x + 15 + 4 = 3x + 19$

Collecting like terms $3x + 19 = 3x + 19$

The equation is identical since both members are the same (identical).

EXERCISES [A]

Tell whether the following equations are identical or conditional:

1. $4x - 8 = 0$

2. $3x - 2 = 10$

3. $2(x - 4) = 2x - 8$

4. $3(m - 9) = 4m + 25$

5. $3(x - 1) = 10x + 11$

6. $3x - 5 = x - 5 + 2x$

7. $2(c - 5) - c = c - 6$

8. $7m - 4 = 4(2m - 1) - m$

9. $5(x + 1) - 4x = x - 5$

10. $3(x + 1) = 3(x - 1) + 6$

Using Parentheses with Algebraic Expressions[A]

In arithmetic all our numbers or expressions can be written as monomials. For example, 5, $2\frac{3}{4}$, and 356 are all monomials. In algebra we cannot write all our numbers or expressions as monomials. For example, $4x$, $7y - 14$, and $c^2 + 6c - 4$ are all in their simplest forms, but only the first expression is a monomial.

When we wish to indicate that we are to multiply or divide an expression such as $7x - 14$ it is necessary to enclose it in parentheses.

For example $\quad 2(7y - 14) = 14y - 28$

But $\quad 2 \times 7y - 14 = 14y - 14$

Also $\quad (7y - 14) \div 7 = y - 2$

But $\quad 7y - 14 \div 7 = 7y - 2$

Example. Write the product of 6 and the sum of $2x$ and y in algebraic symbols.

Solution. Since the sum of $2x$ and y is to be multiplied by 6, we must write the $2x + y$ in parentheses. Then our expression becomes

$$6(2x + y)$$

EXERCISES[A]

Express the following, using algebraic symbols:

1. The sum of x and 3; the sum of $2x$ and -3; the sum of $4m$ and $m + 4$.

2. The product of x and y; the product of 4 and $3x - 1$; the product of -6 and $y + 5$; the product of $2x$ and $x + 8$.

3. The square of a; twice the square of a; the cube of b; the square of $3c - 1$.

4. If a truck can haul a load of 4 tons each trip, how many tons can it haul in 2 trips? in 6 trips? in x trips? in $(x + 6)$ trips?

5. In 8 trips how many tons can a 2-ton truck haul? In x trips how many tons can a y-ton truck haul? In $(y + 3)$ trips how many tons can a 7-ton truck haul?

6. What is the value of 4 pints of milk at 10 cents a pint? at c cents a pint? at $(c+1)$ cents a pint?

7. The product of two numbers is $8x^2 + 4$, and one of them is 4. What is the other number?

Letting x stand for the number, write the following, using algebraic symbols:

8. The sum of four times a certain number and 3 is 35.

9. Four times the sum of a certain number and 3 is 35.

10. Four times the sum of 12 and a certain number is 60.

11. 9 is as much more than a certain number as 4 times the number is more than 1.

12. If to a certain number 3 is added and the sum is multiplied by 5, the result is 20.

13. If 6 is subtracted from a certain number and the remainder is multiplied by 8, the result is 16.

14. If a certain number is subtracted from 11 and the remainder is multiplied by 4, the result is 24.

15. 5 times a number which is 10 larger than x is 20.

16. If a number is decreased by 6 and then multiplied by 5, the result is 2 more than the number.

EXERCISES. PROBLEMS

Example. One number is 8 less than another. If 3 times the smaller is subtracted from 4 times the larger, the remainder is 47. What are the numbers?

Solution. Let $x =$ the larger number

Then $x - 8 =$ the smaller number

and
$$4x - 3(x-8) = 47$$
$$4x - 3x + 24 = 47$$
$$4x - 3x = 47 - 24$$
$$x = 23, \text{ the larger number}$$
$$x - 8 = 15, \text{ the smaller number}$$

Check. 15 is 8 less than 23. Three times the smaller number is 45. Four times the larger number is 92. $92 - 45 = 47$.

A

1. One number is 7 larger than another. If twice the larger is added to the smaller, the sum is 65. What are the numbers? 17 / 24

2. The difference of two numbers is 10. If twice the larger is subtracted from 5 times the smaller, the remainder is 22. What are the numbers? 14/2

3. A certain truck can haul 4 tons of freight at a time, and a larger truck can haul 6 tons at a time. How many trips must each truck make to haul 58 tons if the smaller truck makes 2 more trips than the larger truck? 7/5

4. The capacity of a certain truck is 3 tons more than the capacity of another. The combined haul of 7 loads of the smaller truck and 5 loads of the larger is 99 tons. What is the capacity of each truck? 7/1

5. One transport carried 1600 more soldiers than another. How many soldiers did each transport carry in each trip if in 8 trips of the smaller and 6 trips of the larger 100,600 soldiers were transported? 6500 / 8100

6. One number is 8 less than another. If 3 times the larger is subtracted from 9 times the smaller, the remainder is 18. Find the numbers. 7/4

7. The difference of two numbers is 13 and their sum is 61. Find the numbers. 24/

8. A boy is 5 years older than his sister. If 3 times the sister's age is subtracted from twice the boy's age, the remainder is 1. Find their ages. 9 yea sist / 14 ye boy

B

9. A farmer sold two steers for $303. If they both sold at 15 cents a pound and the difference of their weights was 140 pounds, what was the weight of each steer?

10. The length of a rectangle is 12 feet more than twice its width. Find the dimensions of the rectangle if its perimeter is 126 feet.

11. The perimeter of a rectangle is 96 inches. Find its dimensions if the length is 10 inches less than twice the width.

12. One angle of a triangle is 2 degrees larger than a second and the third angle is 16 degrees less than the sum of the other two. Find the number of degrees in each angle.

Exceeds and Excess [A]

The word "exceeds" is often used in place of the words "is greater than." Thus the expression "7 exceeds 5 by 2" is used instead of the expression "7 is greater than 5 by 2." The amount of the exceeding is called the *excess*. The excess is the difference of the numbers. The number that does the exceeding is the larger number and the number that is exceeded is the smaller number.

Some pupils have difficulty in forming equations for problems which contain the words "exceeds" or "excess." It is helpful in such cases to substitute the words "is greater than" for "exceeds," and the words "difference of" for "excess over."

There are three common ways of expressing the relationship of the larger number, the smaller number, and their difference. These three ways are indicated briefly as follows:

1. *Larger* − *smaller* = *difference*
2. *Larger* = *smaller* + *difference*
3. *Larger* − *difference* = *smaller*

EXERCISES [A]

1. John's age exceeds Frank's by 6 years. Let $x =$ the number of years in Frank's age. Is John's age greater or less than Frank's age? Which of the following expressions will represent John's age? $x - 6$; $6 - x$; $x + 6$.

2. Write the equation $x = y + 3$ so that both x and y are in the left member. Write the equation so that only y is in the right member.

3. Write the expression "8 exceeds 6 by 2" in equation form three different ways.

4. A's age exceeds B's age by 4 years. If x stands for B's age, what will represent A's age?

5. The excess of B's age over A's age is 10 years. If x stands for A's age, what will represent B's age?

6. The weight of one stone exceeds three times the weight of another by 7 pounds. If x represents the weight of the smaller stone, what will represent the number of pounds in the weight of the larger?

7. Write the expression "b exceeds c by 9" in equation form three different ways.

8. One number exceeds twice another by 4. Express the larger number in terms of m when m stands for the smaller number.

9. The rate of one train exceeds by 18 twice the rate of another. If x is the rate of the slower train in miles per hour, what is the rate of the faster train in miles per hour?

EXERCISES [A]. NUMBER PROBLEMS

Example. The sum of two numbers is 40 and the larger exceeds three times the smaller by 4. What are the numbers?

Solution. Let $x =$ the smaller number

Then $3x + 4 =$ the larger number

Then

$$\begin{aligned} x + 3x + 4 &= 40 \\ x + 3x &= 40 - 4 \\ 4x &= 36 \\ x &= 9, \text{ the smaller number} \\ 3x + 4 &= 31, \text{ the larger number} \end{aligned}$$

Check. The sum of 9 and 31 is 40. Also 31 is 4 more than 3 times 9.

1. One number is represented by x and another by $x - 4$. If the sum of the two numbers is 320, find the numbers.

2. Separate 800 into two parts such that one part will exceed the other by 32.

3. The sum of two numbers is 92 and one exceeds the other by 40. What are the numbers?

4. One of two numbers exceeds the other by 10 and their sum is 126. What are the numbers?

5. Separate 147 into two parts such that the larger will exceed three times the smaller by 27.

6. Separate 119 into three parts such that the second will exceed the first by 8 and the third one will exceed twice the second by 3.

7. The sum of two numbers is 42. Find the numbers if the excess of the larger over 3 times the smaller is 2.

The Word Sum [A]

Let us consider the following problem: "The sum of two numbers is 36. If 3 times the larger is added to 4 times the smaller, the result is 123. What are the numbers?" Suppose we let x represent the smaller number. What will represent the larger number? The answer to this question will be easier after you have done the following exercises.

ORAL EXERCISES [A]

1. If the sum of two numbers is 8 and one of them is 3, what is the other?

2. The sum of two numbers is 13 and the smaller is 5. What is the larger?

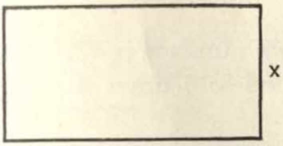
3. The sum of two numbers is 20 and the larger is 11. What is the smaller?

4. The sum of two numbers is 25. If you know one of the numbers, how can you find the other? In finding the second number, do you subtract the sum of the numbers from the first number, or do you subtract the first number from the sum?

5. The sum of two numbers is 20 and x is one of the numbers. Is the other number $x - 20$? Is it $20 - x$? Does $x - 20 = 20 - x$?

6. If 15 is the sum of two numbers and one of them is m, what is the other?

7. The perimeter of the rectangle shown is 80. Represent its length in terms of x.



8. A radio and a violin cost $200. If the cost of the radio is r dollars, what is the cost of the violin?

9. If x is one of two complementary angles, what is the other?

10. If one of two supplementary angles is $3c$, what is the other?

11. The perimeter of a rectangle is P and the length is l. Represent the width in terms of P and l.

12. How would you start the solution of problems beginning as follows?

a. The sum of two numbers is 35.

b. A boy and girl together own 50 rabbits.

c. There are 32 boys and girls in an algebra class.

EXERCISES. WRITTEN PROBLEMS

Example. The sum of two numbers is 36. If 3 times the larger is added to 4 times the smaller, the result is 123. What are the numbers?

Solution. Let $x =$ the smaller number

Then $36 - x =$ the larger number

and

$$3(36 - x) + 4x = 123$$
$$108 - 3x + 4x = 123$$
$$-3x + 4x = -108 + 123$$
$$x = 15, \text{ the smaller number}$$
$$36 - x = 21, \text{ the larger number}$$

A

1. The sum of two numbers is 45. If 3 times the larger is increased by 4 times the smaller, the sum is 155. Find the numbers.

2. The sum of two numbers is 47. What are the numbers if twice the larger added to 3 times the smaller makes a sum of 106?

3. The sum of two numbers is 250. If 4 times the smaller is added to the larger, the sum is 592. What are the numbers?

4. The sum of two numbers is 138. If 4 times the smaller is subtracted from 3 times the larger, the remainder is 99. Find the numbers.

5. Separate 1200 into two parts so that one part will be 800 less than 3 times the other.

6. A farmer sold 425 bushels of wheat and oats for $562. How many bushels of each did he sell, if he received $1.80 a bushel for the wheat and 64 cents a bushel for the oats?

B

7. The second of three numbers exceeds the first by 13, and a third number exceeds the second by 11. What are the numbers if their sum is 91?

8. The sum of two numbers is 154. If the larger exceeds 4 times the smaller by 44, what are the numbers?

9. The larger of two numbers is 7 more than the smaller, and exceeds twice the smaller by 1. What are the numbers?

10. Separate 62 into two parts such that if the larger is added to 18 and the smaller to 32 the two results will be equal.

11. Separate 140 into two parts such that the sum of the larger and 100 will exceed twice the smaller by 36.

C

12. The sum of two numbers is 108. If twice their difference is subtracted from the smaller, the remainder is 9. What are the numbers?

13. Divide 76 into two parts such that if the smaller be taken from 43 and the larger from 61, the remainders are equal.

14. The sum of two numbers is 40. The difference of 5 times the larger and 4 times the smaller exceeds 7 times the smaller by 8. Find the numbers.

15. The sum of two numbers is 156. The excess of 4 times the larger over 4 times the smaller is 75. What are the numbers?

COIN PROBLEMS[A]. ORAL

1. How many cents are there in 3 nickels? in 4 nickels? in x nickels? in $2x$ nickels? How do you find the number of cents in a given number of nickels?

2. How many cents are there in 4 dimes? in 10 dimes? in x dimes? in d dimes? How do you find the number of cents in any number of dimes?

3. How many cents are there in 3 quarters? in 5 quarters? in x quarters? in $5x$ quarters? How do you find the number of cents in a given number of quarters?

4. How many cents are there in $3? in $3.15? in x dollars? in $4y$ dollars?

5. How many cents are there in 3 half-dollars? in x half-dollars?

6. How many cents are there in $(x - 3)$ dimes?

7. How many cents are there in $(2x - 4)$ nickels?

8. How many cents are there in $2(x + 3)$ quarters?

9. Harry has 12 more nickels than dimes and $2.70 in all. If d = the number of Harry's dimes, what does $d + 12$ represent? What does $10d$ represent? What does $5(d + 12)$ represent? How many cents does Harry have? Does $d + d + 12 = 2.70$? Does $d + d + 12 = 270$? Does $10d + 5(d + 12) = 270$?

COIN PROBLEMS

A

1. A man has 2 more nickels than dimes. He has \$1.15 in all. How many coins of each kind has he?

Let $x =$ the number of dimes
Then $x + 2 =$ the number of nickels
and $10x + 5(x + 2) = 115$

Why is x multiplied by 10? Why is $x + 2$ multiplied by 5? Why is 1.15 multiplied by 100? Complete the solution.

2. A boy has 3 more dimes than quarters. The value of all his dimes and quarters is \$1.70. How many coins of each kind has he?

3. A boy has twice as many nickels as dimes and 5 more half-dollars than nickels. If the total value of the coins is \$6.10, how many coins of each kind has he?

4. A girl has 2 more nickels than dimes, and 3 more quarters than nickels, having in all \$3.35. How many coins of each kind has she?

5. A purse contains 21 coins, consisting of nickels and dimes. Find the number of coins of each kind if their total value is \$1.65.

6. A safe contains 160 coins whose value is \$10. The coins are nickels and dimes. Find the number of coins of each kind.

7. A sum of \$14 consists of 92 dimes and quarters. Find the number of dimes and the number of quarters.

8. The initiation fee for joining a woman's club is \$7.50, and the monthly dues are 25 cents. How many years has a woman been a member when she has spent \$31.50 for initiation and membership dues?

B

9. A sum of money amounting to \$10.70 consists of nickels, dimes, and quarters. There are 10 more quarters than nickels, and the number of dimes is 2 less than 3 times the number of nickels. How many coins of each kind are there?

10. Dick gave the grocer \$3.60, consisting of half-dollars, dimes, and nickels. If there were 3 times as many nickels as half-dollars and 2 more nickels than dimes, how many coins of each kind were there?

AGE PROBLEMS [A]. ORAL

1. If John is 10 years old now, how old will he be in 6 years? in 7 years? in x years? in $2x$ years?

2. If Emily is x years old now, how old will she be in 4 years? in 7 years? How old was she 3 years ago? 10 years ago?

3. A is x years old. Represent B's age if B is 4 times as old as A. How old will A be in 3 years?

4. Sarah is m years old. How old is a person 4 times as old? How old is a person 7 years older than Sarah? 6 years younger than Sarah?

5. A is x years old and C is 3 times as old as A. How old was each 8 years ago?

6. The difference in the ages of two men is $(m - 4)$ years. If the age of the older is $10 + m$, what is the age of the younger?

7. John is 9 years old and Patricia is 1 year old. One year from now John's age will be how many times Patricia's age? Compare their ages 3 years from now. Was John's age ever 1000 times Patricia's age? Will her age ever equal his age?

AGE PROBLEMS [A]

Example. A man is 9 times as old as his son. In three years the father will be only 5 times as old as his son. What is the age of each?

Solution. It is usually advisable to make four preliminary statements in the solution of an age problem.

Let $x =$ the number of years in the son's age now

Then $9x =$ the number of years in the father's age now

$x + 3 =$ the number of years in the son's age in 3 years

$9x + 3 =$ the number of years in the father's age in 3 years

(cont. on next page)

Many pupils prefer to arrange these four statements in box form, as follows:

PERSON	AGE NOW	AGE IN 3 YEARS
son	x	$x + 3$
father	$9x$	$9x + 3$

Then
$$\begin{aligned} 9x + 3 &= 5(x + 3) \\ 9x + 3 &= 5x + 15 \\ 9x - 5x &= -3 + 15 \\ 4x &= 12 \\ x &= 3, \text{ the son's age now} \\ 9x &= 27, \text{ the father's age now} \end{aligned}$$

1. Henry is 10 years older than James. In 8 years twice Henry's age will equal 3 times James's age. How old is each at present?

2. Walter is 8 years older than Ray. In 6 years 5 times Walter's age will equal 9 times Ray's age. How old is each at present?

3. The sum of the ages of Mary and her mother is 60 years. In 20 years twice Mary's age, increased by her mother's age (then), will equal 138 years. How old is each?

4. Robert is 14 years old and his father is 38 years old. How many years ago was the father exactly seven times as old as Robert?

5. Two years ago a man was 4 times as old as his son. Three years from now the father will be only three times as old as the son. How old is each at present?

Suggestion. Let x and $4x$ represent their ages 2 years ago.

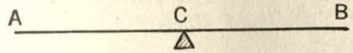
6. A man was 30 years of age when his son was born. The father's age now exceeds three times the son's age by 6 years. How old is each at present?

7. Frank is 4 times as old as Carl. In 10 years he will be only twice as old as Carl is then. How old is each?

8. Elsie said to the class, "The sum of Mary Jane's age and my age is 29 years. Four times my age 10 years ago exceeded Mary Jane's age 5 years ago by 2 years. How old is each?"

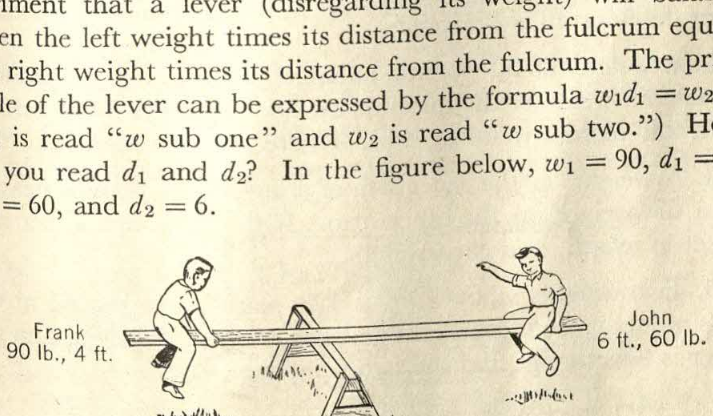
Principle of Levers[A]

A *lever* is a stiff board or bar which is balanced upon a support called the *fulcrum*. The parts of the lever on either side of the fulcrum are called the *lever arms*. In the figure, AB is a lever, AC and BC are the lever arms, and C is the fulcrum.



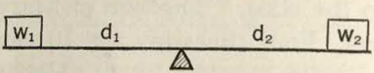
When equal weights are placed at the ends A and B, the lever will balance when C is the middle of the lever.

Most of you have played on a teeterboard, or seesaw, and know that two people will balance only when the heavier of the two sits nearer the fulcrum. Thus if Frank weighs 90 pounds and John weighs 60 pounds, they will balance on the teeterboard when Frank sits 4 feet from the fulcrum and John sits 6 feet from the fulcrum ($90 \times 4 = 60 \times 6$). It has been found by experiment that a lever (disregarding its weight) will balance when the left weight times its distance from the fulcrum equals the right weight times its distance from the fulcrum. The principle of the lever can be expressed by the formula $w_1d_1 = w_2d_2$. (w_1 is read "w sub one" and w_2 is read "w sub two.") How do you read d_1 and d_2? In the figure below, $w_1 = 90$, $d_1 = 4$, $w_2 = 60$, and $d_2 = 6$.


Frank
90 lb., 4 ft.
John
6 ft., 60 lb.

* ORAL EXERCISES [A]

1. Will this lever balance when $w_1 = 4, d_1 = 6, w_2 = 3, d_2 = 8$?



2. Will the lever balance if $w_1 = 7,\ d_1 = 7,\ w_2 = 8$, and $d_2 = 6$? If they do not balance, which end will rise?

3. Will the lever balance when $w_1 = 60, w_2 = 40, d_1 = 5$, and $d_2 = 7\frac{1}{2}$?

4. Find w_1 when $w_2 = 100, d_2 = 4, d_1 = 5$, and w_1 balances w_2.

5. Find d_1 when $w_1 = 50, w_2 = 80, d_2 = 5$, and w_1 balances w_2.

LEVER PROBLEMS

A

Make a sketch for each problem, and solve.

1. Jack sits 6 feet from the fulcrum and balances Carl who sits 8 feet from the fulcrum. If Jack weighs 70 pounds, how much does Carl weigh?

2. Mary, who weighs 135 pounds and sits 4 feet from the fulcrum, balances Imogen, who weighs 67.5 pounds. How far from the fulcrum is Imogen?

3. A weight of 30 pounds balances a weight of 50 pounds on the other side of the fulcrum. If one weight is 2 feet farther from the fulcrum than the other, where is each weight placed?

4. A weight on the short arm of a lever is 27 pounds heavier than the weight on the other arm. If the arms are 6 feet and 8 feet in length, find each weight.

5. Two weights whose sum is 128 pounds are placed at the ends of a balanced lever. If one lever arm is 7 feet long and the other is 9 feet long, find the two weights.

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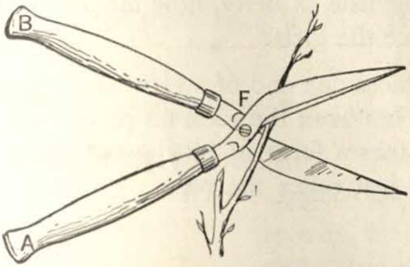
* Disregard the weights of the levers in these problems.

6. Where is the fulcrum of a lever 93 inches long if a weight of 170 pounds at one end balances a weight of 54 pounds at the other end?



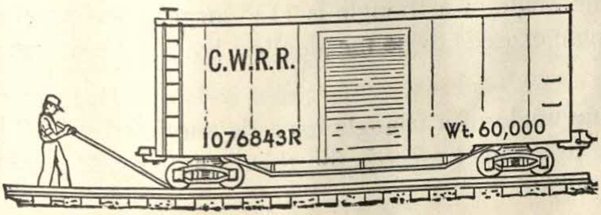
7. When cutting a branch of a hedge a man applied a force of 40 pounds on the handles *A* and *B* of the shears shown here. If *AF* is 24 inches and the branch is $1\frac{1}{2}$ inches from *F*, what force was exerted on the limb?

B

8. Two girls weighing 80 pounds and 120 pounds respectively wish to sit at the ends of a 10-foot teeterboard and teeter. How far from the larger girl should the fulcrum be placed?

9. A lever is 30 feet long. How far from each end must a fulcrum be placed so that a $7\frac{1}{2}$-pound weight at one end will balance a 6-pound weight at the other end?

10. A man who can exert a force of 170 pounds on a pinch bar is just able to move a freight car weighing 60,000 pounds. If the pinch bar is 5 feet long and the short end is $1\frac{1}{4}$ inches from the fulcrum, what force is applied to the wheel of the car?



11. Betty, Fern, and Bob, weighing 115 pounds, 120 pounds, and 140 pounds respectively, balance on a teeterboard. If Betty sits 6 feet from the fulcrum and Fern sits $4\frac{1}{2}$ feet from the fulcrum on the same side as Betty, how far from the fulcrum must Bob sit to balance the girls?

12. A man places one end of a crowbar $5\frac{1}{2}$ feet long under a rock. At $4\frac{1}{2}$ inches from the rock he places a stone to act as a fulcrum. If he uses a force of 160 pounds on the other end of the crowbar, he just raises the rock. What weight of rock has he lifted?

GEOMETRY PROBLEMS

A

1. The perimeter of a rectangle is 170 feet. If the length exceeds 5 times the width by one foot, what is its area? (First find the width and the length.)

2. One angle of a triangle exceeds another by 12° and the third angle exceeds twice the smallest by 48°. How large is each angle?

3. Five times one of two complementary angles equals 4 times the other. How large is each?

4. Two angles are supplementary. If 7 times the smaller is subtracted from 3 times the larger, the remainder is 190°. How large is each angle?

5. Find the size of the angle whose supplement is 3 times as large as its complement.

B

6. One angle of a triangle is 115° larger than another. The third angle exceeds twice the smallest by 9°. How large is each angle?

7. One angle of a triangle exceeds another by 2°. The sum of these two angles exceeds the third angle by 28°. How large is each angle?

8. How large is a rectangular field whose length exceeds twice the width by 10 rods and whose perimeter is 1 mile?

9. One angle of a triangle exceeds another by 23°. Twice the smallest angle exceeds the third angle by 11°. How large is each angle?

WORD LIST

Be sure you can spell all these words:

area (p. 49)
circle (p. 54)
circumference (p. 54)
conditional equation (p. 178)
exceeds (p. 182)
excess (p. 182)
formula (p. 191)
fulcrum (p. 191)
geometry (p. 194)
identical equation (p. 178)
lever (p. 191)
monomial (p. 13)
parenthesis (p. 176)
polynomial (p. 13)
quotient (p. 88)
radius (p. 54)
trinomial (p. 13)

CHAPTER REVIEW [A]

Solve:

1. $(x + 3) + 2 = 7$
2. $2(6\,c - 5) - 2 = 24$
3. $12 - 9(2 - m) = 30$
4. $9(1 + 2\,y) - 8\,y - 69 = 0$
5. $4(x - 20) + 2(x + 6) + 77 = 0$
6. $2(5\,c - 1) - 6 = 3(c - 7) - 14$
7. $13\,x - (3\,x + 2) = 12\,x - (5\,x - 4)$
8. $6\,x + 7(4 + x) - 6(3 - 4\,x) = 0$

9. Is $5(x + 2) = 10 + 5\,x$ a conditional or an identical equation?

10. Is $5(x + 2) = 3\,x - 12$ a conditional or an identical equation?

11. Mary has \$3.45 in quarters and dimes. She has 10 more dimes than quarters. How many dimes and how many quarters has she?

12. Find two numbers if their sum is 45 and their difference is 13.

13. A man carries two packages weighing 28 and 42 pounds by fastening them to the ends of a 5-foot pole which he places over his shoulder. At what point does the pole rest on his shoulder when the packages balance?

14. The capacities of two trucks are 3 tons and 4 tons respectively. If the larger truck makes 3 more trips than the smaller, its total haul is 20 more tons than that of the smaller. How many trips does each truck make?

15. The capacities of two trucks are 3 tons and 5 tons respectively. If the smaller truck makes 18 more trips than the larger, it can deliver 12 more tons of freight than the larger. How many trips does each truck make?

16. A is 11 years old and B is 68 years old. In how many years will B be just 4 times as old as A?

17. Mrs. West is 14 years younger than her aunt. If Mrs. West's age in years is as much below 60 as her aunt's age is over 40, how old is each?

18. Jim is 11 years older than his brother John. In 5 years Jim will be twice as old as John. How old is each boy at present?

19. The perimeter of a rectangle is 256 inches. How long are the sides if the length exceeds the width by 16 inches?

20. One angle of a triangle exceeds another by 30° and the third angle exceeds the smallest by 15°. How many degrees are there in each angle?

21. One of two complementary angles exceeds the other by 22°. How large are the angles?

GENERAL REVIEW

A

In the following twelve sentences certain words have been omitted. On your paper write the words which best fit the blank spaces:

1. $x = 3$ is a --?-- of the equation $4x = 12$.

2. The equation $4x = 12$ is called a --?-- equation because it has only one --?--.

3. $3(x+1) = 3x + 3$ is called an --?-- equation because it has an infinite number of --?--.

4. The --?-- of 8 divided by 2 is 4.

5. The --?-- of 8 and 2 is 16.

6. The --?-- of 8 and 2 is 10.

7. The --?-- of 8 and 2 is 6.

8. 9 --?-- 7 by 2.

9. The --?-- of 8 over 6 is 2.

10. To find the number of cents in a given number of dimes, we --?-- the number of dimes by --?--.

11. The equation $w_1d_1 = w_2d_2$ expresses the principle of the --?--.

12. The supplement of an angle of 50° is an angle of --?-- degrees.

13. Indicate the following by algebraic expressions:

a. m increased by twice x.

b. The square of x decreased by the cube of y.

c. Four more than twice the number x.

d. Eight times the sum of x and $3y$.

If $a = 4$, $b = 5$, $c = -7$, and $d = 0$, find the value of

14. $a^3 - b^3$

15. $abd - d$

16. $c^3 - ab$

17. $2ab + d^2$

18. $bc - a^2$

19. $-4b + 5c$

20. $\dfrac{3d - 5a}{b - a}$

21. $\dfrac{a^3 + b^2 + c}{5c - b - 2}$

22. If $5x - 3y = 9$, find y when $x = 0$.

23. If a newspaper costs 4 cents a copy, write a formula for the number of copies that can be bought for x dollars.

24. To have some handbills printed one must pay \$3 for the first 100 copies and 20 cents per hundred for each additional hundred. Write a formula for the total cost (c) of n handbills, assuming that n is greater than 100.

25. $C = 2\pi r$. Find C when $\pi = 3.14$ and $r = 16$.

26. $A = \pi r^2$. Find A when $\pi = 3.14$ and $r = 12$.

27. $C = \frac{5}{9}(F - 32)$. Find C when $F = 77$.

28. A room is 30 feet long, 24 feet wide, and 11 feet high. How many cubic feet are there in the room? If 40 pupils are in the room, how much space is there in it for each pupil?

29. A cylindrical silo 40 feet high has an inside diameter of 8 feet 6 inches. What is its volume? ($V = \pi r^2 h$, $\pi = 3.14$.)

30. A spherical balloon has a diameter of 27 feet. What is the volume? ($V = \frac{4}{3}\pi r^3$, $\pi = 3.14$.)

31. From $x^2 - 2x + 3$ take $2x^2 + 3x - 2$.

32. Take $1 - x$ from $3x^2 - 4x$.

33. From the sum of $2x^2 - 5x - 3$ and $x^3 - x^2 + 4$ take $2x^3 - x^2 + 8$.

34. Write $xxxx$ using an exponent.

35. Do as indicated:

a. x^2x^3

b. $-4(6a)$

c. $5a^2(-3b)$

d. $(4x)^3$

e. $(6x) \div 2$

f. $x^2(x^2 - 5x)$

g. $7(x^2 + y^2)$

h. $x(1 - x)$

i. $(3x - 6) \div (-2)$

j. $(-26x^2y^3) \div (2x^2y)$

k. $2(5x - 3)$

l. $(15a^2 - 10a) \div 5a$

m. $(7x^3 - 14x^2 + 21x) \div (-7)$

n. $(4x^4 - 12x^3 - 20x^2 - 2x) \div (2x)$

36. One number is 7 larger than another. The sum of the numbers is 43. What are they?

37. The sum of three numbers is 63. The second is 8 larger than the third and the first is 5 more than 5 times the third. What are the numbers?

38. One number is 3 less than another. Six times the larger decreased by 7 times the smaller is 7. What are the numbers?

39. If 4 times the complement of a certain angle is subtracted from twice its supplement, the result is 104°. How large is the angle?

40. Ann is 4 years older than Jane. Eight years ago Ann was twice as old as Jane. How old are the girls?

B

41. Simplify: $2a + [a - 3a] - \{6 - 5a\} - (-4 - 3a)$

42. Simplify:

$$(2x^2 - 3x + 9) - (-5x^2 - 3) + (-7x^2 - 19x - 3)$$

43. Simplify: $4(\frac{1}{4}a - \frac{1}{2}b + 3c) - 6(-\frac{1}{3}a - \frac{1}{12}b + 3\frac{1}{2}c)$

44. Simplify: $\frac{1}{3}(3a - 4b - 6c) - \frac{1}{4}(6a - 5b + 9c)$

45. $A = 4\pi r^2$. Find A to the nearest tenth if $\pi = 3.1416$ and $r = 4.63$.

46. $l = a + (n - 1)d$. Find l if $a = 5$, $d = -2$, and $n = 24$.

47. $S = \frac{n}{2}(a + l)$. Find S if $a = 5$, $n = 24$, and $l =$ the value found in Exercise 46.

Solve:

48. $x + 1.2x = 560$

49. $.2y - 19 = 10 - .3y$

50. $x - 4944.6 + .05x = 0$

51. $(2x + 3) - (x - 6) + (3x + 7) = 18$

52. $10x - [.2x - 6] = 8x + [2.4x - 9]$

53. In 7 years William will be 3 times as old as Helen. What are their present ages, if William's age 7 years ago was 6 times Helen's age 8 years ago?

54. If the supplement of a certain angle is subtracted from 9 times its complement and the remainder is multiplied by 2, the product is 15° less than the angle itself. How many degrees are there in the angle?

55. There are 3 consecutive integers such that the sum of the first, twice the second, and 3 times the third is 98. What are the integers? (Represent the integers by x, $x+1$, and $x+2$.)

56. There are 4 consecutive odd integers whose sum exceeds twice the largest by 102. Find the integers. (Represent the integers by x, $x+2$, $x+4$, and $x+6$.)

57. In an election there were 5760 votes cast for Brown, Jones, and Smith. Jones received 5 times as many votes as Brown. If Smith had received 1320 more votes and Jones had received 1460 less votes, Smith would have received twice as many votes as Jones. How many votes did each receive?

CHAPTER TEST [A]

Part I. Equations

Solve:

1. $3(x+2)=15$

2. $3y-4(y+2)=5$

3. $6c+2(c+1)=-14$

4. $4b-1=2(b+3)$

5. $x-7-(x+2)=3x$

6. $m-3(m+1)=-7$

7. Is $2(x-1)=2x-2$ a conditional or an identical equation?

8. Show by checking that -3 is a root of $2(x-1)-(x+1)=-6$.

Part II. Problems

1. One number is 5 less than another. If 4 times the smaller is subtracted from 3 times the larger, the remainder is 17. Find the numbers.

2. Write in three ways in equation form the fact that x exceeds y by 4.

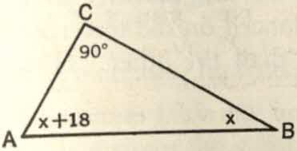
3. If n stands for the number of degrees in one of two complementary angles, what will represent the number of degrees in the other angle?

4. The sum of two numbers is 15. If 4 times the smaller is added to 3 times the larger, the sum is 49. Find the numbers.

5. A is 3 times as old as B. Five years ago A was 8 times as old as B. How old is each now?

6. A purse contains 4 more dimes than nickels. Find the number of nickels and the number of dimes if their value is $1.60.

7. Find the number of degrees in $\angle A$ and in $\angle B$.



8. A weighs 38 more pounds than B. They find that they balance on a teeterboard when A's distance from the fulcrum is 6 feet and B's distance from the fulcrum is 9 feet. What is the weight of each?

CHAPTER TEST [B]

Part I. Equations

Solve:

1. $10 - (x - 4) = 1$

2. $3(c + 2) - c = 4$

3. $2(x + 1) - 3(2x + 3) = 3x$

4. $4(y + 2) - 6(y + 3) = y - 4$

5. $2(c - 3) = 8 - 3(c - 2)$

6. $4(x + 9) - 18x = 4(1 - 2x) - 5(2 - 3x)$

7. Complete: An equation that is true for only one value of the letter is called $\left.\begin{matrix}\text{a}\\\text{an}\end{matrix}\right\}$ --?-- equation.

Part II. Problems

1. The sum of two numbers is 13. If s represents the smaller number, represent the larger number in terms of s.

2. A's age exceeds B's by 12 years. Eight years ago A was twice as old as B. How old is each?

3. The value of 40 coins consisting of dimes and quarters is \$6.70. Find the number of coins of each kind.

4. The length of a tennis court for singles is 3 feet less than 3 times its width. Find the dimensions of the court if its perimeter is 210 feet.

5. A teeterboard 20 feet long is balanced at its middle. One child weighs 40 pounds and another weighs 56 pounds. When the children are balanced on the teeterboard, one child is 2 feet nearer the fulcrum than the other. How far is each from the fulcrum?

CUMULATIVE TEST (CHAPTERS I–VI). Time, 40 Minutes

19–20 correct, excellent; 16–18, good; 13–15, fair.

1. Simplify $8b - 5b^2 + 2b - 3b^2 - 10$.

2. Subtract $3a - 2b + 5c$ from $8a - b - 2c$.

3. Divide $-8x^8y^4$ by $-4x^4y^2$.

4. Multiply $-6a^6b^4$ by $2a^2b^4$.

5. If $x = -5$ and $y = -5$, find the value of

a. xy^2

b. $4xy$

c. $-x^3$

d. $4x - 2y$

e. $\frac{x^2}{xy}$

6. Solve $3x - 2 - 4x = 0$.

7. Add $x^2 - x + 3$, $3x^2 + 6x - 5$, and $x^2 + x - 4$.

8. Divide $28x^7 - 40x^5 + 24x^3 - 4x$ by $-4x$.

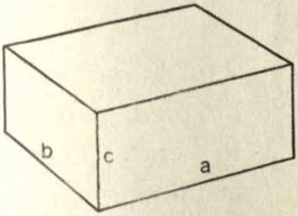
9. Multiply: $-3y^2(2y^3 - 3y^2 + 6y)$.

10. If $x = -5$ and $y = 10$, find the value of $2x^3 - xy - y^2$.

11. Find the area of a triangle if its height is 10.6 inches and its base is 6.4 inches.

12. One morning the temperature was $-14°$ Fahrenheit. What was the temperature centigrade? $C = \frac{5}{9}(F - 32)$.

13. Write a formula for the total surface, T, of a rectangular solid a inches long, b inches wide, and c inches high.



14. $x = 10 - 4y$. If y increases, how does x change?

15. The perimeter of a rectangle is 120 inches. Find its width and length if the length is 4 times the width.

16. A boy has 22 coins consisting of nickels and dimes. If their total value is $1.85, how many coins of each kind has he?

17. A man is 6 times as old as his son. In 6 years the father will be 3 times as old as his son. What are their present ages?

18. An 8-pound weight and a 6-pound weight are balanced on a lever. If the distance between the weights is 28 inches, how far is the smaller weight from the fulcrum?

19. If 8% of a man's income is $192, what is his income? ($P = br$)

20. Solve: $3(x - 6) - (x - 2) = 0$.

MATHEMATICS IN AERONAUTICS

The mathematics essential to an air pilot, navigator, or one wishing to study aeronautics consists of arithmetic, algebra, plane and solid geometry, and trigonometry. Anyone contemplating being a designer in aeronautics should continue the study of mathematics in an engineering college.

Arithmetic is needed to perform the various computations in aeronautics. Algebra is needed to solve formulas and equations. Plane geometry is needed to understand the forces acting upon the airplane, for finding the radius of action, ground speed, bearing, interception, course, and other necessary figures. Trigonometry is used to find the angle of bank, the height of a cloud, components of forces and velocities, and gliding and climbing angles.

In the Second World War many young men were unable to become pilots and navigators because they were deficient in mathematics. Those who were accepted as candidates for positions as pilots and navigators were given intensive training in the parts of algebra, geometry, and trigonometry that pertained to flying.

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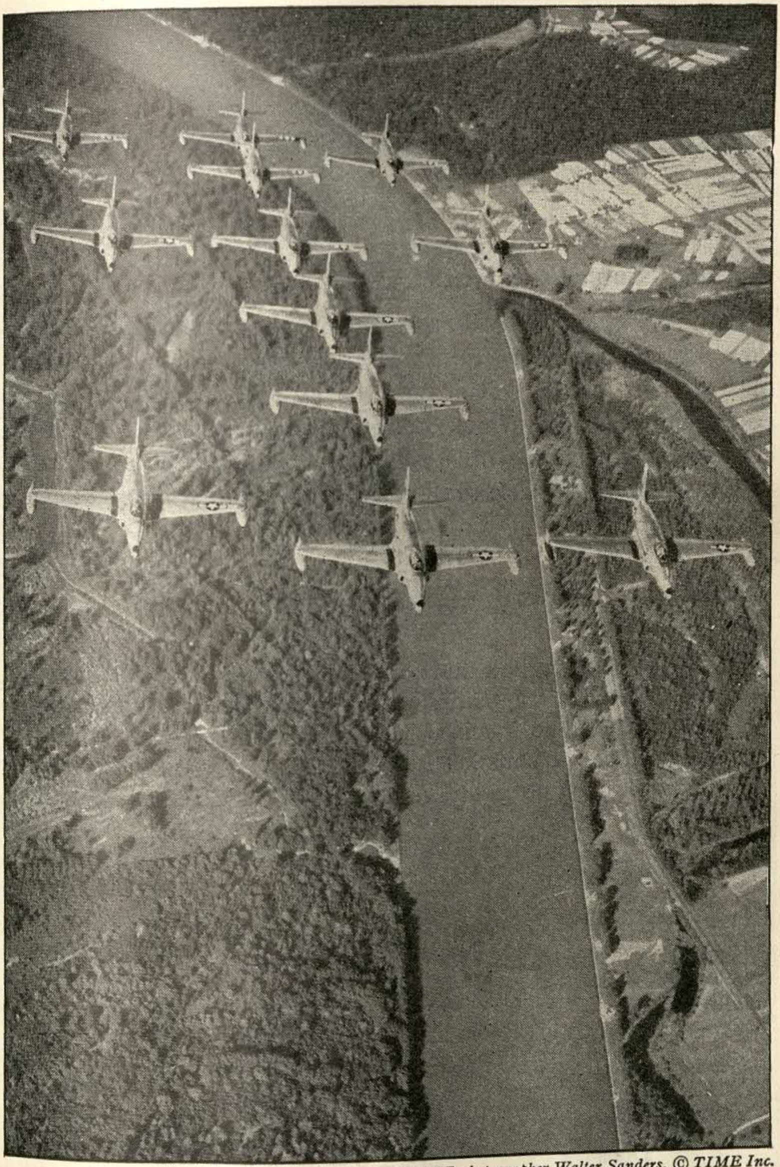


By LIFE photographer Walter Sanders, © TIME Inc.

F–80 Jet Planes Flying in Formation

CHAPTER VII

Equations Containing Fractions

Clearing Equations of Fractions [A]

In this chapter you will make a more thorough study of equations which contain fractions. In most cases equations containing fractions can be solved more easily by changing them into equivalent equations which do not contain fractions. When you change an equation with fractions into one without fractions, you are *clearing the equation of fractions.* Study the following examples:

Example 1. Solve $\frac{x}{3} - \frac{x}{5} = 6$.

Solution. The smallest number that can be exactly divided by 3 and 5 is 15. If we multiply each member of the equation by 15, there will be no fractions left in the equation. When the left member of the equation is multiplied by 15, each term of the equation must be multiplied by 15. The solution is set down as follows:

$$\frac{x}{3} - \frac{x}{5} = 6$$

M_{15} $$15\left(\frac{x}{3}\right) - 15\left(\frac{x}{5}\right) = 15(6)$$

Then $$5x - 3x = 90$$

$$2x = 90$$

D_2 $$x = 45$$

Check. Does $\frac{45}{3} - \frac{45}{5} = 6$?

Does $15 - 9 = 6$? Yes.

The parentheses are used to denote multiplication. They make the solutions more easily understood.

Example 2. Solve $\frac{1}{3} + \frac{5x}{6} = \frac{7}{12}$.

Solution. The smallest number which can be exactly divided by the denominators 3, 6, and 12 is 12. It is called the *lowest common denominator*, abbreviated L.C.D. The solution should be written as follows:

$$\frac{1}{3} + \frac{5x}{6} = \frac{7}{12}$$

M_{12} $$12\left(\frac{1}{3}\right) + 12\left(\frac{5x}{6}\right) = 12\left(\frac{7}{12}\right)$$

$$4 + 10x = 7$$

$$10x = 7 - 4$$

$$10x = 3$$

D_{10} $$x = \tfrac{3}{10}$$

Check. Does $\frac{1}{3} + \frac{5}{6}(\frac{3}{10}) = \frac{7}{12}$?

Does $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$?

Does $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$? Yes.

You should now learn the steps in solving equations containing fractions and parentheses.

1. If an equation contains one or more fractions, clear it of fractions by multiplying each of its members by the lowest common denominator.
2. Remove parentheses.
3. Change the equation so that all terms containing the unknown letter are in one member and all other terms are in the other member.
4. Simplify each member.
5. Divide each member by the coefficient of the unknown letter.
6. Check the solution by substituting the root in the original equation.

NOTE. In some cases it is advisable to remove parentheses in an equation before clearing it of fractions.

EXERCISES [A]

Solve and check:

1. $\frac{x}{4} = \frac{1}{3}$

2. $\frac{3}{4}x = \frac{2}{3}$

3. $\frac{5m}{7} = \frac{1}{14}$

4. $3\frac{1}{7}r = 44$

5. $\frac{2y}{5} - 7 = y$

6. $p - \frac{5}{6}p = 3$

7. $\frac{k}{3} - \frac{k}{4} = 8$

8. $\frac{x}{3} + \frac{2x}{5} = \frac{11}{15}$

9. $\frac{2c}{9} - 4 = \frac{c}{6}$

10. $\frac{b}{3} + \frac{b}{2} = 5$

11. $\frac{x}{3} + \frac{x}{4} = 1$

12. $\frac{m}{2} + 1 = \frac{2m}{5}$

13. $\frac{5x}{18} = \frac{x}{6} - 4$

14. $\frac{2x}{7} - \frac{x}{3} = 4$

15. $\frac{3x}{4} - 2 = \frac{x}{5}$

16. $\frac{7x}{6} - \frac{5}{3} = \frac{3x}{4}$

17. $\frac{3h}{4} = \frac{2h}{5} + \frac{7}{20}$

18. $\frac{2x}{3} + \frac{5x}{4} = \frac{23}{24}$

19. $y = 14 + \frac{y}{3} + \frac{y}{5}$

Example 3. Solve and check: $\frac{2x}{3} + \frac{3}{4}(28 - x) = 20.$

Solution. The L.C.D. is 12.

M_{12}

$$\overset{4}{\cancel{12}}\left(\frac{2x}{\cancel{3}}\right) + \overset{3}{\cancel{12}}\left(\frac{3}{\cancel{4}}\right)(28 - x) = 12(20)$$

$$8x + 9(28 - x) = 240$$

$$8x + 252 - 9x = 240$$

$$8x - 9x = 240 - 252$$

$$-x = -12$$

$$x = 12$$

Check. Does $\frac{2}{3}(12) + \frac{3}{4}(28 - 12) = 20$?

Does $8 + 12 = 20$? Yes.

EXERCISES

A

1. $\frac{1}{5}(x - 2) = 4$

2. $\dfrac{c + 6}{7} = \dfrac{c - 3}{8}$

3. $\frac{1}{3}(2x - 1) = \frac{1}{7}(11 - 8x)$

4. $\dfrac{h + 1}{2} = 1 - \dfrac{1 - 2h}{5}$

5. $\frac{1}{4}(3c + 5) - \frac{1}{2}(2c + 3) = \frac{1}{2}$

6. $\frac{2}{3}x + \frac{3}{4}(35 - x) = 25$

7. $\frac{1}{6}y - \frac{1}{8}(20 - y) = 1$

8. $\dfrac{c}{6} - \dfrac{2}{9}(90 - c) = 1$

9. $\frac{1}{2}(x - 7) = \frac{1}{5}(7 - x)$

10. $\dfrac{x - 4}{5} - \dfrac{3(x - 2)}{10} = 0$

— stop

Caution. $10 \times 0 = 0$

11. $\dfrac{c - 4}{9} - \dfrac{c + 2}{6} = \dfrac{1}{3}c$

12. $x - 2 = \dfrac{5x}{3} - \dfrac{2(x - 1)}{9}$

13. $\frac{1}{2}(y + 1) - \frac{1}{3}(y - 1) = \frac{7}{3}$

14. $\dfrac{3x - 1}{6} - \dfrac{x - 3}{4} = \dfrac{4}{3}$

15. $12 - \dfrac{2k + 9}{5} = \dfrac{k + 3}{7}$

16. $\dfrac{x + 4}{3} - \dfrac{x + 12}{6} = \dfrac{x - 2}{12}$

17. $2k - \dfrac{11 + k}{2} = \dfrac{19 + k}{3}$

18. $\dfrac{3c - 3}{4} - \dfrac{3}{2} + \dfrac{5c - 7}{4} = 0$

19. $\dfrac{2x + 3}{5} - \dfrac{x - 3}{3} = 2$

20. $\dfrac{10x + 7}{8} = \dfrac{11x + 4}{2} - \dfrac{5x}{4}$

B

21. $\dfrac{x - 12}{10} - \dfrac{3(2x + 1)}{3} + \dfrac{3(x - 2)}{2} = 0$

22. $\dfrac{x + 6}{7} - \dfrac{x + 7}{8} + \dfrac{x + 8}{9} = 1.$ The L.C.D. $= 7 \times 8 \times 9.$

23. $\dfrac{1}{4}x + 20 = \dfrac{x}{8} - \dfrac{5}{6}x + \dfrac{1}{2}x + 7$

24. $5y + \frac{1}{7}(3y - 4) + \frac{1}{3}(5y + 3) = 43$

25. $\dfrac{x - 10}{5} + \dfrac{x - 8}{6} - \dfrac{x - 5}{10} = \dfrac{x - 11}{3} - \dfrac{5}{2}$

26. $\dfrac{2x - 1}{3} - \dfrac{x + 10}{6} = \dfrac{10 - 3x}{5} - \dfrac{x + 40}{2}$

Equations Containing Decimal Fractions[A]

Equations containing decimal fractions are often solved more easily by clearing them of decimals.

A decimal fraction is a fraction whose denominator, being a power of 10, is not written but is indicated by a decimal point. Thus .3 means $\frac{3}{10}$; 1.23 means $1\frac{23}{100}$ or $\frac{123}{100}$; and 4.162 means $4\frac{162}{1000}$ or $\frac{4162}{1000}$. Notice that there are as many digits to the right of the decimal point in the decimal as there are zeros in the denominator of the equal common fraction.

Example 1. Solve $.2\,x = 50$.

olution. The L.C.D. is 10.

$$\begin{aligned} & .2\,x = 50 \\ M_{10} \qquad & 2\,x = 500 \\ D_2 \qquad & x = 250 \end{aligned}$$

Check. Does $.2(250) = 50$?

Does $50 = 50$? Yes.

Example 2. Solve $.3(2\,x - 4) - .025(x - 3) = 3.475$.

Solution. Either the parentheses may be removed and the equation then cleared of decimals, or the equation may be cleared of decimals and the parentheses then removed. This example will be solved by the second method.

The L.C.D. is 1000.

$$\begin{aligned} & .3(2\,x - 4) - .025(x - 3) = 3.475 \\ M_{1000}, \quad & 1000(.3)(2\,x - 4) - 1000(.025)(x - 3) = 1000(3.475) \\ & 300(2\,x - 4) - 25(x - 3) = 3475 \\ & 600\,x - 1200 - 25\,x + 75 = 3475 \\ & 600\,x - 25\,x = 3475 + 1200 - 75 \\ & 575\,x = 4600 \\ & x = 8 \end{aligned}$$

Check. Does $.3(16 - 4) - .025(8 - 3) = 3.475$?

Does $3.6 - .125 = 3.475$? Yes.

EXERCISES

A

Solve and check:

1. $.3x = 12$
2. $.05y = 6$
3. $.08p = 16$
4. $1.25h = 12$
5. $.2r = 4$
6. $.003k = 1.5$
7. $2.3b = 11.5$
8. $.004t = .02$
9. $.08y = 12$
10. $.012m = .06$
11. $1.04y = 52$
12. $1.20p = 9$
13. $.4r = 32$
14. $.625h = 10$
15. $.375x = 24$
16. $4x - 2.3 = 3x - .5$
17. $7y - 1.7 = 2y - 4.2$
18. $.4x + 5 = 9$
19. $.09y - 10.8 = -11$
20. $5.2x - .68 = 1.8$
21. $3.47x - 8 - 1.2x = 39.6$
22. $1.2y - 17 = 8 + .7y$
23. $p + .12p = 280$
24. $c + .04c = 228.80$
25. $1.3x + 604 = 2.7x + 142$

B

In solving some equations it is advisable to remove parentheses before clearing the equation of decimals. Use your own judgment when solving the following equations.

26. $2.17x + (1.5x - 3) = 2.14x + 2.4$
27. $0.006x + 0.296 = 0.15x - 0.07x$
28. $1.25x - .25(2x + 1) = 1.25$
29. $x = 4.25 - .25(2x - 1)$
30. $0.25x = .3125 - 1.25(x + 1)$
31. $3(0.4y + 0.5) + 4(0.5y - 0.15) = 26$
32. $0.5(c + 0.2) + 0.07 - (0.3 - c) = 0.02$

Meaning of the Word "of" [A]

Many pupils are unable to solve written problems because they do not *understand* the problems. The failure in understanding a problem is often due to not knowing the meaning of a word. One word which is much used in mathematics and should not be misunderstood is *of*. The word *of* very often means "times" and pupils should learn when its meaning is "times." For example, "$\frac{1}{3}$ of x" means "$\frac{1}{3}$ times x," and can be written $\frac{1}{3} \times x$, $\frac{1}{3} \cdot x$, $\frac{1}{3}x$, or $\frac{x}{3}$. Similarly "$\frac{2}{5}$ of c" can be written $\frac{2}{5} \times c$, $\frac{2}{5} \cdot c$, $\frac{2}{5}c$, or $\frac{2c}{5}$. Likewise "$\frac{2}{5}$ of the sum of x and 3" can be written $\frac{2}{5}(x + 3)$ or $\frac{2(x + 3)}{5}$. The following exercises will assist you in solving some problems that will appear later.

EXERCISES [A]

Express the following, using algebraic symbols:

1. $\frac{2}{3}$ of m.

2. Three fourths of k.

3. One half of c.

4. Two thirds of h.

5. Four sevenths of y.

6. The sum of a and b.

7. The sum of m and -5.

8. One half the sum of x and y.

9. Four fifths of $(2x - 3)$.

10. The sum of p and $\frac{2}{3}q$.

11. Is $\frac{3}{4}x + 5$ the sum of $\frac{3}{4}x$ and 5, or is it $\frac{3}{4}$ the sum of x and 5?

12. Is $\frac{1}{4}(m + 7)$ the sum of $\frac{1}{4}m$ and 7, or is it $\frac{1}{4}$ the sum of m and 7?

13. What is the difference in meaning between $\frac{2}{5}x + 15$ and $\frac{2}{5}(x + 15)$?

Let x stand for the unknown number in exercises 14–17 and express

14. $\frac{5}{7}$ of the sum of a number and -14.

15. The sum of $\frac{5}{7}$ of a certain number and -14.

16. Six added to $\frac{3}{8}$ of a number.

17. Four added to one ninth of a certain number.

18. If x stands for the smaller of two numbers and their sum is 10, what will stand for the other?

19. The sum of two numbers is 20 and the larger one is y. What is the smaller? What is $\frac{3}{5}$ of the smaller?

20. John is x years old. Frank is 4 years older than John. How old is Frank? What is $\frac{2}{3}$ of John's age? What is $\frac{3}{4}$ of Frank's age? What was John's age 7 years ago? What was $\frac{3}{4}$ of John's age 7 years ago?

21. The perimeter of a rectangle is 40 feet. What is its length if its width is w? What is $\frac{5}{6}$ of its width? What is $\frac{7}{8}$ of its length?

22. A board 24 feet long had x feet cut off. What is $\frac{2}{3}$ of the part that is left?

Form equations, letting x equal the number:

23. Two thirds of a number is 60.

24. Three fourths of a number is 45.

25. Seven eighths of a number, increased by six, equals 20.

26. The sum of one seventh of a number and one fourth of the number is 10.

27. If $\frac{1}{6}$ of a number is subtracted from $\frac{1}{4}$ of it, the remainder is 1.

EXERCISES. PROBLEMS

A

1. One half of a number is 15. Find the number.

2. The difference between one third of a number and one fourth of it is 4. Find the number.

3. The width of a rectangle is $\frac{3}{4}$ of its length, and the perimeter is 210 feet. What are the length and width of it?

4. The perimeter of a rectangle is 234 inches and the width is $\frac{4}{5}$ of the length. Find the dimensions of the rectangle.

5. The sum of two numbers is 60. What are the numbers if the sum of $\frac{3}{5}$ of the smaller and $\frac{2}{7}$ of the larger is 25?

Let $s =$ the smaller

Then $60 - s =$ the larger

$$\frac{3s}{5} + \frac{2}{7}(60 - s) = 25$$

Complete the solution.

6. The difference of two numbers is 9. If $\frac{2}{3}$ of the larger is added to $\frac{1}{6}$ of the smaller, the sum is 21. What are the numbers?

7. One of two complementary angles is $\frac{4}{5}$ as large as the other. How large is each angle?

8. One of two supplementary angles is one fourth as large as the other. How large is each?

9. Two angles are supplementary. If $\frac{2}{9}$ of the smaller is added to $\frac{3}{4}$ of the larger, the sum is 116°. How large is each angle?

10. If $\frac{1}{6}$ of an angle is subtracted from $\frac{1}{4}$ of its complement, the remainder is 5°. How large is each angle?

11. Find the angle such that the sum of one fourth of its complement and one seventh of its supplement is 27°.

12. A girl, after having spent one fifth of her allowance for candy and one half of what was left for flowers, had 72 cents remaining. How many cents did she have at first?

13. The numerator of a certain fraction is 3 less than the denominator. If 1 is added to the numerator, the value of the fraction becomes $\frac{3}{4}$. Find the fraction.

SUGGESTION. Let $x =$ the denominator

Then $x - 3 =$ the numerator

and $\frac{x-3}{x} =$ the fraction

$$\frac{x-2}{x} = \frac{3}{4}$$

The L.C.D. is $4x$.

14. John's age now is $\frac{2}{5}$ of what it will be in 15 years. How old is he?

15. Mary's age now is $\frac{2}{3}$ of what it will be in 6 years. How old is she now?

16. Dick is one third as old as his father. In 6 years he will be three sevenths as old as his father. How old is Dick now?

17. A man is 6 years older than his brother. In 8 years the sum of one fifth of his age and one half of his brother's age will be 32 years. How old is each man?

18. Separate 100 into two parts such that the sum of one ninth of the smaller part and one fourth of the larger part is 20.

19. A farmer keeps $\frac{1}{20}$ of his wheat for seed, gives $\frac{2}{5}$ of it to his landlord, and sells the remainder. If the amount he sells is 462 bushels, how many bushels are there in his whole crop?

20. The width of a rectangular building is $\frac{5}{7}$ of its length, and its perimeter is 560 feet. Find its dimensions.

21. A man has $4.50, consisting of quarters, nickels, and dimes. There are half as many dimes as quarters, and three times as many nickels as dimes. How many coins of each kind has he?

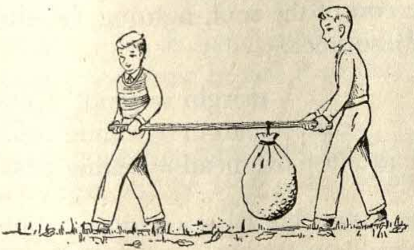
22. Elsie has 80 cents and Tillie has 60 cents. How many cents must Elsie give Tillie so that she will have $\frac{17}{18}$ as much money as Tillie?

23. What number divided by 13 will give 40 for a quotient and 3 for a remainder?

24. The perimeter of a triangle is 44 inches. One side is $\frac{2}{3}$ of the longest side and the third side is $\frac{3}{7}$ of the longest side. How long is each side?

25. A lever is 20 feet long. How far from one end must the fulcrum be placed if a $7\frac{1}{2}$-pound weight at this end is to balance a 5-pound weight at the other end?

26. Two boys wish to carry a bag of hickory nuts on a pole 5 feet long. Where shall they place the bag so that the smaller boy may carry $\frac{2}{3}$ as much as the larger boy?



27. One number is $\frac{3}{5}$ of the other. Find the numbers if their sum is 35 less than 3 times the smaller.

B

28. Dick is half as old as his father. He was one third as old as his father 12 years ago. How old is each?

29. The difference of two numbers is 12. If one half their sum is subtracted from four fifths of the larger, the remainder is zero. Find the numbers.

30. Hazel is $\frac{2}{3}$ as old as Elizabeth. In 9 years she will be $\frac{3}{4}$ as old as Elizabeth. How old are they?

31. Frank has \$20 more than Elmer. After Frank gives Elmer \$5 he finds that he has only $\frac{7}{6}$ as much money as Elmer. How much has Frank at first?

32. The difference of two numbers exceeds (is more than) the smaller by 4. Find the numbers if the larger exceeds one half their sum by 4.

33. A father is 12 times as old as his son. In 6 years the son's age will lack 2 years of being one third of the father's age. How old is the father?

Solving Business Problems by Algebra[A]

As was shown in Chapter IV, many problems which confront a businessman are more easily solved by algebra than by arithmetic. In business the words *overhead* and *margin* are frequently used.

The *margin* is the sum of the profit and the overhead; and the *overhead* is the cost of doing business. It includes among other items the cost of the rent, lighting, heating, depreciation, telephone, and insurance.

Then

$$\begin{aligned} \text{margin} &= \text{profit} + \text{overhead} \\ \text{cost} + \text{margin} &= \text{selling price} \\ \text{cost} + \text{profit} + \text{overhead} &= \text{selling price} \end{aligned}$$

Example 1. A merchant buys a dress wholesale for \$31.20 and wishes to retail it so that his profit will be 15% of the selling price. He knows that his overhead is 20% of the retail price. At what price shall he retail the dress?

Solution. He cannot multiply the retail price by 15% to find his profit and by 20% to find his overhead. Why? Let us see how the problem is solved by algebra.

Let $s =$ the number of dollars in the selling price

Then $.15\,s =$ the number of dollars in the profit

and $.20\,s =$ the number of dollars in the overhead

$$31.20 + .15\,s + .20\,s = s$$

M_{100} $$3120 + 15\,s + 20\,s = 100\,s$$

$$15\,s + 20\,s - 100\,s = -3120$$

$$-65\,s = -3120$$

D_{-65} $$s = 48$$

Check. 15% of \$48 = \$7.20; 20% of \$48 = \$9.60; \$7.20 + \$9.60 + \$31.20 = \$48.00.

Example 2. At a 20% discount sale an article sold for \$8.96. What was the selling price before the discount was deducted?

Solution. Let $x =$ the number of dollars in the original selling price

Then $x - .20\,x = \$8.96$

M_{100}, $100\,x - 20\,x = 896$

$$80\,x = 896$$

$x = 11.20$, the number of dollars in the original price

EXERCISES. PROBLEMS

A

1. A dining room suite is purchased by a furniture dealer for \$147. At what price must he sell it so that his margin will be 30% of the cost price?

2. At what price must the merchant (in Problem 1) sell the suite so that his margin will be 30% of the selling price?

3. At what price must an article be purchased in order that he may sell it for \$12 and make a margin of 25% of the selling price?

4. At what price must an article be purchased so that it may sell for $12, with a margin of 25% of the cost?

5. A retail furniture dealer paid $13.32 for a chair. At what price must he mark its sale ticket if his profit is to be 12% and his overhead is 18%, both based upon the selling price?

6. The regular price of a chair was reduced 10% for a special sale. If the original price was $17.82, what was the sale price?

7. The regular price of a chair was reduced 10% for a special sale. If the sale price of the chair was $17.82, what was the regular price of it?

8. A grocer wishes to sell goods at 20% more than the cost price. At what price must he buy goods that are to sell at $28.80?

9. At a 15% discount sale a rocking chair sold for $8.67. What was the original price of the chair?

10. After a reduction of 30% was made, an automobile sold for $735. Find the original price.

B

11. A man purchased a farm for $8450 and sold it for $9564. His expenses of selling were $79.50. What was his margin? What was his profit? His profit was what per cent of the cost? What per cent of the selling price was his profit?

12. The present population of a certain city is 785,755. In the last decade it increased 10%. What was its population 10 years ago?

Interest [A]

Interest is money paid for the use of money. The money that is borrowed is the *principal*. To find the simple interest for any length of time, we multiply the principal by the rate per year, and then multiply this product by the number of years. The formula for simple interest is $i = prt$. This formula contains four letters. When the values of any three of them are known, the value of the fourth one can be found by substituting the values of the known letters in the formula and solving the resulting equation for the fourth letter.

Example 1. Find the simple interest on \$400 at 4% for $2\frac{1}{2}$ years.

Solution 1. Write the formula, $i = prt$

2. Substitute the known values. $i = 400 \times \frac{4}{100} \times \frac{5}{2}$

3. Find the value of the fourth letter. $i = 40$

The simple interest is \$40.

Example 2. At what rate will \$125 produce \$16.25 simple interest in 4 years 4 months?

Solution. Write the formula, $i = prt$

Substitute the values of the known letters.

$$16.25 = 125 \times r \times \frac{13}{3}$$

$$\text{M}_3 \qquad 3(16.25) = \cancel{3} \times 125 \times r \times \frac{13}{\cancel{3}}$$

$$48.75 = 1625\,r$$

$$-1625\,r = -48.75$$

$$r = .03, \text{ or } 3\%$$

Note. If the formula had been written $prt = i$, negative numbers would have been avoided.

EXERCISES

A

1. Find i when $p = \$400$, $r = 5\%$, and $t = 3$ yr.
2. Find i when $p = \$350$, $r = 4\frac{1}{2}\%$, and $t = 3$ yr. 3 mo.
3. Find p when $i = \$300$, $r = 5\%$, and $t = 4$ yr.
4. Find t when $i = \$94.50$, $p = \$450$, and $r = 6\%$.
5. Find r when $p = \$800$, $i = \$110$, and $t = 2$ yr. 6 mo.
6. Find p if $t = 3$ yr., $r = 5\%$, and $i = \$8.25$

7. Find r when $p = \$260$, $i = \$23.40$, and $t = 2$ yr.

8. Find t when $p = \$375$, $r = 8\%$, and $i = \$90$.

9. Find i when $p = \$40$, $r = 6\%$, and $t = 6$ mo. 21 da.*

10. Find i when $p = \$625$, $r = 4\frac{1}{2}\%$, and $t = 3$ yr. 4 mo. 24 da.*

B

11. Find the interest on $375.40 for 1 yr. 8 mo. at 6%.

12. Find the principal if the interest is $19.01, the time is 3 yr. 5 mo., and the rate is 6%.

13. At what rate will $560 produce $106.40 simple interest if the time is 2 yr. 4 mo. 15 da.?

14. How long will it take $456 to produce $79.04 interest if the rate is 5%?

The Amount [A]

In interest problems the *amount* means the sum of the principal and interest. There are two formulas used in connection with amounts. One is $A = p + i$ and the other is $A = p + prt$. Each one states that the amount is equal to the principal plus the interest.

Example. If $A = \$3500$, $r = 3\%$, and $t = 4$ years, find p.

Solution. We are asked to find p, so we shall write the formula with p in the left member. Is this necessary?

$$p + prt = A$$

$$p + \tfrac{3}{100} \times 4 \times p = 3500$$

$$p + \tfrac{12}{100}p = 3500$$

$$\text{M}_{100} \qquad 100p + 12p = 350{,}000$$

$$112p = 350{,}000$$

$$p = 3125.$$

The principal is $3125.

---

* In figuring time, count 30 days to the month and 12 months to the year.

EXERCISES [A]

Find A, p, r, or t:

1. $p = \$40$, $t = 6$ years, and $r = 6\%$.
2. $p = \$750$, $t = 1$ year, and $r = 4\frac{1}{2}\%$.
3. $p = \$650$, $A = \$676$, and $t = 1$ year.
4. $A = \$1234$ and $p = \$1000$. Find i.
5. Find p when $A = \$1756.50$ and $i = \$324.50$.
6. Find t when $p = \$425$, $r = 5\%$, and $A = \$488.75$.
7. Find i when $A = \$57{,}120$, $t = 4$ years, and $r = 5\%$.
8. Find t when $p = \$375$, $r = 8\%$, and $A = \$465$.
9. Find t when $A = \$1675$, $p = \$1200$, and $r = 10\%$.
10. How long will it take any principal at simple interest to double itself at 6%?

Baseball Problems [A]

Batting averages are usually expressed as decimals. Thus if a player makes 3 hits in 10 times at bat, he is batting .300. The batting average equals the number of hits divided by the number of times at bat.

$$\text{B.A.} = \frac{\text{hits}}{\text{times at bat}}.$$

At one time Lou Gehrig, who represented the best that is in baseball, made 140 hits and had at that time a batting average of .373. Suppose we want to know how many times he had been at bat.

Let $x =$ the number of his times at bat

Then $$\frac{140}{x} = .373$$

M_x $$x\left(\frac{140}{x}\right) = .373(x)$$

$$140 = .373\,x$$

M_{1000} $$140{,}000 = 373\,x$$

$$375 = x$$

The value of x is not exactly 375.

EXERCISES [A]

Find the missing numbers in the following:

	PLAYERS	TIMES AT BAT	HITS	BATTING AVERAGE
1.	Walker	272	100	?
2.	Musial	300	?	.350
3.	Williams	?	97	.349
4.	DiMaggio	?	89	.352
5.	Vernon	264	96	?
6.	Keller	265	?	.328
7.	Hopp	230	83	?

8. During a baseball season Morton High School had 810 times at bat and made 192 hits. Clark High School was at bat 836 times and made 198 hits. What was each team's batting average for the season?

Investments [A]

One of the most common forms of business organization is the *corporation*. The *capital stock* of the corporation is the amount invested in it. This capital stock is divided into *shares* of equal value, say \$100 each. The *par value* of a share is the value stated in the stock certificate. Not all stock has a par value. The true value of stock is usually not its par value; the prices of stocks vary from day to day. A person owning one or more of these shares is a *stockholder*. The profits distributed by a corporation to its stockholders are known as *dividends*.

If a corporation or a government needs to borrow money, it may do so by selling bonds. A *bond* is a kind of promissory note. Bonds are issued at a *face value*, or *par value*, usually \$1000 each, but the selling price may fluctuate. The rate of interest on a bond is based upon the face value. However, this rate does not always represent the actual yield. For example, a 6% \$1000 bond pays interest of \$60 per year, but if the bond was bought at 120 (meaning \$120 for each \$100 of face value), the yield is only \$60 ÷ \$1200, or 5%.

EXERCISES [A]

1. Why is it sometimes necessary for a school board to issue and sell school bonds? Who buys these bonds?

2. A man bought ten \$1000 6% bonds at par value. How much did they cost him? What is his annual dividend from them?

3. Frank Robinson bought 60 shares of stock at \$90 a share, and receives a yearly dividend of \$243 from them. What rate of income does he receive on his investment? If the par value of a share is \$100, what rate of dividend is declared?

4. Find the cost of 80 municipal \$100 bonds that sell at $103\frac{1}{2}$.

5. A man invests x dollars in bonds and \$800 more than this in stocks. How much does he invest in stocks?

6. What is 4% of \$$x$? What is m% of \$30?

7. Find 5% of $(x + 30)$ dollars.

8. A man invests \$3000 in stocks and bonds. If x dollars of this amount is invested in stocks, how much is invested in bonds? If the stocks pay 6% on the investment and the bonds pay 5% on the investment, how much income does he receive on both investments?

EXERCISES. PROBLEMS

A

1. A farmer invested part of his money at 8% and \$500 more than this at 5%. If his income on both investments was \$545, how much was invested at each rate?

2. A widow invested \$10,000, part at 4% and the remainder at 3%. Her yearly income from these investments was \$360. How much was invested at 4%?

3. One sum of money was invested at 5% and \$1500 more than this sum was invested at 3%. If the total income from the two investments was \$325, how much was invested at each rate?

4. Part of \$7000 was invested at 6% and the other part at 4%. If the 4% investment yielded \$160 more than the other, what was the amount of each investment?

5. A man invested some money at 5% and a sum \$1300 less at 4%. If the first investment produces \$80 more interest per year than the second, how much is invested at each rate?

6. How can \$8000 be invested, part at 4% and the remainder at 6%, so that both investments produce the same income?

7. A man lends some money at 4% and an equal amount at $2\frac{1}{2}$%. If the income from the $2\frac{1}{2}$% loan is \$75 less than the 4% loan, how much money did the man loan at each rate?

B

8. How can \$6000 be divided so that one part can be loaned at 4% and the other part at 7%, making an average of 6% on the \$6000?

9. A part of \$8500 is invested at 4% and the remainder at $5\frac{1}{2}$%. If the income from the first investment is \$102.50 more than that from the second investment, what is the amount invested at each rate?

10. How much can one afford to pay for eight \$1000 4% bonds in order to yield $3\frac{1}{2}$% on his investment?

11. A United States Series E bond can be bought for \$18.75. The value of this bond in 10 years is \$25. The rate at compound interest is about 2.9%. What is the rate at simple interest?

Literal Equations [A]

A *literal equation* is one in which at least one of the known numbers is represented by a letter. From this definition any formula is a literal equation. For example, the formula $i = prt$ is used to find the unknown number i when the known numbers are p, r, and t.

Literal equations are solved like numerical equations.

Example 1. Solve for x: $3x = 6c$

Solution.

$$3x = 6c$$

D_3

$$x = 2c$$

Example 2. Solve for x: $2(x - 3a) = 6b$

Solution. $2(x - 3a) = 6b$

$2x - 6a = 6b$

$2x = 6b + 6a$

D_2 $x = 3b + 3a$

Example 3. Solve for C: $F = \frac{9}{5}C + 32$

Solution. $F = \frac{9}{5}C + 32$

M_5 $5(F) = 5(\frac{9}{5}C) + 5(32)$

$5F = 9C + 160$

$-9C = -5F + 160$

D_{-1} $9C = 5F - 160$

D_9 $C = \dfrac{5F - 160}{9}$, or $\dfrac{5}{9}(F - 32)$

Notice that the original formula tells how to find F when C is known and that the last formula tells how to find C when F is known.

ORAL EXERCISES [A]

Solve for x, explaining each step of the solution:

1. $bx = b$
2. $x + 5 = a$
3. $x - 5 = a$
4. $x + 3 = b$
5. $3x = a$
6. $x - b = a$
7. $2ax = 4ab$
8. $bx = b^3$
9. $m - n = x$
10. $x - h = 0$
11. $\dfrac{x}{4} = p$
12. $\dfrac{x}{5} = c$
13. $b - a = -x$
14. $dx = c$
15. $mx = m^2 - 2m$
16. $5ax = 10ab$
17. $\dfrac{x}{4} = \dfrac{1}{c}$
18. $\frac{1}{2}x = c$
19. $\frac{1}{3}x = v$
20. $2x - 5a = 7a$
21. $3x - 4x = a$
22. $3bx = 12b^2$
23. $\frac{1}{4}x - 5 = 0$
24. $\dfrac{x}{c} = \dfrac{a}{b}$
25. $\dfrac{x - a}{c} = b$

EXERCISES [A]

Solve for x or y:

1. $b + x = a$
2. $b - x = c$
3. $-b + x = h$
4. $-b - x = a$
5. $3x + 2a = 8a$
6. $3x + 6a = -9a$
7. $3y - 9c = 16c$
8. $2y - 3b = 5y - 9b$
9. $4m - 3y = 5y - 8m$
10. $7x - (3x - 2a) = 8a$
11. $2(x - 3k) = 6k$
12. $3(x - 4c) - 6c = 0$
13. $6(5b - x) = 7b$
14. $3y - 10d = 4(d - y)$
15. $2(a - y) = 5y - 19a$
16. $b(x - 3) = a$
17. $5bx - a = b(b + 2x)$
18. $a(x + 2 - a) = 1$
19. $ax - (a - 2) = 2$
20. $b(x + b) = b^2 - ab$

Solve the following formulas for the letters indicated:

21. Solve $A = lw$ for l; for w.
22. Solve $d = rt$ for r; for t.
23. Solve $A = \frac{1}{2} lw$ for l; for w.
24. Solve $c = \pi d$ for π; for d.
25. Solve $S = 180(n - 2)$ for n.
26. Solve $V = \frac{1}{3} Bh$ for h.
27. Solve $S = \frac{1}{2} gt^2$ for g.
28. Solve $A = p + prt$ for r; for t.
29. Solve $l = a + d(n - 1)$ for a.
30. Solve $pv = k$ for p.
31. Solve $A = \frac{h}{2}(b + b')$ for b.

Ratios [A]

The *ratio* of one number to another is the *first* number divided by the second. Thus the ratio of 4 to 8 is $\frac{4}{8}$, or $4 \div 8$, or $4 : 8$; the ratio of x to y is $x \div y$, or $\frac{x}{y}$, or $x : y$; and the ratio of 7 inches to 9 inches is $\frac{7}{9}$, or $7 \div 9$, or $7 : 9$.

There can be no ratio between two unlike quantities. Thus we cannot find the ratio of 4 pounds to 6 feet. If two quantities of the same kind are expressed in different units of measurement, they must be changed to the same units of measurement before we can find their ratio. For example, to find the ratio of 4 ounces to 3 pounds, we can change 3 pounds to ounces and find the ratio of 4 ounces to 48 ounces, which is $\frac{4}{48}$, or $\frac{1}{12}$.

ORAL EXERCISES [A]

1. State these ratios in their simplest forms:

a. 4 to 12

b. 12 to 4

c. 10 in. to 15 in.

d. a^2 to $3\,a^2$

e. $3\,x$ to $8\,x$

f. 12 in. to 1 ft.

g. 40 cents to $1

h. 6 yd. to 4 ft.

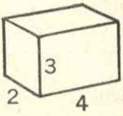
i. 1 in. to 1 ft.

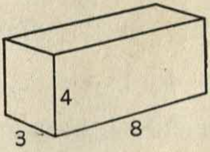
j. 1 in. to 1 yd.

k. 75 to 100

l. 2 ft. 3 in. to 1 yd.

2. Below are two rectangular solids. What is the ratio of their lengths? of their heights? of their widths? of their volumes?





3. If x and y are in the ratio of 3 to 5, then $\frac{x}{y} = ?;\ x : y = ? : ?$.

4. Two numbers are in the ratio $2 : 3$. They may be represented by $2\,x$ and $3\,x$, for $\frac{2\,x}{3\,x} = \frac{2}{3}$. Represent two numbers in the ratio of

a. $3 : 8$

b. 4 to 5

c. 6 to 7

Proportions [A]

A *proportion* is an equation whose two members are ratios. Thus $\frac{4}{5} = \frac{12}{15}$, $\frac{x}{3} = \frac{1}{4}$, and $2 : 8 = 6 : x$ are proportions. If the ratio of two numbers is equal to the ratio of two other numbers, the four numbers are said to be in proportion.

Since the proportion $\frac{a}{b} = \frac{c}{d}$ may be written in the form $a : b = c : d$, it is easy to remember the names of the four parts. The *first term* is a, the *second term* is b, the *third term* is c, and the *fourth term* is d. Since a and d are at the ends of the proportion, they are called the *extremes*. The second and third terms are the *means*, meaning the middle terms.

ORAL EXERCISES [A]

1. Complete: In the proportion $\frac{x}{4} = \frac{5}{6}$, x is the --?-- term, --?-- is the third term, and --?-- is the fourth term.

2. Complete: The means of the proportion $\frac{x}{4} = \frac{5}{6}$ are --?-- and --?--, and its extremes are --?-- and --?--.

3. Complete: In the proportion $\frac{3}{7} = \frac{5}{m}$, the fraction $\frac{3}{7}$ is a --?--.

4. Which of the following are proportions?

(Remember that the two ratios of a proportion are equal.)

a. $\frac{2}{3} = \frac{10}{15}$

b. $\frac{4}{5} = \frac{8}{10}$

c. $\frac{10}{15} = \frac{9}{6}$

d. $\frac{10}{11} = \frac{11}{12}$

e. $\frac{7}{8} = \frac{21}{24}$

f. $\frac{3}{4} = \frac{15}{19}$

5. Explain why $x = \frac{2}{3}$ may be written $\frac{x}{1} = \frac{2}{3}$. Change $y = \frac{3}{4}$ into a proportion.

6. Is $\frac{x}{7} = \frac{4}{5}$ a proportion for all values of x?

7. Can you write $3x = 7$ in the form of a proportion?

8. Solve the following proportions for x:

a. $\frac{x}{3} = \frac{1}{4}$

b. $\frac{x}{4} = \frac{2}{1}$

c. $\frac{3x}{7} = \frac{4}{5}$

d. $\frac{5}{6} = \frac{3}{x}$

e. $\frac{1}{x} = \frac{2}{8}$

f. $\frac{3}{7} = \frac{x}{14}$

9. Is $\frac{x}{15} = \frac{12}{15}$ a proportion for one value of x? What is the product of its means? What is the product of its extremes?

10. Is $\frac{10}{15} = \frac{12}{18}$ a proportion? Is the product of its means equal to the product of its extremes?

WRITTEN EXERCISES [A]

1. $\frac{a}{b} = \frac{c}{d}$. Clear the equation of fractions by multiplying both members by bd. What is the product of the means? What is the product of the extremes?

Copy and complete:

In a proportion the product of the --?-- equals the --?-- of the extremes.

2. Using the principle in the rectangle above, tell whether or not the following are proportions:

a. $\frac{1}{3} = \frac{4}{12}$

b. $\frac{7}{2} = \frac{2}{7}$

c. $7 : 2 = 13 : 4$

d. $\frac{9}{4} = \frac{7}{3}$

e. $\frac{7}{4} = \frac{12}{7}$

f. $\frac{125}{20} = \frac{25}{4}$

3. Solve the following proportions for y:

a. $\frac{1}{4} = \frac{6}{y}$

b. $\frac{y}{8} = \frac{3}{24}$

c. $\frac{4}{y} = \frac{2}{5}$

d. $\frac{3}{y} = \frac{6}{1}$

e. $\frac{1}{3} = \frac{4}{y}$

f. $\frac{y}{a} = \frac{b}{c}$

g. $\frac{m}{y} = \frac{1}{3}$

h. $\frac{1}{y} = \frac{7}{1}$

i. $\frac{6}{1} = \frac{y}{9}$

4. Find two numbers whose sum is 80 and which have the ratio 2 : 3.

(Let $2x =$ the smaller number. Then $3x =$ --?--.)

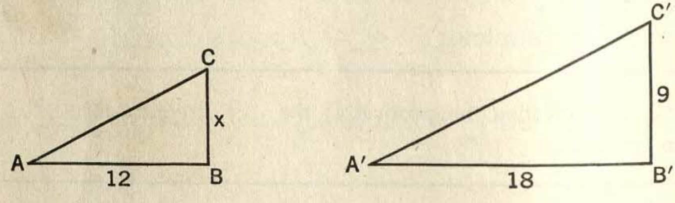
5. A board 12 feet long is to be cut into two parts having the ratio 1 : 2. How long should each part be?

6. If Bill earns \$3.15 in 9 hours, how much will he earn in 11 hours? $\left(\frac{3.15}{x} = \frac{9}{11}\right)$

7. If 7 yards of muslin cost 63 cents, how many yards can be bought for 99 cents?

8. If 2 inches on a map represent 150 miles, how many inches will represent 500 miles?

9. $\triangle ABC$ is similar to $\triangle A'B'C'$. In similar triangles the corresponding sides are proportional. Then $\frac{x}{9} = \frac{12}{18}$. Find BC.



10. A tree casts a shadow of 65 feet when a boy 5 feet 4 inches tall casts a shadow of 7 feet. How high is the tree?

WORD LIST

Be sure you know these words:

amount (p. 220)
bond (p. 222)
corporation (p. 222)
dividend (p. 222)
extremes (p. 228)
interest (p. 218)
literal equations (p. 224)
lowest common denominator (p. 207)
means (p. 228)
par value (p. 222)
ratio (p. 226)
principal (p. 218)
proportion (p. 227)

CHAPTER REVIEW

A

1. How do we clear an equation of fractions?

Solve:

2. $\frac{c}{4} - \frac{c}{3} = 5$

3. $\frac{c}{3} + \frac{2c}{4} = 1$

4. $\frac{3p}{5} = 1 + \frac{p}{2}$

5. $\frac{y+5}{2} - \frac{2y+4}{3} = 0$

6. $\frac{2k-1}{5} - \frac{k+1}{2} = -1$

7. $\frac{1}{3}(2m-1) + \frac{1}{7}(8m-11) = 0$

8. $.007x = 21$

9. $c + .18c = 35.4$

10. $x = 583 - .06x$

11. $.6x + 1.1 = .2x + 1.9$

12. $.03y + .14 = .09y + .08$

13. $4.2 = .7s - .7$

14. If a tree 75 feet high casts a shadow of 54 feet, how long will the shadow of a 50-foot tree be?

15. If a number is increased by 8% of itself, the result is 297. Find the number.

16. Find the simple interest when the principal is \$800, the rate is 3%, and the time is 1 year 2 months.

17. At a 20% discount sale, a chair was sold for \$19.96. What was the original price of the chair?

B

18. A furrier buys a coat for \$234. At what price shall he sell it so that he will have a margin of 40% of the selling price?

19. Solve for c: $\frac{2c-3}{8} - \frac{5c+2}{6} = \frac{8c-1}{9} - \frac{3c+5}{12}$

20. Solve: $.5(x + .2) + .07 - (.3 - x) = .02$

21. Solve: $.25(5 - 2x) - x = -2 - .5(6x - 8)$

22. A woman wishes to invest \$5000, part at 5% and the remainder at 4%, so as to produce an income of \$221.60. How much must she invest at each rate?

C

23. A merchant marked his goods so that he had a margin of 40% of the cost. What per cent of the selling price was his margin?

24. A man buys eggs at x cents apiece and sells them at y cents a dozen. What is his profit? What is his per cent of profit based on the cost?

GENERAL REVIEW

A

1. Simplify $4x^2 - 6x - 3 - 5x^2$.

2. From $2a - 5b + 6c$ take $-2a + 3b - c$.

3. Find the products:

a. $x^3 \cdot x^7$

b. $-4c \cdot c^5$

c. $x^3(xy)$

d. $(-x)(x^2)(x^3)$

e. $\frac{a}{b} \times \frac{b}{a^2}$

f. $\frac{1}{2x} \times \frac{8x}{3}$

4. Find the powers:

a. $(x^2)^2$ **b.** $(-3)^3$ **c.** $(.1p)^3$ **d.** $(-2a^2)^4$

5. Do as indicated:

a. $8x^2 \div (-4x)$

b. $(-63y^2) \div (-9y^2)$

c. $8 \div \frac{1}{2}$

d. $a \div \frac{1}{a}$

e. $4x(x^2 - 5x - 1)$

f. $(7c - 3d + 4) \div (-1)$

6. Change $\frac{3}{20}$ to a per cent.

7. 25 is what per cent of 75?

8. If $a = -2$, $b = 1$, and $c = 0$, find the value of $\frac{a^3 + 3bc + 1}{a}$.

9. What is the name of the value of the unknown in an equation?

10. By substitution determine which of the numbers $+2$, -2, $+1$, and -1 are roots of the equation $x^2 - x = 2$.

Solve:

11. $13m + 7 = 49 + 6m$

12. $3d + 2 = 9d + 5$

13. $6(7y + 3) = 60$

14. $3(k + 2) - 5(k + 1) = -1$

15. A girl has 12 coins consisting of nickels and dimes. If the total value of the coins is \$1, how many coins of each kind has she?

16. A boy is 10 years old and his sister is 18 years old. How soon will the boy be $\frac{3}{4}$ as old as his sister? 10 + x = 18 + x

CHAPTER TEST [A]

Part I. Equations (50%)

Solve:

1. $\frac{x}{3} = 10$

2. $.04x = 2$

3. $\frac{1}{5}(c - 5) = 7$

4. $\frac{y}{2} - \frac{y}{3} = 4$

5. $\frac{3}{4} = \frac{12}{x}$

6. $x + .04x = 78$

7. $\frac{m}{2} - 2 = \frac{2}{3}m - 1$

8. $\frac{c}{5} + 1 = \frac{c}{4} - 1$

9. $\frac{1}{3}(x + 7) = \frac{1}{4}(x - 2)$

10. $\frac{1}{4}(x - 5) - \frac{1}{3}(2x + 1) = 3$

Part II. Problems (50%)

1. Three fifths of a number is 54. Find the number.

2. In a football practice a player lost 7 pounds, which was 4% of his weight. What was his weight just before the practice?

3. The difference between $\frac{3}{4}$ of a number and $\frac{1}{7}$ of it is 17. Find the number. 4x=1

4. One number exceeds another by 18. What are the numbers if $\frac{2}{3}$ of the smaller increased by $\frac{1}{9}$ of the larger equals 23?

5. John is one fourth as old as his father, who is 60 years old. How soon will he be one half as old as his father?

CHAPTER TEST [B]

Part I. Equations (40%)

Solve:

1. $.125\,c = 7$
2. $\frac{3y}{5} = \frac{8}{15}$
3. $x + .08\,x = 59.4$
4. $\frac{3}{4}y - \frac{2}{3}y = \frac{1}{6}y + 1$
5. $\frac{3}{4}p - 2 = \frac{5}{8}p$
6. $3\,h - 7.5 = 12.5 + 0.5\,h$
7. $\frac{2x-7}{3} - \frac{x-5}{6} = 1$
8. Solve for y: $cy - 3\,c = 3\,c^2$
9. Solve for n: $l = a + d(n-1)$
10. Solve: $0.5(y-1) + 2(3y+4) = 5.875$

Part II. Problems (60%)

1. The width of a rectangle is $\frac{5}{6}$ of its length. What are the dimensions of the rectangle if its perimeter is 154 inches?

2. One of two complementary angles is $\frac{3}{5}$ as large as the other. How large is each angle?

3. Dick has $2.85 consisting of nickels, dimes, and quarters. How many coins of each kind has he if the number of nickels is one third the number of dimes, and the number of quarters is one half the number of nickels?

4. In 1948 Newton Heights had a population of 297, which was an increase of 8% over the population in 1947. What was the population in 1947?

5. Part of $10,000 was invested at 5% and the remainder at 4%. The income on the 5% investment exceeded the income on the 4% investment by $185. How much was invested at 5%?

6. What principal at 3% simple interest will amount to $127.20 in 2 years?

CUMULATIVE TEST (CHAPTERS I–VII). Time, 40 Minutes

1. $(2\,x^2)(-3\,x^4) = ?$
2. $x^{12} \div x^2 = ?$
3. $(-4\,m)^3 = ?$
4. $(15\,c - 25\,d) \div 5 = ?$
5. $-c^2(c^2 - c + 1) = ?$
6. $(2\,x - 5) - (7 - 6\,x) = ?$

7. Simplify: $7a - 4b - b + 6a$

8. If $x = -2$ and $y = 4$, find the value of $x^3 - 5y$.

9. $(8x^2y - 4xy - 32y^2) \div (-4y) = ?$

10. Solve: $\frac{3}{4}x = \frac{2}{5}x - 7$

11. Solve: $3(m - 2) - 7 = 5(2m + 5) - 3$

12. $ab(a^2 + ab + b^2) = ?$

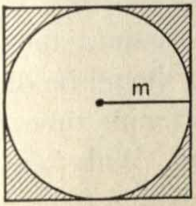
13. Write the interest formula.

14. $24(\frac{2}{3} - \frac{3}{4}x + \frac{1}{2}x^2) = ?$

15. Subtract $c^2 - c + 1$ from $3c + 6$.

16. John is 9 years older than Henry. In 3 years, 7 times John's age will equal 10 times Henry's age. How old is each?

17. Write a formula for the shaded area of the square below.



18. Find the altitude of a triangle whose area is 200 square inches and base 40 inches.

19. At an election among three candidates Brown had twice as many votes as Smith and Douglas had 10,319 more votes than Smith. If all three had a total of 130,747 votes, how many did each receive?

20. The larger of two numbers is 5 more than 6 times the smaller. Find the numbers if their sum is 75.

21. In a triangle the second angle is 3 times the first and the third is 5 degrees larger than the first. How many degrees are there in each angle?

22. Solve: $2(y - 9) = 7(4 - y) - 6(y + 2)$

23. Solve: $1.1x - 8.2 = 4.7 - 1.9x$

24. From the sum of $x - y$ and $2x - y$ take $7 - 4x$.

25. Find the value of $x^2 + 4x - 8$ when $x = -3$.

MATHEMATICS AND ASTRONOMY

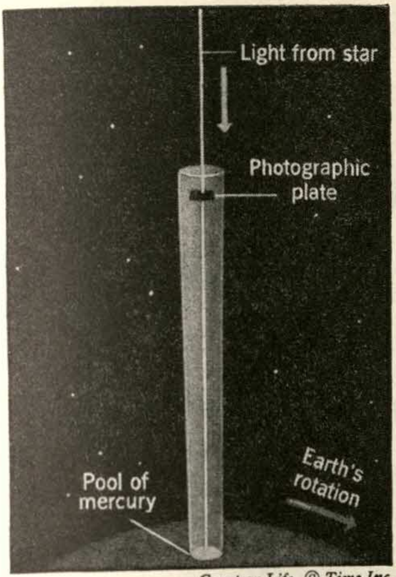
Astronomy is the science which makes a study of the heavenly bodies, including our own solar system. The study of astronomy becomes more interesting and fascinating as the student acquires more knowledge of the subject.

Much of the world's progress can be attributed to astronomy. It is through astronomy that we can tell to the minute when an eclipse of the sun or moon will appear. Astronomers prepare star maps and almanacs, which are used in air and sea navigation.

To obtain the correct time, the astronomer determines when a particular star should be directly overhead. Then with a photographic zenith tube he observes when this star is exactly overhead. With this information the correct time can be found to within 0.002 second.

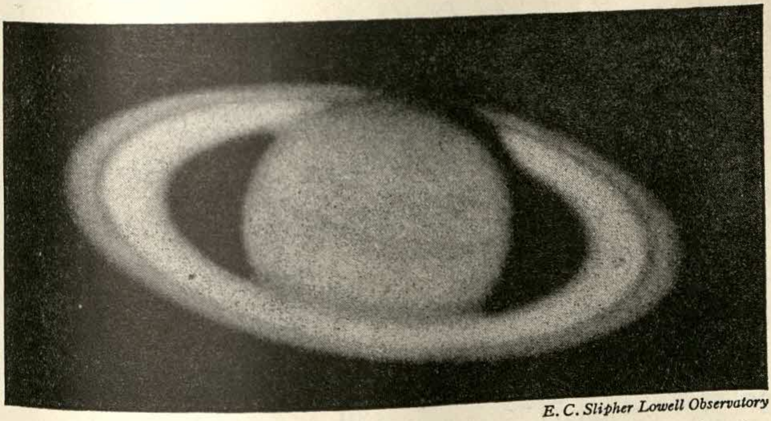
Astronomers have learned much about the stars and planets. They know the sizes and shapes of the planets, how far they are from the sun, what paths they travel in going around the sun, and many other interesting facts.

One does not need very much mathematics to begin the study of astronomy, but to make an intensive study of the subject much mathematics is needed. Besides the mathematics learned in high school, one needs college algebra, trigonometry, analytic geometry, calculus, and mechanics to understand advanced astronomy.



Courtesy Life, © Time Inc.

Mechanics of Zenith Tube



E. C. Slipher Lowell Observatory

Saturn and Its Rings · Saturn has three rings around it. The outer ring has an outside diameter of 172,400 miles and a width of 10,900 miles

CHAPTER VIII

Multiplication and Division of Polynomials

Review of Multiplying by Monomials [A]

In order that you may learn how to multiply one polynomial by another quickly and accurately, a review of multiplying by monomials will now be given.

Example 1. $5^3 \cdot 5^4 = 5^7$

Example 2. $(a^3)^2 = a^3 \cdot a^3 = a^6$

ORAL EXERCISES

Find the indicated products:

1. $x^2 \cdot x^3$
2. $x^2 \cdot x$
3. $m^3 \cdot m$
4. $c^6 \cdot c$
5. $b^3 \cdot b^7$
6. $(-a^3)(-a^5)$
7. $(-6x)(4x)$
8. $(-x^3)(-x^4)$
9. $a(-a^2)$
10. $4x^2(-3y)$
11. $(2x)(3x^2)(-x)$
12. $-4a^2(2a^2)(5a)$
13. $(ab)(bc)c$ ab^2c^2
14. $3(-x)(-y)4$
15. $(5a^2bc)(2abc^3)$
16. $4^2 \cdot 4^3$
17. $5^2 \cdot 5^2$
18. $10^2 \times 10^5$
19. $7^4 \cdot 7$ 7^5
20. $3^6 \cdot 3$
21. $8^2 \cdot 8$
22. $(x^4)^2$
23. $(a^5)^2$
24. $(-y^6)^2$
25. $(-m^3)^3$
26. $(c^5)^3$
27. $(-x^5)^4$
28. $3(x^2 - 4x + 7)$
29. $-4x(x^2 - 6x + 9)$
30. $-4a(a^2 - 2a + 1)$
31. $5ab(a^2 - ab + b^2)$

32. $xy^2(a+b+c)$

33. $18\left(\frac{x}{3}-2\right)$

34. $10\left(\frac{a^2}{5}+3a-\frac{1}{10}\right)$

35. $3x^2(3x^2-7x-5)$

36. $-1(a-b+c)$

37. $m^2n^2(m^2-m+1)$

38. $a(ab-bc-ca)$

39. $\frac{1}{2}xy(x^3-2xy+6y^2)$

40. In finding the products of three or more numbers, which two numbers should first be multiplied?

Multiplying a Polynomial by a Polynomial[A]

The method of multiplying one polynomial by another will be illustrated by four examples.

Example 1. Multiply $x+2$ by $x+3$.

Solution.

$$\begin{array}{rl}
x+2 & \text{multiplicand} \\
\underline{x+3} & \text{multiplier} \\
x^2+2x\phantom{{}+6} & \text{first partial product} \\
\underline{+3x+6} & \text{second partial product} \\
x^2+5x+6 & \text{product}
\end{array}$$

Notice that each term of the multiplicand is multiplied by *each term* (x and 3) of the multiplier, and that like terms of the partial products are placed in the same column to make combining terms easier.

Example 2. $(4a-2b)(3a+2b)=?$

Solution.

$$\begin{array}{rl}
4a-2b\phantom{{}-4b^2} & \\
\underline{3a+2b}\phantom{{}-4b^2} & \\
12a^2-6ab\phantom{{}-4b^2} & \text{This product is } 3a(4a-2b) \\
\underline{+8ab-4b^2} & \text{This product is } 2b(4a-2b) \\
12a^2+2ab-4b^2 & \text{product}
\end{array}$$

Example 3. Multiply $a+b$ by $c+d$.

Solution.

$$\begin{array}{rl}
a+b\phantom{{}+ad+bd} & \\
\underline{c+d}\phantom{{}+ad+bd} & \\
ac+bc\phantom{{}+ad+bd} & \\
\underline{+ad+bd} & \text{In this example no terms of the} \\
ac+bc+ad+bd & \text{partial products are alike.}
\end{array}$$

To multiply one polynomial by another, multiply each term of the multiplicand by each term of the multiplier and (if possible) combine the partial products.

Example 4. Multiply $a^2 - 3\,b^2 + 4\,ab$ by $2\,a^2 - ab + 4\,b^2$.

Solution. The multiplicand and multiplier, when it is possible, should be arranged according to either the descending or the ascending powers of *one* letter. We shall arrange these polynomials in the descending powers of a. Notice that the term containing the highest power of a in either polynomial is written first, the term containing the next highest power of a is written second, and so on.

$$\begin{array}{rrrrr}
a^2 & +\,4\,ab & -\,3\,b^2 & & \\
2\,a^2 & -\,ab & +\,4\,b^2 & & \\
\hline
2\,a^4 & +\,8\,a^3b & -\,6\,a^2b^2 & & \\
 & -\,a^3b & -\,4\,a^2b^2 & +\,3\,ab^3 & \\
 & & +\,4\,a^2b^2 & +\,16\,ab^3 & -\,12\,b^4 \\
\hline
2\,a^4 & +\,7\,a^3b & -\,6\,a^2b^2 & +\,19\,ab^3 & -\,12\,b^4
\end{array}$$

Checking Multiplication[A]

The three methods of checking multiplication are as follows:

1. Performing the multiplication a second time.
2. Performing the multiplication by interchanging the multiplier and multiplicand.
3. Substituting numerical values for the letters.

We shall now check Example 4 above by substitution.

Let $a = 1$ and $b = 2$.

$$\begin{array}{lrl}
\text{Then} & a^2 + 4\,ab - 3\,b^2 = 1 + 8 - 12 & = -\,3 \\
\text{and} & 2\,a^2 - ab + 4\,b^2 = 2 - 2 + 16 & = +\,16 \\
\cline{2-3}
 & 2\,a^4 + 7\,a^3b - 6\,a^2b^2 + 19\,ab^3 - 12\,b^4 & = -\,48 \\
 & 2 + 14 - 24 + 152 - 192 & = -\,48 \\
 & -\,48 & = -\,48
\end{array}$$

Is it possible for an incorrect solution to check? Is it possible for a correct solution not to check?

EXERCISES

A

Multiply:

1. $x + 1$ by $x + 1$
2. $x + 2$ by $x + 2$
3. $a + 3$ by $a + 1$
4. $c + 3$ by $c + 4$
5. $d + 7$ by $d + 3$
6. $g + 6$ by $g + 7$
7. $h - 1$ by $h - 2$
8. $x - 3$ by $x - 3$
9. $c + 10$ by $c - 1$
10. $c - 5$ by $c + 5$
11. $b - 4$ by $b - 4$
12. $x - 7$ by $x + 7$
13. $x + 3$ by $x - 8$
14. $a + b$ by $a + b$
15. $x + y$ by $x + y$
16. $x + y$ by $x - y$
17. $x - y$ by $x - y$
18. $c + 2b$ by $c + 3b$
19. $x - 3y$ by $2x + y$
20. $m - 3n$ by $2m - 5n$
21. $3x - 4y$ by $4x - 3y$
22. $8x + 2y$ by $8x - 2y$
23. $5m + 3n$ by $m - 5n$
24. $a + b$ by $m + n$
25. $m - n$ by $x + y$
26. $x - 3y$ by $5x + 2$

Multiply as indicated:

27. $(x^2 - x + 1)(x + 2)$
28. $(c^2 - 3c + 2)(2c + 3)$
29. $(3x + 5 - x^2)(x + 3)$
30. $(x + y)(x^2 - xy + y^2)$
31. $(-2x + x^2 + 3)(x - 2)$
32. $(2x - 1)(2x^2 + 4x - 1)$
33. $(x^2 + x + 1)(x - 1)$
34. $(c^2 + 2c + 4)(c - 2)$
35. $(c^2 - 2c + 1)(c - 1)$
36. $(2x - 5y)^2$

SUGGESTION.
$(2x - 5y)^2 = (2x - 5y)(2x - 5y)$

37. $(3x + 1)^2$
38. $(2x - 9y)^2$
39. $(\frac{1}{2}x - 3)(\frac{1}{2}x + 3)$
40. $(x^2 - 4x + 6)^2$

Simplify:

41. $2x(x - 3) - 4x(2x - 1)$
42. $2a(a - 3) - 5a(a - 1)$
43. $5a(a + 4) + 2a(a - 5)$
44. $2y(3y - 6) - (2y^2 - 5)$

45. Find the area of a rectangle whose base is $2x + 1$ and whose altitude is $x - 2$.

46. The base of a parallelogram is $x + 2y$ and its altitude is $x - 2y$. What is its area?

47. Find the area of a triangle if the base is $4x - 6$ and the altitude is $2x - 3$.

48. Find the area of a square if one side is $2x + 3$.

49. Find the cost of $(x + 3)$ articles if each cost $(x - 6)$ cents.

50. How far will an automobile go in $(h + 3)$ hours at an average rate of $(m + 1)$ miles an hour?

B

Multiply as indicated:

51. $(x^2 - y^2)(x^2 + y^2)$

52. $(4x - 6)(\frac{1}{2}x + 1)$

53. $(.4x - 3)(5x + 1)$

54. $(7x^2 - 3xy + y^2)(2x + 3y)$

55. $(x^2 - x + 1)(x^2 + x + 1)$

56. $(2x - 1)(x^3 - x^2 + x - 1)$

57. $(2y^2 + y - 1)(2 - 3y + y^2)$

58. $(x^2 - x + 1)(2 - x + x^2)$

59. $(a + b - c)(a - b + c)$

60. $(x + y + 3)(x + y - 3)$

61. $(3x^2 - 4 - 3x)(2x^2 + 3x)$

62. $(4x^2 - 4x + 3)(2 - x)$

63. $(x + y + z)^2$

64. $(x + y)^3$

65. $(a - b)^3$

66. $(a^2 + b^2)(c + d)$

Example.[B] Simplify $(3x - 1)(5x + 2) - (x + 4)(3x - 2)$.

Solution.

$$\begin{array}{r} 3x - 1 \\ 5x + 2 \\ \hline 15x^2 - 5x \qquad \\ +6x - 2 \\ \hline 15x^2 + x - 2 \end{array} \qquad\qquad \begin{array}{r} x + 4 \qquad \\ 3x - 2 \qquad \\ \hline 3x^2 + 12x \qquad \\ -2x - 8 \\ \hline 3x^2 + 10x - 8 \end{array}$$

Then

$$\begin{aligned} &(3x - 1)(5x + 2) - (x + 4)(3x - 2) \\ &= (15x^2 + x - 2) - (3x^2 + 10x - 8) \\ &= 15x^2 + x - 2 - 3x^2 - 10x + 8 \\ &= 12x^2 - 9x + 6. \text{ Ans.} \end{aligned}$$

When simplifying expressions containing the products of polynomials, always place the products in parentheses, and then simplify the resulting expression.

EXERCISES [B]

1. Simplify $(x - 1)(2x - 1) - (3x + 1)(x - 2)$.
2. Simplify $(2y + 5)(y - 6) - (3y + 1)(y - 8)$.
3. Simplify $(3c - 1)^2 - (2c + 4)^2$.
4. Simplify $(3x + 2)^2 - 3(x + 4)(2x - 3)$.

Equations [A]

On page 176 you learned how to solve equations such as Example 1, below.

Example 1. Solve $3 - 2(3x + 2) = 7$.

Solution. The quantity $3x + 2$ is to be multiplied by -2. Then we have

$$\begin{aligned} 3 - 6x - 4 &= 7 \\ -6x &= 7 - 3 + 4 \\ -6x &= 8 \\ x &= -1\tfrac{1}{3} \end{aligned}$$

There is a slight difference between Example 1 and Example 2.

In Example 1, $3x + 2$ is to be multiplied by 2 and the result is to be subtracted from 3. You were able to do this a short way by multiplying $3x + 2$ by -2 and adding the result to 3.

In Example 2, $(x + 2)$ is to be multiplied by $(x - 5)$ and the result is to be subtracted from 8. Pupils often forget to subtract as indicated. To prevent errors of this kind you should always place the products of the two binomials in parentheses.

Example 2. Solve $8 - (x - 5)(x + 2) = 3 - x^2$.

Solution.

$$\begin{aligned} 8 - (x - 5)(x + 2) &= 3 - x^2 \\ 8 - (x^2 - 3x - 10) &= 3 - x^2 \\ 8 - x^2 + 3x + 10 &= 3 - x^2 \\ -x^2 + x^2 + 3x &= -8 - 10 + 3 \\ 3x &= -15 \\ x &= -5 \end{aligned}$$

$$\boxed{\begin{array}{r} x - 5 \phantom{{}- 10} \\ x + 2 \phantom{{}- 10} \\ \hline x^2 - 5x \phantom{{}- 10} \\ + 2x - 10 \\ \hline x^2 - 3x - 10 \end{array}}$$

Check. Does $8 - (-5 - 5)(-5 + 2) = 3 - (-5)^2$?

Does $8 - (-10)(-3) = 3 - (25)$?

Does $8 - 30 = -22$? Yes.

EXERCISES

A

Solve, and check as directed by your teacher:

1. $9x - (3x - 18) = 36$

2. $-(5x + 4) - 6x = 3$

3. $y + (2y - 1) = 2$

4. $3(2 - y) = 5(3 - y)$

5. $3c - (c + 2) = 2$

6. $14 = 5x + 3(2 - x)$

7. $3(3x - 1) = 13 + 2(x + 6)$

8. $2(x + 14) = 3 - 7(x - 1)$

9. $2(3x + 1) - 2(15 - 4x) = 0$

10. $2(x - 40) - 3(50 - x) = 20$

11. $x + 5 = 7 - (2x + 1)$

12. $10x - 17 = 2 - (7x - 15)$

13. $(x - 2)(x + 3) = x(x + 4)$

14. $(2x - 1)(2x + 3) = 4x^2 + 9$

15. $(x - 2)(x - 6) - (x - 8)(x - 3) = 0$

Caution. Do not forget to place the products in parentheses.

16. $(x - 1)(3x + 4) - (x - 3)(3x + 2) = 0$

17. $(x - 1)^2 - (x + 1)^2 = 4$

18. $9x(x + 3) - (3x - 1)^2 = 32$

19. $(4x - 1)(x + 1) - (2x - 5)(2x + 4) = 4$

20. $(2x - 1)(3x + 2) = 16 + (3x - 1)(2x + 5)$

B

21. $(5x + 7)(6x - 2) - 15x(x - 3) = 35 + 15x^2$

22. $(3x + 5)(4x - 1) - 2(3x - 1)(2x + 3) = 0$

23. $(3x - 1)(3x + 1) - (3x + 2)^2 = 7$

24. $(x - 3)^2 - (2x - 1)^2 = 3(2 - x^2)$

25. $3(4x - 3)^2 - (x + 6)(x - 5) = 47x^2 - 16$

26. $(4x + 1)(2x - 5) - 2(6x - 1)(6x + 1) + 37 = -64x^2$

27. $4(x - 6)^2 - (x - 1)^2 - 3x(x - 8) = -11$

28. $(x - 2)(x + 2)(x - 3) = (x - 1)^3 - 1$

Motion Problems [A]

If an automobile goes 50 miles an hour, how far will it go in 3 hours? This is a simple motion problem. It is sometimes called a *time, rate, distance problem.* In a problem of this kind three things are involved — time, rate, and distance. *Rate* is the distance traveled in one unit of time. The following oral exercises are intended to assist you in solving the written problems which follow.

ORAL EXERCISES [A]

1. How far will a man walk

- **a.** In 4 hours at the rate of 2 miles an hour?
- **b.** In 5 hours at the rate of 3 miles an hour?
- **c.** In 7 hours at the rate of r miles an hour?
- **d.** In t hours at the rate of r miles an hour?
- **e.** In $2x$ hours at the rate of 4 miles an hour?
- **f.** In 6 hours at the rate of $(x + 2)$ miles an hour?
- **g.** In $(x + 3)$ hours at the rate of 5 miles an hour?

2. If d stands for the distance, r for the rate, and t for the time, tell what $d = rt$ means.

3. How is the formula $r = \frac{d}{t}$ obtained from the formula $d = rt$? How is the rate found when the distance and time are known?

4. How is the formula $t = \frac{d}{r}$ obtained from the formula $d = rt$? How is the time found when the distance and rate are known?

5. How much time is required for an automobile to go

- **a.** 120 miles at 40 miles an hour?
- **b.** 12 miles at 60 miles an hour?
- **c.** $4x$ miles at x miles an hour?
- **d.** $x - 3$ miles at 8 miles an hour?

6. Find the missing numbers in the table below.

	TIME	RATE	DISTANCE
a.	3	?	12
b.	4	$x + 2$	?
c.	$x + 5$	3	?
d.	?	a	$8ab$

	TIME	RATE	DISTANCE
e.	$2t$	$t - 4$	?
f.	$t + 1$	7	?
g.	?	8	$8t$
h.	?	x	$x^2 - 4x$

7. If x stands for the rate in miles an hour of one speed boat, what will represent the rate of another boat that travels

a. 4 miles an hour faster?

b. 3 miles an hour less than twice as fast?

c. 4 miles an hour more than three times as fast?

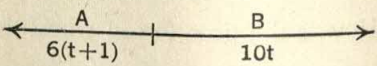
d. half as fast?

Example. A and B start from the same point and travel in opposite directions, A at the rate of 6 miles an hour and B at the rate of 10 miles an hour. If A starts an hour before B, how many hours has each been traveling when they are 54 miles apart?

Solution. Let t = the number of hours B travels
Then $t + 1$ = the number of hours A travels
$10t$ = the number of miles B travels
$6(t + 1)$ = the number of miles A travels

Many pupils prefer to write these four preliminary statements in box form. A diagram is also useful.

	TIME	RATE	DISTANCE
B	t	10	$10t$
A	$t+1$	6	$6(t + 1)$



In filling in the spaces in the box, the known quantities are inserted first. Here the rates of both B and A are known and the 10 and 6 are inserted first. Next the quantities to be found are inserted. In this problem the times of A and B are to be found, so the times of B and A are represented by t and $t + 1$ respectively and placed in the box. Then since distance = rate × time, the spaces under "Distance" can be filled in.

The equation can be written as soon as the spaces in the box are filled.

The sum of A's distance and B's distance = 54 miles.

$$\begin{aligned}6(t+1)+10t&=54\\6t+6+10t&=54\\6t+10t&=54-6\\16t&=48\\t&=3, \text{ } B\text{'s time in hours}\\t+1&=4, \text{ } A\text{'s time in hours}\end{aligned}$$

WRITTEN EXERCISES

A

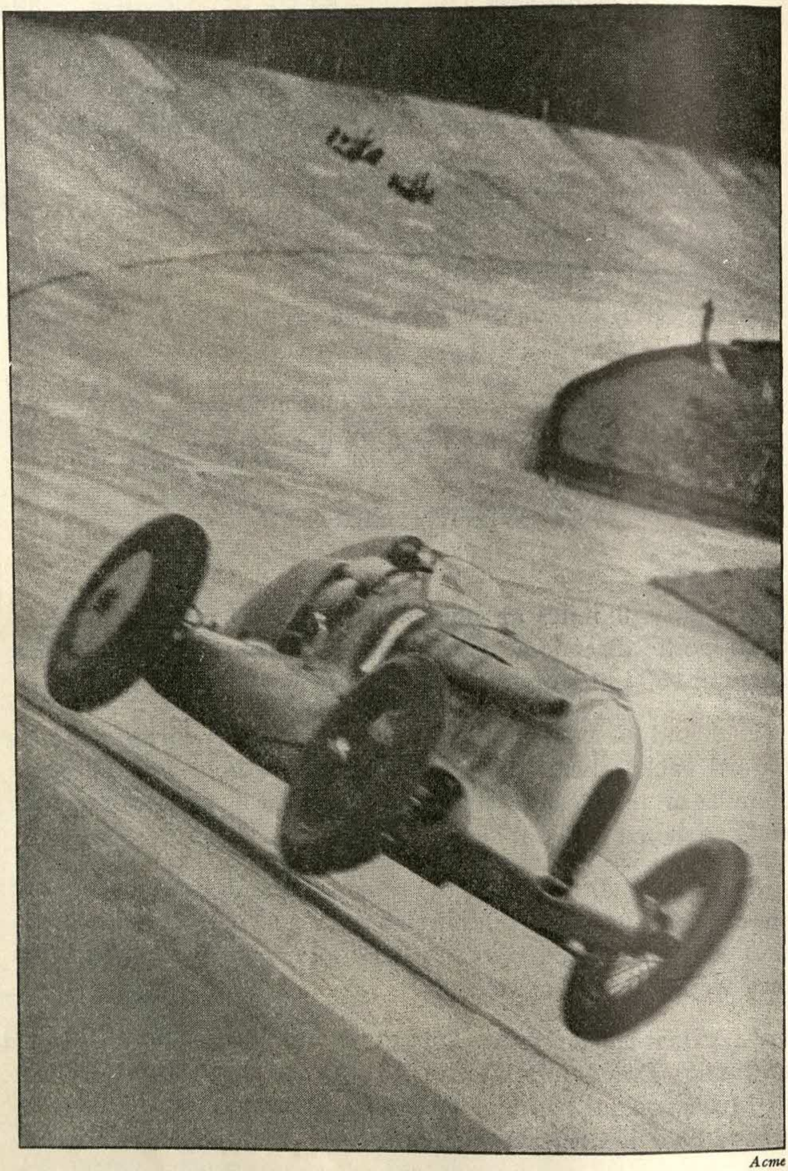
1. A goes 8 miles an hour and B goes 20 miles an hour in the opposite direction. How many hours has B traveled when they are 180 miles apart? (A and B travel the same time, t.)

2. A and B, who are 566 miles apart, start in automobiles toward each other. A drives 40 miles an hour and B drives 36 miles an hour. How soon will they meet if A has an accident and is delayed an hour?

3. A and B started from the same place and went in opposite directions, A traveling 30 miles an hour and B 10 miles an hour. If B started 3 hours before A, how long had A traveled when they were 90 miles apart?

4. A boat started from Chicago toward Muskegon at 10 miles an hour. Two hours later another boat started from Chicago to Muskegon at 14 miles an hour. How long did it take the second boat to overtake the first?

5. Horace started west on road U. S. 50, riding a bicycle at an average rate of 9 miles an hour. Four hours later Charles started after Horace on a motorcycle and overtook him in 6 hours. What was Charles's rate?



Acme

The Degree of Banking in an Automobile Race Track Is Determined Mathematically

6. Portland, Oregon, and Seattle, Washington, are about 188 miles apart by road. Driver A travels at the rate of 40 miles an hour and driver B travels at the rate of 34 miles an hour. Driver A leaves Portland for Seattle at 8 o'clock and driver B leaves Seattle for Portland at 9 o'clock. When will the drivers pass each other?

SUGGESTION. Let $t =$ the number of hours B travels before they meet.

7. A train traveling 50 miles an hour left a station 30 minutes before a second train running 55 miles an hour. How soon did the second train overtake the first?

8. A boat which travels 10 knots (nautical mi./hr.) is followed six hours later by another boat which travels 14 knots. In how many hours will the second boat overtake the first?

9. Two trains start at the same time from towns 385 miles apart and meet in 5 hours. If the rate of one train is 7 miles an hour less than the rate of the other, what is the rate of each?

B

10. Dick can run the 440-yard dash in 55 seconds, and Jack can run it in 88 seconds. How great a handicap must Dick give Jack in order that the boys may finish the race at the same time?

11. The Indianapolis Speedway is $2\frac{1}{2}$ miles around. How long will it take a driver going 106.25 miles an hour to gain a lap on a driver going 100 miles an hour?

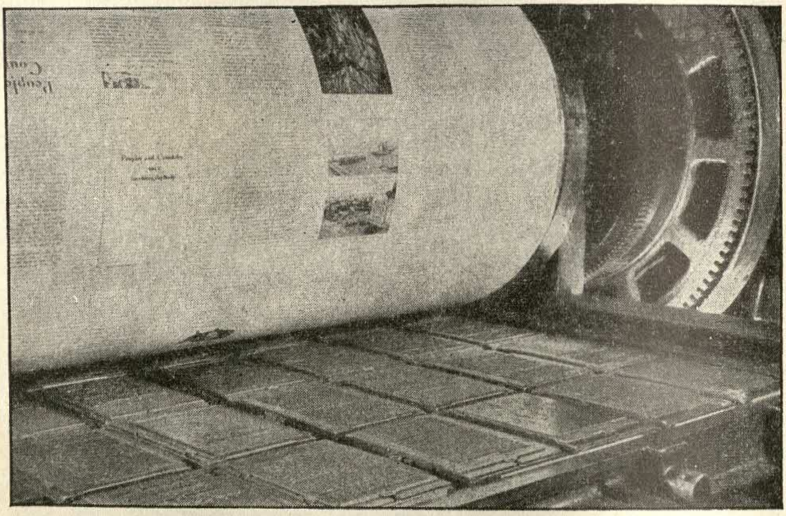
12. Mr. Jackson drove his automobile from his home to Boston at a rate of 35 miles an hour and returned at a rate of 40 miles an hour. Find his time going and returning if the time returning was one hour less than the time going.

13. How far can a plane go, and return in 2 hours 42 minutes, if the rate going is 280 miles an hour and the rate returning is 350 miles an hour?

14. A farmer drove to town at 30 miles an hour and returned at 24 miles an hour. What was his time in each direction if his total driving time was 2 hours 15 minutes?

Work Problems [A]

Work problems usually deal with the amount of time needed to do a piece of work.



Many Pages of a Book Are Printed in One Operation

Example 1. One printing press can print 6000 copies an hour, and another can print 4000 an hour. After the second press has been running 2 hours, the first press is started. How soon after the first press is started will 40,000 copies be finished?

Solution.

Let $x =$ the number of hours the first press runs

$x + 2 =$ the number of hours the second press runs

Then $6000\,x + 4000(x + 2) = 40{,}000$

$$6000\,x + 4000\,x + 8000 = 40{,}000$$

$$6000\,x + 4000\,x = 40{,}000 - 8000$$

$$10{,}000\,x = 32{,}000$$

$x = 3\frac{1}{5}$, the number of hours that the first press runs

Example 2. Jim can do a piece of work in 3 days, and Harry can do the same work in 2 days. How long will it take them to do the work if they work together?

Solution. In solving a problem like this we first find what part of the work each individual can do in one unit of time.

If Jim can do all the work in 3 days, he can do $\frac{1}{3}$ of the work in one day. If Harry can do all the work in 2 days, he can do $\frac{1}{2}$ of the work in one day.

Let x = the number of days required to do the work when they work together

Then $x\left(\frac{1}{3}\right)$ or $\frac{x}{3}$ = the part of the work that Jim does in x days. Why?

And $x\left(\frac{1}{2}\right)$ or $\frac{x}{2}$ = the part of the work that Harry does in x days.

Since both together do the whole of the work,

$$\frac{x}{3} + \frac{x}{2} = 1$$

$$2x + 3x = 6$$

$$5x = 6$$

$x = 1\frac{1}{5}$, the number of days required

EXERCISES [A]. WORK PROBLEMS

1. Frank can mow a lawn with one lawn mower in 2 hours and Jessie can mow it with another lawn mower in 3 hours. How long will it take them to mow the lawn if they work together?

2. *A* can do a piece of work in 8 days and *B* can do the same work in 10 days. How long will it take them working together to do the work?

3. One printing press can print 4000 copies an hour and another press can print 3000 copies an hour. After the first press has been going 3 hours, the second press is started. How soon after the start of the second press will 50,500 copies be printed?

4. John can build a bird house in 24 hours and Wilbur can build one like it in 30 hours. After John has worked 10 hours on the construction of a bird house, Wilbur begins to help him. How many more hours are needed for the boys to complete the house?

5. Mary can grade a set of papers in 45 minutes, Jane can grade the set in 50 minutes, and Frances can grade the set in 40 minutes. How long will it take all three girls working together to do the grading?

6. Frank can dig a trench in 15 hours and his father can dig it in 9 hours. How long will it take Frank and his father working together to dig the trench?

7. Bob can do a task in 10 hours and Roy can do it in 12 hours. After Bob has worked on the job for 3 hours, how many hours are needed for both boys to complete the job?

Mixture Problems [A]

In the past some pupils have found mixture problems difficult. If you will make sketches and imagine that you are actually making the mixtures, you will be able to solve mixture problems easily.

Example. A seedsman has clover seed worth 21 cents a pound and timothy seed worth 9 cents a pound. How many pounds of each should he use to make a mixture of 200 pounds worth 12 cents a pound?

Solution.

Clover x lb. 21¢	+	Timothy $(200 - x)$ lb. 9¢	=	Mixture 200 lb. 12¢

The equation is formed from the fact that

$$\begin{array}{ccccc} \text{Value of clover} & + & \text{value of timothy} & = & \text{value of mixture} \\ 21x & + & 9(200 - x) & = & 12(200) \end{array}$$

$$21x + 1800 - 9x = 2400$$

$$21x - 9x = 2400 - 1800$$

$$12x = 600$$

$$x = 50, \text{ the no. of lb. of clover seed}$$

$$200 - x = 150, \text{ the no. of lb. of timothy seed}$$

EXERCISES. MIXTURE PROBLEMS

A

1. How many pounds of English walnuts worth 38 cents a pound must be mixed with 24 pounds of almonds worth 45 cents a pound in making a mixture worth 40 cents a pound?

2. A grocer wishes to make a mixture of 100 pounds of coffee to sell at 35 cents a pound. He has two grades of coffee, one worth 40 cents a pound and another worth 32 cents a pound. How many pounds of each grade should he use to make the mixture?

3. A miller wishes to make 120 bushels of a mixture of corn and oats to sell at 50 cents a bushel. If a bushel of oats is worth 38 cents and a bushel of corn is worth 70 cents, how many bushels of each should he use?

4. Maple syrup worth $6.00 a gallon and corn syrup worth 80 cents a gallon are used to make a mixture worth $2.36 a gallon. How many gallons of each kind of syrup are needed to make 50 gallons of the mixture?

5. Some Girl Scouts wish to make a mixture of chocolates and hard candy to sell at 25 cents a pound. They will use 12 pounds of chocolates and 18 pounds of hard candy. If chocolates are worth 20 cents more a pound than hard candy, how much a pound is each kind of candy worth?

6. A grocer has one brand of tea worth 50 cents a pound and another brand worth 80 cents a pound. How many pounds of each kind must he use to make a 60-pound mixture worth 72 cents a pound?

7. How many pounds of coffee worth 32 cents a pound must be added to 24 pounds of coffee worth 38 cents a pound to make a mixture worth 36 cents a pound?

8. An order of candy cost $14. It contained one kind of candy worth 60 cents a pound and another kind worth 50 cents a pound. If there were 5 more pounds of the 60-cent candy than of the 50-cent candy, how many pounds of each kind were in the order?

B

9. In an alloy of copper and zinc there are 75 pounds of copper and 25 pounds of zinc. How many pounds of pure copper must be added to the alloy so that it will be 80% copper?

Suggestion. Make a diagram comparing the amounts of copper.

Pure copper x lb.	+	100 lb. of old alloy 75 lb. copper	=	$(x + 100)$ lb. of new alloy 80% copper
x	+	75	=	$.80(x + 100)$

Notice that the equation is based on the fact that the two amounts of copper used in making the new alloy equal the amount of copper in the new alloy. Complete the solution.

10. Nickel bronze is an alloy of copper and nickel. How much nickel must be melted with 100 pounds of nickel bronze which is 25% nickel to produce an alloy that is 40% nickel?

11. A 10% solution of iodine contains 10% iodine and 90% alcohol. How much alcohol must be added to one quart of a 10% solution of iodine to produce a 6% solution of iodine?

12. A 3-gallon antifreeze solution of water and alcohol is 20% alcohol. How much alcohol must be added to make the solution 40% alcohol?

13. Some United States silver coins are $\frac{9}{10}$ pure silver or "$\frac{9}{10}$ fine." How much pure silver must be melted with 200 ounces of silver $\frac{3}{5}$ fine to make it of the standard fineness for coinage?

14. How much pure gold must be added to 120 ounces of gold 16 carats fine ($\frac{16}{24}$ pure) to make an alloy 22 carats fine?

15. How many tons of 8% copper ore and of 3% ore should be mixed to make 300 tons of 6% ore?

16. How many gallons of cream containing 40% fat must be mixed with 160 gallons of skim milk to make milk containing 8% fat?

Review of Dividing by Monomials[A]

In the next article we shall learn how to divide one polynomial by another. The following exercises are intended to give you the necessary preparation for this new work.

ORAL EXERCISES[A]

Divide as indicated:

1. $x^3 \div x$

2. $6x \div 6$

3. $-30a \div 6$

4. $\dfrac{12x^2}{-2x}$

5. $-\dfrac{x^7}{x^3}$

6. $(8x^2) \div (8x^2)$

7. $4xy \div (2y)$

8. $-x^3y^2 \div (xy^2)$

9. $\dfrac{-18ab}{-9b}$

10. $\dfrac{x^3y^2z}{xy^2}$

11. $(x^3y^5) \div (x^2y)$

12. $(4.5a^2b^4) \div 1.5$

13. $\left(\dfrac{1}{2}x^3y\right) \div (2xy)$

14. $(20x^7) \div (-4x)$

15. $\dfrac{10^3}{10}$

Divide:

16. $4x - 8$ by 4

17. $18r + 12s$ by -6

18. $8x^2 - x$ by x

19. $25ce + 30e$ by $5e$

20. $9cz - 18c^2z$ by cz

21. $12x^4 - 18x$ by $-6x$

22. $y^3 + y$ by y

23. $8a^2b^2 - 6ab$ by $-2ab$

24. $7c^2 - 14c^5$ by $(-7c^2)$

25. $(\frac{1}{2}x^3 - \frac{1}{3}y)$ by $\frac{1}{6}$

26. $(\frac{2}{3}c + \frac{3}{2})$ by $\frac{5}{6}$

27. $(.5a + .15)$ by $.5$

28. $(a^2c - ac^2)$ by $(-ac)$

29. $(x^3 - x^2 + x)$ by $.1x$

Dividing a Polynomial by a Polynomial[A]

The division of one polynomial by another is very similar to long division in arithmetic. As you are learning how to divide by a polynomial, think how you divide by long division in arithmetic. Compare the two methods on page 256.

Example 1.

$$
\begin{array}{rll}
 & x - 5 & \text{quotient} \\
\text{Divisor } x - 3 & \overline{\big)\, x^2 - 8x + 17} & \text{dividend} \\
 & \underline{x^2 - 3x} & \\
 & -5x + 17 & \\
 & \underline{-5x + 15} & \\
 & 2 & \text{remainder}
\end{array}
$$

$$(x^2 - 8x + 17) \div (x - 3) = x - 5 + \frac{2}{x - 3}$$

Check.

$$
\begin{array}{ll}
x - 5 & \text{quotient} \\
\underline{x - 3} & \text{divisor} \\
x^2 - 5x & \\
\underline{\quad -3x + 15} & \\
x^2 - 8x + 15 & \\
\underline{\qquad\qquad 2} & \text{remainder} \\
x^2 - 8x + 17 & \text{dividend}
\end{array}
$$

$$
\begin{array}{rr}
 & 34 \\
231 & \overline{\big)\,7896} \\
 & \underline{693} \\
 & 966 \\
 & \underline{924} \\
 & 42
\end{array}
$$

$$7896 \div 231 = 34\tfrac{42}{231} \text{ or } 34\tfrac{2}{11}$$

Check.

$$
\begin{array}{r}
231 \\
\underline{34} \\
924 \\
\underline{693} \\
7854 \\
\underline{42} \\
7896
\end{array}
$$

To Divide by a Polynomial:

1. Arrange the terms of both dividend and divisor in descending (or ascending) powers of one letter.
2. Divide the *first term* of the dividend by the *first term* of the divisor to find the first term of the quotient.
3. Multiply the *whole* divisor by this term of the quotient and subtract the result from the dividend to obtain a new dividend.
4. Continue this process until the first term of the dividend cannot be divided exactly by the first term of the divisor.

Example 2. Divide $x^3 - 17xy^2 + 66y^3 - 5x^2y$ by $x - 6y$.

Solution.

STEP 1. Arrange both dividend and divisor in descending powers of x (pay no attention to powers of y).

$$x - 6y\ \overline{\big)\, x^3 - 5x^2y - 17xy^2 + 66y^3}$$

STEP 2. Divide x^3 by x, obtaining x^2 as the first term of the quotient.

$$\begin{array}{rl} & x^2 \\ x-6y & \big)\overline{x^3-5x^2y-17xy^2+66y^3} \end{array}$$

STEP 3. Multiply the divisor $x-6y$ by x^2, obtaining x^3-6x^2y. Subtract, obtaining $x^2y-17xy^2+66y^3$ as a new dividend.

$$\begin{array}{rr} & x^2 \\ x-6y & \big)\overline{x^3-5x^2y-17xy^2+66y^3} \\ & \underline{x^3-6x^2y} \\ & x^2y-17xy^2+66y^3 \end{array}$$

STEP 4. Continue Steps 2 and 3, obtaining xy as the second term of the quotient and $-11xy^2+66y^3$ as another new dividend.

$$\begin{array}{rr} & x^2+xy \\ x-6y & \big)\overline{x^3-5x^2y-17xy^2+66y^3} \\ & \underline{x^3-6x^2y} \\ & x^2y-17xy^2+66y^3 \\ & \underline{x^2y-6xy^2} \\ & -11xy^2+66y^3 \end{array}$$

STEP 5. Continue Steps 2 and 3, obtaining $-11y^2$ as the last term of the quotient. The complete division is shown here.

$$\begin{array}{rr} & x^2+xy-11y^2 \\ x-6y & \big)\overline{x^3-5x^2y-17xy^2+66y^3} \\ & \underline{x^3-6x^2y} \\ & x^2y-17xy^2+66y^3 \\ & \underline{x^2y-6xy^2} \\ & -11xy^2+66y^3 \\ & \underline{-11xy^2+66y^3} \\ & 0 \end{array}$$

Checking Division.

A division may be checked in three ways:

1. By performing the division again.
2. By substitution.
3. By multiplying the quotient by the divisor and adding the remainder to the product to obtain the dividend.

EXERCISES

A

Divide, and check as directed by your teacher:

1. x^2-6x+9 by $x-3$

2. $m^2+8m+12$ by $m+2$

3. $c^2+11c+24$ by $c+3$

4. $x^2-7x+10$ by $x-5$

5. $x^2 - 9x + 20$ by $x - 4$

6. $y^2 - 9y + 15$ by $y - 2$

7. $x^2 + 4x - 12$ by $x + 6$

8. $y^2 - 2y - 15$ by $y - 5$

9. $t^2 - 3t - 28$ by $t - 7$

10. $x^2 - x - 42$ by $x + 6$

11. $2x^2 - 5x + 72$ by $x - 7$

12. $x^2 - 4x - 140$ by $x + 10$

13. $4c^2 - 8c + 3$ by $2c - 3$

14. $3x^2 + x - 1$ by $x - 2$

15. $8x^2 + 3x + 4$ by $x - 3$

16. $x^2 - 1$ by $x - 1$

Notice that there is no x term in the dividend; arrange the dividend as $x^2 + 0x - 1$.

17. $m^2 - 9$ by $m + 3$

18. $x^2 - 4$ by $x + 2$

19. $x^2 - 16$ by $x - 4$

20. $x^2 - y^2$ by $x - y$

21. $h^2 - 64y^2$ by $h - 4y$

22. $12x^2 - 11xy - 36y^2$ by $4x - 9y$

23. $m^2 - 3mn - 88n^2$ by $m + 8n$

24. $x^4 + 6x^2 - 27$ by $x^2 - 3$

25. $12x^3 - 11x^2 + 8x - 4$ by $3x - 2$

26. $24x^2 + 15 - 38x$ by $6x - 5$. (Rearrange dividend.)

27. $2y^3 - 6y - 5 - 11y^2$ by $y - 5$

28. $a^3 - 17a - 5a^2 + 66$ by $a - 6$

29. $a^3 - 6a^2 - 8 + 12a$ by $a - 2$

30. One factor of $x^3 - 5x^2 + 7x - 2$ is $x - 2$. What is the other factor?

B

31. $2a^3b - 6a^2b + 18ab$ by $a^2 - 3a + 9$

32. $2x^3 + 11x - 3 - 9x^2$ by $2x - 3$

33. $4a^4 - 9a^2 - 6a - 1$ by $2a^2 - 1 + 3a$

34. One factor of $6x^3 - 18x + 12$ is $x - 1$. What is the other factor?

35. $a^3 - 1$ by $a - 1$
$a^3 - 1 = a^3 + 0a^2 + 0a - 1$

36. $a^3 + 1$ by $a + 1$

37. $a^3 + 8$ by $a + 2$

38. $a^3 - 8$ by $a - 2$

39. $x^3 - y^3$ by $x - y$

40. $x^3 + y^3$ by $x + y$

41. $x^3 - 27$ by $x - 3$

42. $x^3 + 64$ by $x + 4$

43. $27x^3 + 8$ by $3x + 2$

C

44. $x^4 - 1$ by $x - 1$

45. $x^5 + 32$ by $x + 2$

46. $x^4 + 1$ by $x + 1$

47. $x^5 - 1$ by $x - 1$

48. $a^4 + a^2b^2 + b^4$ by $a^2 - ab + b^2$

49. $-7x^3 + 21 + 8x + 18x^2 + 6x^4$ by $3x^2 - 5x + 7$

50. One factor of $x^5 + 1$ is $x + 1$. What is the other?

51. $x^6 - y^6$ by $x^4 + x^2y^2 + y^4$

52. $a^3 - b^3 + c^3 - 3abc$ by $a + b + c$

WORD LIST

At this time you should know and be able to spell the following words:

algebra (p. 3)
decimal (p. 210)
dividend (p. 256)
equation (p. 178)
literal equation (p. 224)
motion (p. 245)
mixture (p. 252)
par value (p. 222)
parentheses (p. 11)
polynomial (p. 13)
principal (p. 218)
quotient (p. 256)
rate (p. 246)
ratio (p. 226)

CHAPTER REVIEW[A]

1. If c is the cost of a dozen oranges, what is the cost of one orange?

2. What is the cost of x dozen pencils at y cents a dozen?

3. Find the cube of $-4x$.

4. Simplify:

a. x^3x^4
b. x^2yx^3
c. $mmmm$
d. $(m^3n^2)mn$
e. $(xy)^4$
f. $c^2d^3(2cd^2)$
g. $a(bc)(ac)$
h. $(b + c)b$
i. $2x(x - 3)$

5. Divide as indicated:

a. $(16c^2) \div (4c)$
b. $(mn^3) \div (mn)$
c. $(32x^4) \div (8x)$
d. $(-18a^2b^3) \div (-6ab^2)$

6. Divide, and check by multiplying the quotient by the divisor:

a. $x^2 - 7x + 12$ by $x - 4$

b. $x^3 - x + 6$ by $x + 2$

c. $y^3 - 3y^2 + 3y + 1$ by $y - 1$

d. $a^2 - b^2$ by $a + b$

e. $x^2 - 9$ by $x - 3$

f. $x^2 + 1$ by $x + 1$

7. Solve:

a. $3x - 2(4 - x) = 2(5x + 8)$

b. $4(y + 1) - 3(y - 3) = 2(y - 5)$

c. $0.4(2x + 1) - 0.3(5 - x) = 0$

d. $51 - (2x + 6) = 3x - 5(3x - 1)$

e. $(x + 2)^2 - (x - 2)^2 = 16$

f. $18 - (3x + 1)(2x + 4) = 57 - (6x - 1)(x - 4)$

8. Multiply:

a. $(x + 7y)(2x - 3y)$

b. $(x^2 - 3x + 1)(x + 2)$

c. $(x^2 - 2xy + y^2)(x - y)$

d. $(x^3 - 2x^2 + 3x + 1)(x - 2)$

9. Simplify:

a. $(3x - 1) + 2(x - 4)$

b. $2x(x + 6) - 3x(x - 5)$

c. $2x(3x + 2) + x(x - 3)$

d. $16 - 2(x + 1) - 2x$

10. Karl and Ben start at the same point and travel in opposite directions, Karl at the rate of 4 miles an hour, and Ben at the rate of 5 miles an hour. If Karl starts an hour before Ben, how many hours has each been traveling when they are 49 miles apart?

11. One automobile goes 36 miles an hour and another automobile goes 48 miles an hour in the opposite direction. If they start from the same place at the same time, how many hours have elapsed when they are 336 miles apart?

12. Sam can wash and polish the family car in 2 hours. It takes Jack 3 hours to do the same job. How long should it take the boys if they work together?

13. Maple syrup worth \$5.00 a gallon and corn syrup worth \$2.00 a gallon are used to make a mixture worth \$3.20 a gallon. How many gallons of each kind of syrup are needed to make 25 gallons of the mixture?

GENERAL REVIEW

A

1. What is the cost of $7c$ books if $2x$ books cost $4cx$ cents?

2. Change the fraction $\frac{2}{5}$ to a per cent.

3. 80 is what per cent of 20?

4. Subtract $\frac{1}{2}$ from $\frac{1}{3}$.

5. Simplify $7a - 3b + 4c - 8b - 9a - c$.

6. Add $4x^3 - 7x^2y - y^3$, $xy^2 - 2x^2y - 4y^3$, and $x^3 - x^2y + 5xy^2 - 10y^3$.

7. From the sum of $3x - 10$ and $x^2 + x - 1$ take the sum of $2x + 6$ and $3x^2 - 5x + 4$.

8. Divide, and check by multiplying the quotient by the divisor:

a. $(4c^3 - 8c^2 + 4) \div 4$

b. $(5a^3 - 10a^2 + 5) \div (-5)$

c. $(x^3 - x^2 - x) \div (-x)$

d. $(12x^3y^2 + 10x^2y^3) \div (2x^2y^2)$

e. $(5a - 6b + 7c) \div (-1)$

f. $(vt - \frac{1}{2}gt^2) \div t$

9. Solve:

a. $4x - 7 - x = -8x$

b. $2(x + 2) = 8$

c. $(6c + 4) - (3c + 1) = 11$

d. $4(2x - 9) = 14x - 36$

e. $x - \dfrac{x + 7}{4} = \dfrac{x - 7}{4}$

f. $\dfrac{3p - 4}{2} - \dfrac{5p - 2}{14} = 5$

g. $\dfrac{3m - 1}{4} - \dfrac{4m - 2}{5} = \dfrac{1}{12}$

10. Solve for x:

a. $3x - 5x = 2a$

b. $4x - 5a = 3a$

c. $cx - a = b$

d. $\dfrac{x}{a} = \dfrac{b}{c}$

e. $2bx - b^2 = 3b^2$

f. $2cx + 3cd = 4cx + 7cd$

g. $5ax - b = 3c - 4ax$

h. $\dfrac{a}{x} = \dfrac{b}{c}$

11. If $x = 3, y = -2$, and $z = 0$, find the value of

a. $x^3 + 4y^3$

b. $y - x^2 - 6$

c. $\frac{xy + z^2 - 6}{y^2}$

d. $xy + yz + xz$

e. $z^2 + x^3 + y^3$

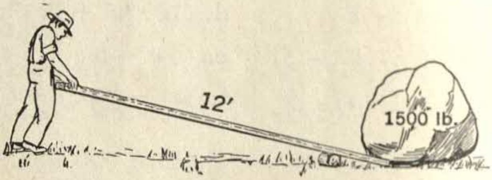
f. $\frac{4x^2 - 5y^2}{6z - 4}$

12. $T = 2\pi r(r + h)$. Find T if $h = 12$, $\pi = 3.14$, and $r = 3$.

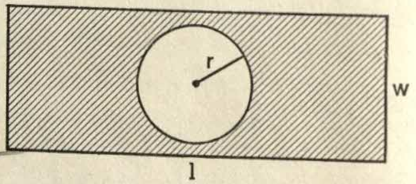
13. $s = vt + \frac{1}{2}gt^2$. Find the value of g if $s = 699.6$, $v = 20$, and $t = 6$.

14. Where is the fulcrum of a lever 31 inches long if a weight of 54 pounds at one end balances a weight of 70 pounds on the other end?

15. A man weighing 160 pounds wishes to use a piece of timber 12 feet long as a lever to lift 1500 pounds. Where should the fulcrum be placed so that all of his weight is needed to lift the 1500 pounds? (Disregard the weight of the timber.)



16. Write a formula for the shaded portion of the figure below. Write a formula for the difference between the perimeter of the rectangle and the circumference of the circle by expressing the difference d in terms of w, l, r, and π.



17. How can a \$5 bill be changed into quarters, dimes, and nickels, so that there will be the same number of nickels as dimes, and $\frac{2}{5}$ as many quarters as dimes?

B

Find the indicated products:

18. $(4 - 5y^2 - 3y)(3 - 2y)$

19. $(3x^2 - xy - 5y^2)(x - 2y)$

20. $(3a^2 - 5 - 3a)(2a^2 + a - 2)$

21. $(a^3 - 4a + a^2 - 2)(2a - 1)$

22. $(a^2 - 2ab + b^2)^2$

Solve:

23. $\frac{k-5}{3} + \frac{2(7k+4)}{8} - \frac{10+3k}{6} = -8\frac{2}{3}$

24. $\frac{x+40}{2} + \frac{2x-1}{3} - \frac{x+10}{6} - \frac{10-3x}{5} = 0$

Simplify:

25. $2x(x^2 - x + 1) + 3x(x^2 + x + 1)$

26. $2c(c^2 - 3c + 4) - c(2c^2 - 1)$

27. $(2y - 1)^2 - (y - 3)^2$

28. $(x - y)^2 - (y - z)^2 + (x + z)^2$

Divide:

29. $a^4 + a^2 + 1$ by $a^2 + a + 1$

30. $a^3 + 64$ by $a + 4$

31. $4a^3 + 15a^2b + ab^2 - 6b^3$ by $a^2 + 3ab - 2b^2$

32. $s = \frac{n}{2}(a + l)$. Find l when $s = 3775$, $n = 50$, and $a = 2$.

33. $A = P(1 + rt)$. Find r when $A = 300$, $P = 200$, and $t = 10$.

34. The length of a rectangle exceeds its width by 11 feet. If the width is decreased by 1 foot and the length is increased by 4 feet, the area remains unchanged. What are the dimensions of the rectangle?

35. A vessel contains 4 gallons of a 3% salt solution. How many gallons of water must be added to make it a 2% solution?

CHAPTER TEST [A]

Part I. Multiplying Polynomials

Multiply:

1. $4c(c^2 - 8c + 1)$

2. $x^2(2x - 3)$

3. $(2m - 1)(m + 5)$

4. $(a - 1)(a + 1)$

5. $(x - 3y)(x + 2y)$

6. $(m - n)^2$

7. $(-3x + 2)(x + 4)$

8. $2\pi r(r + h)$

9. $(m^2 - m + 1)(m + 1)$

10. $(h^2 - 2h + 2)(1 + h)$

Part II. Division of Polynomials

Divide:

1. $(6x^2 - 4x) \div (2x)$

2. $(p + prt) \div p$

3. $(6y^2 - 24) \div (-3)$

4. $(x^2 - 8x + 3) \div (x - 1)$

5. $(x^2 - 25) \div (x + 5)$

6. $(2x^2 + 3x - 20) \div (2x - 5)$

7. $(x^2 + 5x + 6) \div (x + 3)$

8. $(c^2 - 4cy - 32y^2) \div (c - 8y)$

9. $(63 - 2x - x^2) \div (7 - x)$

10. $(x^3 - 1) \div (x + 1)$

Part III. Equations and Problems

1. How far will a plane fly in x hours at an average speed of $(2x + 30)$ miles an hour?

2. Find the area of a rectangle if its base is $2s - 3$ and its altitude is $s + 1$.

3. Solve $2x + 1 = 16 - 3(2x - 3)$.

4. Solve $5(2x + 5) - (4x - 1) = 44$.

5. Two trains start at the same time from stations 600 miles apart and meet in 5 hours. If the rate of one train is 10 miles an hour faster than that of the other, what is the rate of each?

6. How many pounds of 30-cent chocolates must be mixed with 50 pounds of 70-cent chocolates to make a mixture worth 60 cents a pound?

CHAPTER TEST [B]

Part I. Multiplying Polynomials

Multiply:

1. $x^2(x^2 - 4x - 6)$

2. $m^2(-m + n)$

3. $(c + 3d)(c - 3d)$

4. $(x^2 - y^2)(x^2 - y^2)$

5. $(x + 1)(2x - x^2 + 3)$

6. $(x - \frac{1}{2})(x + \frac{1}{2})$

7. $(2x^2 - 1 + 3x)(x + 2)$

8. $(a^2 - 2ab - b^2)(a + b)$

9. Simplify $(x - 3)^2 - (x - 1)(x + 1)$.

10. Simplify $2x(x - y) - y(y - x)$.

Part II. Division of Polynomials

Divide:

1. $(x^3y - xy^2) \div (-xy)$

2. $(a^2 - 4ab + b^2) \div (a - b)$

3. $(x^2 + xy + y^2) \div (x + y)$

4. $(x^3 - 1) \div (x - 1)$

5. $(x^3 + y^3) \div (x + y)$

6. $(4x^3 - 17x - 12) \div (2x + 3)$

7. $(6x^3 - 10x + 5x^2 - 12) \div (3x + 4)$

8. $(a^4 + a^2 + 1) \div (a^2 + a - 1)$

Part III. Equations and Problems

1. Solve $3y - (y + 4) = 6$.

2. Solve $(x - 3)^2 - (x + 1)(x + 2) + 11 = 0$.

3. Solve $18 - 2y(5 - 7y) = 7y(2y - 1)$.

4. The area of a triangle is $2c^2 - 4c - 16$ and its base is $c + 2$. Find its altitude.

5. Plane A can make a flight from one airport to another in $2\frac{1}{2}$ hours. Plane B can fly the same distance in 2 hours. Find the rate of each plane if the speed of one is 60 miles an hour more than that of the other.

CUMULATIVE TEST (CHAPTERS I–VIII). Time, 80 Minutes

See how many of these you can do correctly in 80 minutes. Find your grade in per cent by the formula $P = 40 + \frac{3}{2} R$, where R is the number right.

1. Combine: $3a + 2b - 5a + 4c - 4b$

2. Multiply: $-5a^3b \cdot 3ab^4$

3. Divide $-35r^3t^2$ by $-7r^2t^2$.

4. If $r = 3$ and $s = 4$, find the value of $\frac{2r^2 + rs}{3}$.

5. Solve $12 - a = a - 12$.

6. The sum of two numbers is 13; one exceeds twice the other by 1. What are the numbers?

7. Add:

$$\begin{array}{rrrr} -5x^3 & +3x^2 & & +5 \\ & -2x^2 & -x & -3 \\ \hline -5x^3 & & +x & -10 \end{array}$$

8. A farmer has 172 rods of fencing. How long will his field be if he makes it 15 rods wide?

9. If $S = \frac{1}{2}gt^2$, find S when $g = 32$ and $t = 5$.

10. Solve $3(4 - x) - 7(x - 2) = 9x + 7$.

11. $a = -\frac{b}{3}$. As b increases does a increase or decrease?

12. Multiply: $-4a^2(5a)(-2a^3)$

13. Fifty coins in nickels and dimes amount to $3.50. How many are there of each kind?

14. Multiply $x^2 - 2xy + y^2$ by $x - y$.

15. Divide $a^3 - 4a^2b + 3ab^2 + 2b^3$ by $a - 2b$.

16. Find the value of $x^2 - 3xy + 4y^2$ when $x = 3, y = -2$.

17. Solve $\frac{5}{6}x + 21 = \frac{2}{3}x - 18$.

18. A man gave to one son $\frac{1}{3}$ of his money, and to another son $\frac{1}{4}$ of his money. He has $10 left. How much had he at first?

19. Write the formula for finding the area of a circle.

20. Subtract:

$$\begin{array}{rrr} -3a & +9b & -6c \\ \hline 2a & +3b & -7c \end{array}$$

21. $y = 3x + 1$. Find y when $x = -5$.

22. 3 is what per cent of 15?

23. Two angles of a triangle are each twice the third angle. How many degrees are there in the third angle?

24. Solve $-8 + 3(y + 2) - (2y - 7) = 7$.

25. How much larger than $8a - 5b$ is $-3a + 5b$?

26. Simplify $4x(2x^2 + 8x - 3) - 5x^2(2x + 3)$.

27. If $a = 3$, $b = 2$, $c = -4$, find the value of $\frac{4a - 10b + 5c}{7b}$.

28. Solve $\frac{3n}{4} + \frac{2n}{5} = \frac{5n}{6} + \frac{19}{3}$.

29. Subtract $x^2 - 3x + 6$ from $x + 6$.

30. Simplify $2a(b - 2c) - 2b(a + c) - ac$.

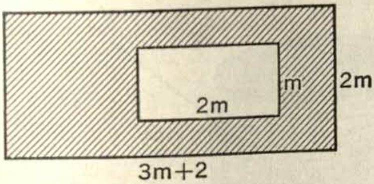
31. Is $x = -4$ the solution of $5x + 12 - 3x = 15 - 2x$?

32. What name is given to the 4 in $3x^4$?

33. Solve $\frac{r + 6}{3} - \frac{r + 1}{5} = \frac{r + 3}{6} + \frac{3}{2}$.

34. A is 5 times as old as B. In 9 years he will be 3 times as old as B. Find their present ages.

35. Write a formula for the perimeter p of the larger of the two rectangles illustrated here.



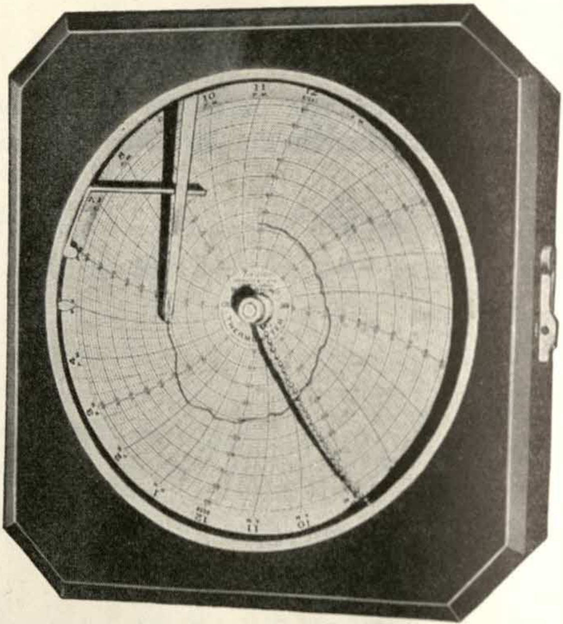
36. Write a formula for the area A of the shaded portion of the figure.

37. Solve $\frac{x - 12}{10} = \frac{6x + 3}{5} - \frac{3x - 6}{2}$.

38. Is $(x + 2)^2 = x^2 + 4x + 4$ an identical equation?

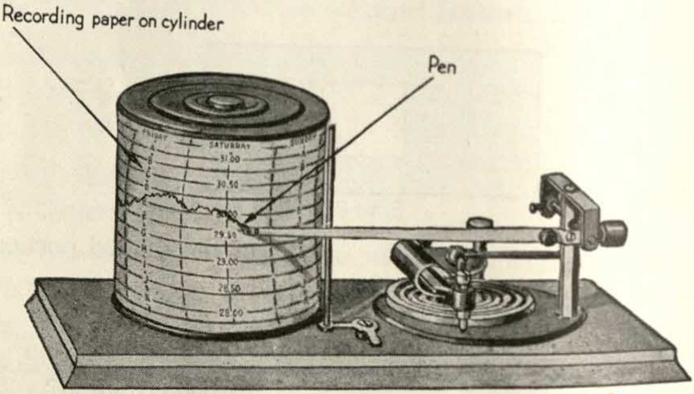
39. What is the root of the equation $7(x - 2) - 4 = x$?

40. Write the formula for the circumference of a circle.



Taylor Instrument Companies

Thermograph · A continuous record of variations in temperature is shown by the graph traced by this instrument. Thermographs are used in powerhouses and by weather bureaus



Barograph · This instrument, called a barograph, is a self-recording barometer. As the drum revolves, the pen traces a graph of the atmospheric pressures

CHAPTER IX

Graphs

Kinds of Graphs [A]

A graph is a diagram which shows the relative sizes of two or more quantities at the same time, or shows the relative sizes of the same quantity at different times.

Graphs may be classified as follows:

1. Statistical graphs
    - *a.* The bar graph
    - *b.* The pictograph
    - *c.* The circle graph
    - *d.* The broken-line graph
    - *e.* The curved-line graph
2. The formula or equation graph

Statistical Graphs [A]

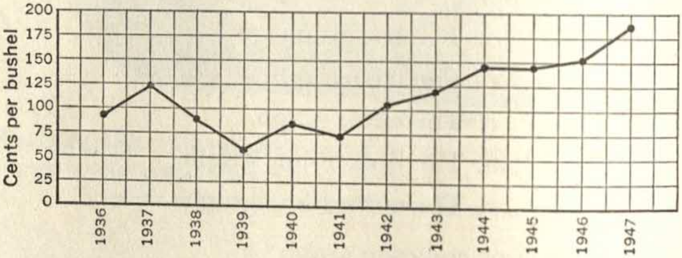
Public and private schools find it necessary to keep records of attendance, of grades, of supplies, of the cost of instruction, and of many other items.

Merchants and manufacturers must keep records to compare their progress with that of previous years. The government keeps thousands of different kinds of records. Among these are records of births, deaths, marriages, divorces, rainfall, number of acres planted in various crops, yields of the various crops, bank deposits, and failures in business. In fact, the government keeps records of nearly all activities which affect the people.

These records are usually written in tables, called *statistical tables*. The following is a statistical table showing the price in cents of a bushel of wheat in the United States from 1936 to 1947.

YEAR	PRICE IN CENTS PER BUSHEL	YEAR	PRICE IN CENTS PER BUSHEL
1936	92.0	1942	106.1
1937	123.6	1943	117.5
1938	88.6	1944	146.0
1939	57.1	1945	146.0
1940	84.5	1946	154
1941	73.0	1947	185

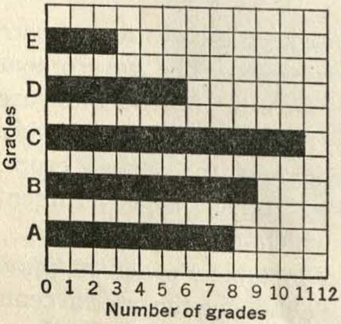
The broken-line graph below shows the same facts as the table. It is more easily read than the table, and presents a clearer picture of what is happening. When was the price of wheat the highest?



Price of Wheat, 1936–1947

The Bar Graph [A]

The bar graph consists of parallel bars of equal width, having lengths proportional to the quantities they represent. These bars may be vertical or horizontal. Usually they are separated from each other by spaces having the same width as the bars.



Grades in an Algebra Class

Here is a bar graph showing the grades made by the pupils of an algebra class. The graph is made on *co-ordinate*, or *squared* paper. Notice that the graph reading matter is written from left to right and from bottom to top.

ORAL EXERCISES [A]

For Exs. 1–5 refer to the bar graph on the opposite page.

1. How many pupils in the class made a grade of *A*? of *B*? of *C*? of *D*? of *E*?

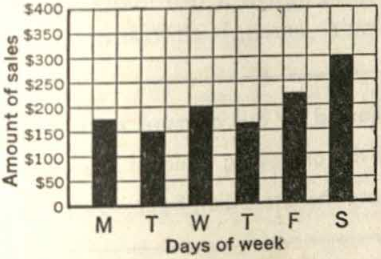
2. How many pupils made a higher grade than *C*? lower than *C*?

3. Without counting, what do you guess the middle grade to be, assuming the grades are arranged in order, with the highest on top and the lowest on the bottom?

4. The number of *D* grades is how many times the number of *E* grades?

5. On which edge of the graph is the "bottom" of each bar as you read the grades?

6. This graph shows the amount of sales of a grocery store for a week. What was the amount of sales on Monday? on Tuesday? on Wednesday? on Thursday? on Friday? on Saturday?



Sales for Week

Why do you think Saturday's sales were the highest?

Rounding Off Numbers [A]

According to the last census there were 51,436 farms in Colorado. Unless a sheet of paper is unusually large it is impossible to represent these 51,436 farms exactly on a graph. For most purposes it is not necessary to represent such large numbers exactly, as most people are not interested in the exact number of things. It is better to say that there were 51,000 farms, which is the number of farms to the nearest thousand. We say 51,000 and not 52,000 because 51,436 is nearer to 51,000 than to 52,000. If the exact number had been 51,536, we should have said the number was 52,000 to the nearest thousand. Then 51,436 *rounded off* to the nearest thousand is 51,000. Other examples of rounding off numbers are as follows:

72,648 rounded off to the nearest 10 is 72,650
72,648 rounded off to the nearest 100 is 72,600
72,648 rounded off to the nearest 1000 is 73,000
72,648 rounded off to the nearest 10,000 is 70,000

So far we have avoided numbers ending in 5, or ending in 5 followed by zeros. Here are some examples.

45 rounded off to the nearest 10 is 40
75 rounded off to the nearest 10 is 80
650 rounded off to the nearest 100 is 600
350 rounded off to the nearest 100 is 400

The rule for rounding off a number ending in 5, or in 5 followed by one or more zeros, is as follows:

If a 5, or a 5 followed by one or more zeros, is dropped,

1. **Keep the preceding figure if it is even or zero,**
2. **Increase the preceding figure by 1 if it is odd.**

By this rule the figure preceding the 5 will always be even after the number is rounded off.

EXERCISES [A]

Read the following numbers rounded off to the nearest hundred:

1. 4765
2. 3475
3. 1862
4. 9450
5. 7550
6. 9327
7. 15,100
8. 7632

Read correct to the nearest thousand:

9. 43,629

10. 84,561

11. 12,501

12. 15,509

13. 72,500

14. 71,500

15. 2352

16. 8947

Read correct to the nearest million:

17. 64,532,876

18. 101,428,635

19. 435,276,180

20. 726,500,000

EXERCISES [A]

Make bar graphs to show the following facts:

1. The attendance at the Grover Cleveland High School for a week was: Monday, 824; Tuesday, 845; Wednesday, 880; Thursday, 858; Friday, 867.

2. The average yearly amount of damage caused in the United States by each of four insects is as follows:

Housefly	$66,500,000	Codling moth	$31,000,000
Clothes moth	$23,000,000	Japanese beetle	$ 3,500,000

3. The distances from Chicago to eight other cities are as follows:

Baltimore,	797 miles	Cincinnati,	285 miles
Bismarck,	844 miles	Indianapolis,	184 miles
Boston,	1033 miles	Denver,	1257 miles
Charleston, S. C.,	1015 miles	Los Angeles,	2397 miles

4. The percentages of increase in population for six states for the period 1930–1940 are as follows:

STATE	PER CENT INCREASE	STATE	PER CENT INCREASE
Florida	29.2	Arizona	14.6
Connecticut	6.4	New Jersey	2.9
California	21.7	Illinois	3.5

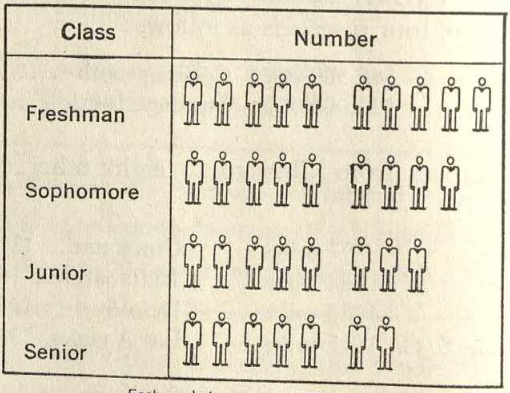
Pictographs [A]

Pictographs are frequently used in newspapers, magazines, and business reports. Other names for pictographs are *picture graphs*, *pictorial graphs*, and *pictograms*.

Some pictographs aim to show the relative sizes of quantities by using symbols of different sizes. The objection to this method of making the pictograph is that the reader does not know whether the heights of the symbols, their areas, or their volumes are to be considered.

In the preferred method of making the pictograph the symbols are of the same size and their number is proportional to the quantities they represent.

Suppose that we wish to make a pictograph to show the relative sizes of the four classes of Ridgeview High School. In this school there are 1024 freshmen, 935 sophomores, 760 juniors, and 728 seniors. These numbers rounded off to the nearest hundreds are 1000, 900, 800, and 700 respectively.



Each symbol represents 100 pupils

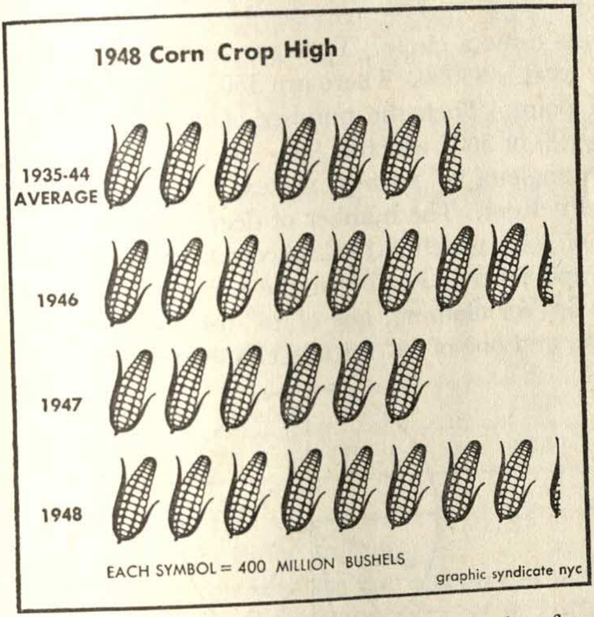
The pictograph above shows the relative sizes of the four classes. Each figure stands for 100 pupils.

ORAL EXERCISE [A]

From the graph above, the number of seniors is what per cent of the number of freshmen?

EXERCISES [A]

1. This pictograph was printed in the daily newspapers. How many bushels of corn are represented by each ear of corn? What does the part of an ear in Row 1 mean? How many bushels were raised in 1946?



2. The chief causes of death for young people of ages 10 to 14 inclusive in one year were as follows: accidents, 31,000; heart disease, 990; tuberculosis, 500.

Make a pictograph for these data, using a cross to represent each 500 deaths.

3. Make a pictograph to show that in cities of less than 10,000 people there is one pedestrian killed for every three nonpedestrians killed in vehicle accidents.

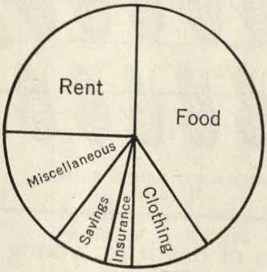
The Circle Graph [A]

Another commonly used graph is the circle graph. It is used to show the relation of each part of a quantity to each of the other parts and to the whole.

We shall now make a circle graph for the budget of a family as given by the following table:

Rent	$750	Savings	$200
Food	$1200	Insurance	$100
Clothing	$300	Miscellaneous	$450

We first draw a circle. Then we add the six items and find that the total is $3000. There are 360° in the sum of the angles about a point. Then the number of degrees in the angle for rent is $\frac{750}{3000}$ of 360° which is 90°. Next with the protractor we make an angle of 90° with its vertex at the center of the circle. We label it Rent. The number of degrees in the angle for food is $\frac{1200}{3000}$ of 360°, which is 144°. Accordingly, we make an angle of 144° and label it Food. In the same manner we make an angle of 36° for clothing, one of 24° for savings, one of 12° for insurance, and one of 54° for miscellaneous.



In some circle graphs the percentage of each item of the whole is placed on the graphs. To make the graph stand out, the parts of the circle may be made in different colors.

EXERCISES [A]

Make circle graphs to show the following data:

1. A city tax dollar was used as follows:

Administration . .	2.5 cents	Health	9.96 cents
Public safety . . .	34.26 cents	Parks	6.18 cents
Payment of debt .	20.6 cents	Sanitation	6.02 cents
Public works . . .	20.48 cents		

2. A family having a monthly income of $200 uses this budget:

Rent	$50	Food	$30
Clothing	$40	Savings	$20
Heat and light	$20	Miscellaneous	$40

The Broken-Line Graph[A]

Line graphs are used to picture related facts. There are three kinds of line graphs: the broken-line graph, the curved-line graph, and the straight-line graph.

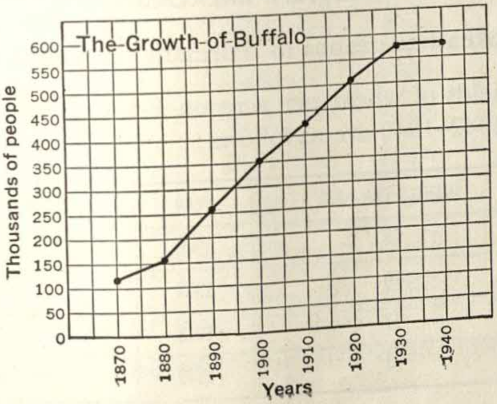
The broken-line graph is used to show the different values of a quantity at different periods of time.

The table below shows the growth of Buffalo from 1870 to 1940.

YEAR	POPULATION	YEAR	POPULATION
1870	117,714	1910	423,715
1880	155,134	1920	506,775
1890	255,664	1930	573,076
1900	352,387	1940	575,901

The population of Buffalo is a quantity that has a different value for each of the eight different times.

We shall now make a broken-line graph from this table. On graph paper we draw two lines perpendicular to each other. These lines are called *axes* (singular, *axis*). The axes cross, or



meet, each other at a point called the *origin.* The intervals of time are marked off on the horizontal axis, allowing two spaces for 10 years. On the vertical axis we let one space represent 50,000 people. On the vertical line of 1870 we place a point about one third the way from the horizontal line of 100,000 to the horizontal line of 150,000. (The population 117,714 rounded off and expressed as thousands is 118 and 118 is about one third the way from 100 to 150.) In the same way the other seven points are *plotted.* These eight points are connected by straight lines.

ORAL EXERCISES [A]

1. When did Buffalo have the smallest increase in population? the largest increase?

2. Predict the population of Buffalo in 1950. Perhaps you can find the actual population and check your estimate.

3. Why did we not let each space on the vertical axis represent 1000 persons?

4. Do the years on the horizontal scale increase from right to left or from left to right? How do the numbers on the vertical scale increase?

WRITTEN EXERCISES [A]

Make broken-line graphs to represent the following:

1. The yields of wheat per acre on Frank Bidwell's farm for the period 1942–1949 are as follows:

YEAR	BUSHELS PER ACRE	YEAR	BUSHELS PER ACRE
1942	15	1946	34
1943	19	1947	17
1944	17	1948	29
1945	24	1949	34

2. The average heights in inches of boys and girls of ages 5 to 16 are given in the following table:

Age	5	6	7	8	9	10	11	12	13	14	15	16
Boys	42	44	46	49	50	52	54	55	57	60	63	65
Girls	41	43	46	48	50	52	54	56	58	60	62	62

Use one set of axes for making the two graphs.

3. Below is a table showing how the price of hogs varies in Indiana.

MONTH	PER CENT OF AVERAGE	MONTH	PER CENT OF AVERAGE	MONTH	PER CENT OF AVERAGE
Jan.	93	May	96	Sept.	113
Feb.	100	June	98	Oct.	105
Mar.	104	July	107	Nov.	94
Apr.	97	Aug.	109	Dec.	86

The first item in this table means that the January price of hogs per pound is 93% of the average price per year.

4. The growth in population of four cities from 1880 to 1940 was as follows:

YEAR	ALBANY	BUFFALO	ROCHESTER	SYRACUSE
1880	90,758	155,134	89,366	51,792
1890	94,923	255,664	133,896	88,143
1900	94,152	352,387	162,608	108,374
1910	100,253	423,715	212,149	137,249
1920	113,344	506,775	295,750	171,717
1930	127,412	573,076	328,312	209,326
1940	130,577	575,901	324,975	205,967

Using the same axes, make four broken-line graphs, one for each city.

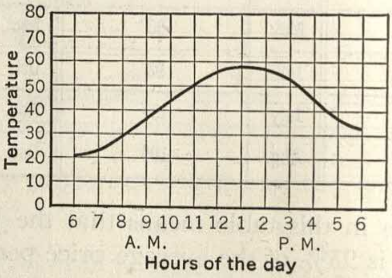
The Curved-Line Graph [A]

A curved-line graph is used to show the different values of a quantity that changes gradually. For example, it is used to show the change of temperature during a day, since temperature changes are usually gradual.

A curved-line graph is made like the broken-line graph, except that the plotted points are joined by a curved line.

EXERCISES [A]

1. Below is a curved-line graph showing the changes in temperature at Elmville on March 19.



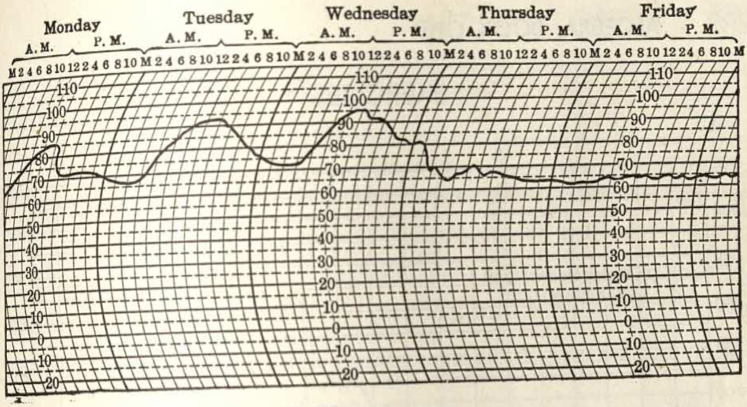
Changes in Temperature

What was the hottest time of the day? What was the temperature at 6 A.M.? at 6 P.M.? Which was the more rapid, the rise of the temperature or its fall?

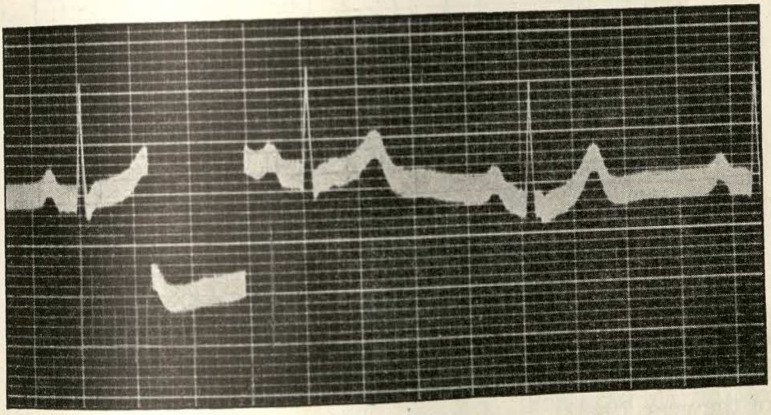
Instruments That Make Graphs [A]

Many curved-line graphs are now made by machines. Records of temperature, air pressure, steam pressure, and horsepower are now usually made by recording instruments.

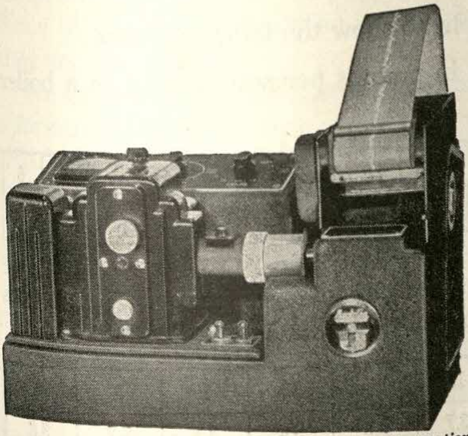
The thermograph records variations of temperature and its graph is called the thermogram. The cardiograph is an instrument that records the force and intervals of heart beats and the cardiogram is the record traced by it. The seismograph is an instrument that registers the earth tremors caused by earthquakes. The graph made by it is called a seismogram.



A Thermogram



A Cardiogram

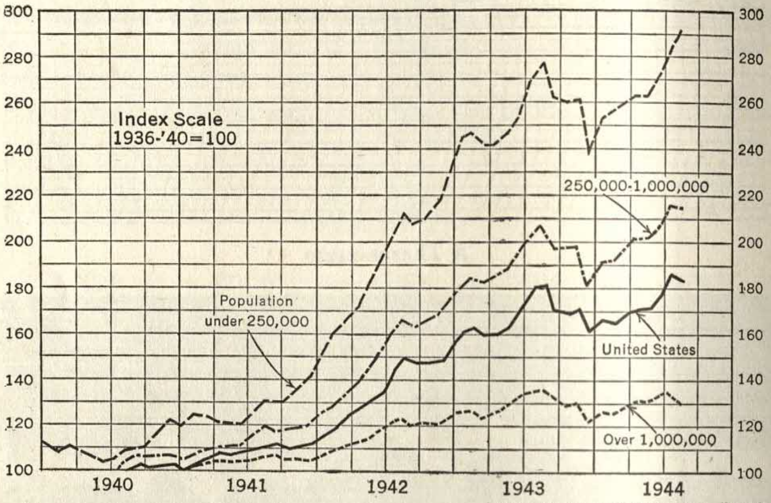


A Cardiograph with a Cardiogram Being Recorded

Courtesy Beck-Lee Corporation

EXERCISES [A]

1.



Automobile Manufacturers Ass., Detroit

Small Cities Show Greatest Increase in Travel

Why are there four spaces used for each year? Which class of cities has the greatest increase in travel? the smallest increase? How do you account for the drop in travel during the last half of the year 1943?

Make curved-line graphs to show the following data:

2. The steam pressure in pounds per square inch in a boiler varied as shown:

Hour	6	7	8	9	10	11	12	1	2	3	4	5	6
Pressure	95	98	100	94	98	95	90	85	88	92	96	100	97

3. A physics class measured the city gas pressure (in ounces per square inch) and recorded their results as follows:

Hour	8	9	10	11	12	1	2	3
Pressure	3	3.5	3.7	3.8	3.4	3.5	3.7	3.8

4. During a summer day the following temperatures were recorded:

TIME	TEMPERATURE	TIME	TEMPERATURE	TIME	TEMPERATURE
6 A.M.	72°	10 A.M.	82°	2 P.M.	90°
7 A.M.	73°	11 A.M.	84°	3 P.M.	91°
8 A.M.	75°	Noon	86°	4 P.M.	90°
9 A.M.	77°	1 P.M.	89°	5 P.M.	87°

5. The depth of a river at a certain station was recorded as follows:

TIME	HEIGHT IN FEET	TIME	HEIGHT IN FEET	TIME	HEIGHT IN FEET
Sun. 6 A.M.	20	Tues. 6 A.M.	28	Thurs. 6 A.M.	30
Sun. 6 P.M.	23	Tues. 6 P.M.	30	Thurs. 6 P.M.	29
Mon. 6 A.M.	26	Wed. 6 A.M.	31	Fri. 6 A.M.	26
Mon. 6 P.M.	27	Wed. 6 P.M.	31	Fri. 6 P.M.	23

The Formula Graph [A]

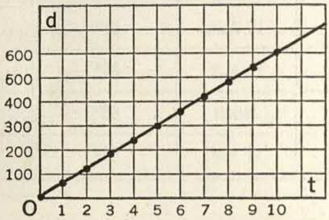
In the broken-line graph and in the curved-line graph there are two quantities that change together. For example, in the temperature graph the temperature and time change together. Since in this graph the time and temperature vary or change, they are *variables.*

We shall now see how easy it is to make a graph or picture of a formula. Let us make a graph of the formula $d = 60\,t$, which tells how many miles d a train will go in t hours if it travels 60 miles an hour. In this formula d and t are variables because they vary. The table below shows the distances gone in 0, 1, 2, 3 · · · 10 hours.

$d = 60\,t.$

t	0	1	2	3	4	5	6	7	8	9	10
d	0	60	120	180	240	300	360	420	480	540	600

The graph of the formula $d = 60t$ is shown below. The different points are plotted in the same way that you plotted points of the broken-line graphs and the curved-line graphs.



Notice that this graph is a straight line. This straight-line graph may be used to find the distance for any time shown on the graph, and to find the time for any distance shown. Suppose we wish to find the distance traveled in $5\frac{1}{2}$ hours. First we find the point halfway between the 5 and 6 points on the t-axis. Then with a ruler we find where the vertical line drawn from this halfway point cuts the graph. Then we use the ruler to find where the horizontal line from this point on the graph cuts the d-axis. We estimate this point on the d-axis to be 330. In other words, when the time is $5\frac{1}{2}$ hours, the distance is 330 miles. Similarly, we can find the time when the distance is known.

To obtain accurate results, it is necessary to use a large graph containing more horizontal and vertical lines.

ORAL EXERCISES [A]

1. Is the graph of the formula $d = 60\,t$ a broken-line graph, a curved-line graph, or a straight-line graph?

2. How is the time axis marked? How is the distance axis marked? The two axes meet at point O. Can you suggest a reason why this point is called the *origin?*

3. From the table estimate the distance traveled in $2\frac{1}{2}$ hours; in $3\frac{1}{3}$ hours.

4. From the graph estimate the distance traveled in $4\frac{1}{2}$ hours; in $7\frac{1}{2}$ hours.

5. Does t have different values in the formula $d = 60\ t$? Does the value of d depend upon the value of t? Does the value of 60 vary in this formula? What are the two variables of this formula?

6. Does t have a negative value in the formula $d = 60\ t$?

7. How many points were plotted in graphing the formula $d = 60\ t$?

8. If two hundred or more points were plotted for this formula, where would they lie?

9. As t increases in value, how does d change? Is the product of any two corresponding values of d and t the same as the product of any two other corresponding values of d and t? If their product remains the same, d and t *vary inversely;* if their quotient does not change, they *vary directly.* Do d and t vary directly, inversely, or neither?

EXERCISES

A

Draw the graphs of the following formulas:

1. $A = 6\ h$
2. $D = 2\ r$
3. $c = w + 4$
4. $c = 8\ w + 1$
5. $c = 3.14\ D$
6. $F = \frac{9}{5}\ C + 32$

7. Apply the tests given in Exercise 9 above, and tell whether the variables in the six exercises just above vary directly or inversely, or in neither of these ways.

B

Draw the graphs of the following formulas:

8. $S = 10\ v$
9. $p = \dfrac{30}{v}$
10. $A = 40\ b$
11. $C = \frac{5}{9}(F - 32)$
12. $v = 32\ t$
13. $S = 180\ n - 360$

Degrees of Equations [A]

The *degree of a monomial* or a term is the sum of the exponents of its letters. Monomials, or terms, such as x, $3\ y$, and $-4\ z$ are of the first degree; monomials, or terms, such as x^2, xy, and

$-4y^2$ are of the second degree; and monomials, or terms, such as x^3, $-x^2y$, mnp, and y^3 are of the third degree.

In determining the degree of a monomial, only the letters which are variables are considered.

The *degree of a polynomial* or the *degree of an equation* is the same as that of its highest-degree term. So to find the degree of an equation we find its highest-degree term. The degree of this term is the degree of the equation.

$4x+8=0$, $x+3y=12$, $x-y=0$ are first degree equations
$x^2=36$, $xy=30$, $x^2-4y=4$ are second degree equations

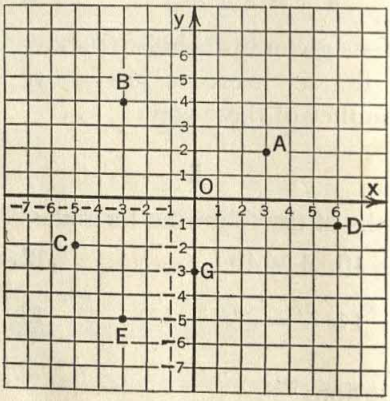
EXERCISES [A]

State the degrees of the following equations:

1. $3x = 10$
2. $x^2 = 25$
3. $x - y = 10$
4. $y - 6 = 4$
5. $xy = 36$
6. $x^2 - x = 6$

Extension of Plotting Points [A]

In order to graph equations whose variables have both positive and negative values, it is necessary to extend the two axes through the origin as shown below.



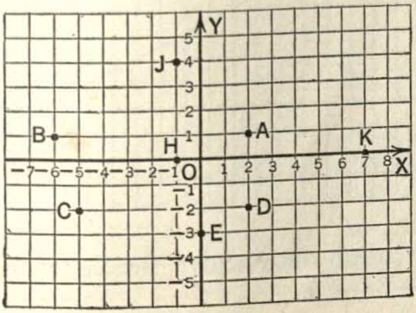
By letting each axis extend in both directions from the origin O, it is possible to locate any point on the paper if its distance and direction from each axis are known. We may think of the two axes as the two principal streets of a small town which meet at right angles at the center of the town. If a stranger at the origin *O* wishes to find a house *A*, he can be directed as follows: "Go three blocks east and two blocks north." To find the point *B* he would go 3 blocks west and 4 blocks north; to find the point *C* he would go 5 blocks west and 2 blocks south; and to find the point *D* he would go 6 blocks east and one block south. If we call the distance east positive, then we should call the distance west negative; and if the distance north is positive, then the distance south should be negative. Then instead of saying "two blocks east" we should say "$+2$"; instead of saying "two blocks north" we should say "$+2$"; instead of saying "two blocks west" we should say "-2"; and instead of saying "two blocks south" we should say "-2."

In stating the location of a point, we always give the horizontal distance first. For example, "two blocks east and 3 blocks south" is written $(+2, -3)$.

The horizontal distance is called the *abscissa*, and the *vertical* distance is called the *ordinate*. The abscissa and ordinate of a point are called the *co-ordinates* of the point. In the graph above, the co-ordinates of *A* are $(3, 2)$; of *B* are $(-3, 4)$; of *G* are $(0, -3)$; and of *E* are $(-3, -5)$.

EXERCISES[A]

1. Remembering that the east or west distances are given first and the north or south distances are given last, give the co-ordinates of *A*; of *B*; of *C*; of *D*; of *E*; of *H*; of *J*; of *K*.



2. Copy and complete: If the co-ordinates of a point are $(5, -2)$, the abscissa of the point is --?-- and the ordinate of the point is --?--.

3. Draw a pair of axes and letter them properly. Let O be the *origin*, the point where the axes cross. Plot the following points:

$A\ (3, 1)$	$C\ (-2, 4)$	$E\ (-6, 2)$	$G\ (0, -5)$
$B\ (3, -1)$	$D\ (-3, -4)$	$F\ (2, 0)$	$H\ (0, 0)$

4. Draw another pair of axes and plot the following points:

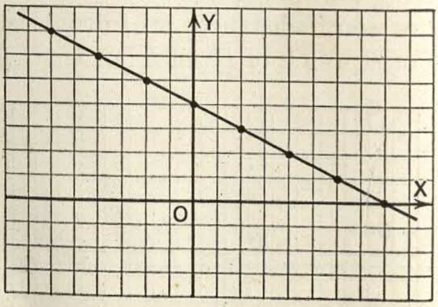
$A(2, -3)$; $B\ (2, 3)$; $C\ (-5, 0)$; $D\ (0, 0)$; $E\ (-4, -5)$

5. Draw a pair of axes, x and y, and plot 5 points which have an abscissa of 6.

6. Copy and complete the table below by finding the corresponding values of y which satisfy the equation $x + 2y = 8$.

x	8	6	4	2	0	− 2	− 4	− 6
y	?	?	?	?	?	?	?	?

The graph for this equation is shown below. For each set of values of x and y there is a point on the graph.



7. Using the graph above, give the value of y when x is 7; 5; 3; 1; − 1; − 3; − 5.

Graphs of First-Degree Equations[A]

If it were possible to plot all the points whose co-ordinates satisfy a given first-degree equation, they would lie on the same straight line. Also, the co-ordinates of any point on this line would satisfy the given equation.

A first-degree equation is called a *linear equation* since its graph is a straight line. Now consider the following example.

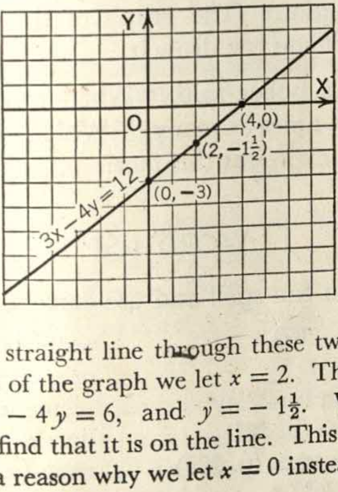
Example. Graph the equation $3x - 4y = 12$.

Solution. Since we know that $3x - 4y = 12$ is a linear equation, we know that its graph is a straight line. Two points are needed to determine a straight line, and a third point is needed to check the graph. Let either x or y have any convenient value. Suppose we let $x = 0$. Then $3(0) - 4y = 12$. $0 - 4y = 12$, and $y = -3$.

$3x - 4y = 12$

x	0	4	2
y	-3	0	$-1\frac{1}{2}$

Then $x = 0$, $y = -3$ is one set of values satisfying the equation. Suppose we let $y = 0$. Then the equation becomes $3x - 4(0) = 12$, or $3x - 0 = 12$, and $x = 4$. That is, another set of values is $x = 4$, $y = 0$. Next we draw the x- and y-axes and plot the points $(0, -3)$ and $(4, 0)$.



Then we draw a straight line through these two points. To check the correctness of the graph we let $x = 2$. Then $3(2) - 4y = 12$, $6 - 4y = 12$, $-4y = 6$, and $y = -1\frac{1}{2}$. We plot the point $(2, -1\frac{1}{2})$ and find that it is on the line. This checks our solution. Can you give a reason why we let $x = 0$ instead of letting it equal some other number?

EXERCISES

A

Draw a graph for each equation:

1. $x = y + 8$

2. $y = 4x$

3. $x - y = 6$

4. $x + y = 0$

5. $x = 2y - 6$

6. $3x = 2y$

7. $-2x + 7y = 0$

8. $4x + y = 12$

9. $a + 6b = -4$

10. $y = 3x - 2$

B

11. $y = 10x - 4$

12. $9x + 5y = 40$

13. $s = 16t$

14. $\frac{2}{3}x - \frac{1}{2}y = 4$

15. $5x - y = 0$

16. $y + 18 = -3x$

17. Draw a graph showing that a certain number is always 4 more than another. ($y = $ --?-- $+ 4$)

18. Draw a graph showing that one number is 5 times another.

19. $A = 3\frac{1}{7}r^2$ is the formula for the area of a circle. Is this a first-degree equation? The graph of this equation is not a straight line. See if you can draw it.

20. A captain of police said, "When traffic is doubled, the accidents are eight times as many." Which of the two formulas, $A = 4t$ or $A = t^3$, expresses this fact?

WORD LIST

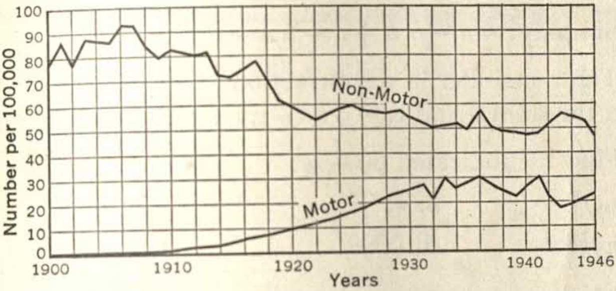
Be sure that you can spell and use correctly these words:

abscissa (p. 287)
axes (p. 286)
circle graph (p. 275)
co-ordinates (p. 287)
degree of a term (p. 285)
graph (p. 269)

ordinate (p. 287)
origin (p. 286)
pictograph (p. 274)
linear (p. 289)
ratio (p. 226)

CHAPTER REVIEW [A]

1. Name five kinds of statistical graphs.



Vehicle Death Rates per 100,000 in the United States

2. Answer the following questions based upon the graph above:

a. When was the highest death rate for non-motor vehicles?

b. Explain why the motor vehicle death rate was so low from 1900 to 1910.

c. Explain why the motor vehicle death rate was lower in 1943 than in 1941.

d. Has any progress been made in reducing the non-motor death rate?

e. The population of continental United States in 1940 was, to the nearest million, 132,000,000. Give the approximate number of deaths in the United States in 1940 due to motor vehicles.

3. Graph the interest formula $i = 0.06\,p$.

4. Recently a survey of tuberculosis sanitarium patients showed that 13% were under 21 years of age, 20% were of ages 21–25, 18% were of ages 26–30, and 12% were of ages 31–35. What per cent were over 35? Make a circle graph for this data.

5. At the beginning of his vacation Frank had savings of $5. At the end of each week he increased his savings by $2. Write a formula for his savings at the end of w weeks.

Graph this formula.

GENERAL REVIEW

A

1. Simplify: $4a - 7b + c - 8a - 4c$

2. The remainder in a subtraction problem is $3x^2 - 4xy + 6y^2$ and the subtrahend is $-4x^2 - xy + y^2$. Find the minuend.

3. Find the indicated powers:

a. $(-5c)^2$

b. $(-3ab^2)^2$

c. $(x^3)^2$

d. $(x^5)^2$

e. $(10^3)^2$

f. $(x^4)^3$

4. Multiply $2x^2 - 3xy - y^2$ by $3x - y$.

5. Divide $16a^3 - 8a^2 - 12a$ by $-4a$.

6. Divide $2m^3 - 11m^2 + 19m - 11$ by $2m - 5$.

7. If $x = -1$ and $y = 2$, find the value of $4x^2y - (xy)^2$.

8. Simplify: $2c - (3c + 1) + c(c - 2)$

9. Remove parentheses in $(2x - 1)(x + 3)$.

Solve:

10. $2x - 6 = 10$

11. $-6 - y = 0$

12. $\frac{7}{8}x = 21$

13. $\frac{4}{9}c = \frac{2}{3}c - 8$

14. $2(p - 3) - 4(2p + 1) = 2$

15. $5(x-4)-3(2x+6)+38=0$

16. Two angles of a triangle are equal and the third angle is twice the sum of the other two. How large is each angle?

17. A farmer sold a number of horses at $70 each, and three times as many cows at $60 each. If the total sales amounted to $2000, how many head of each did he sell?

18. Liquid carbon dioxide boils at $-79°$ centigrade. Find its boiling point, Fahrenheit. $(F = \frac{9}{5}C + 32)$

19. Bismuth melts at 518° Fahrenheit. Find its boiling point, centigrade.

20. Graph the equation $2x - y = 12$.

B

21. Divide $10\,a^3b^4 - 12\,a^2b^3 - 8\,a^2b^5$ by $2\,a^2b^3$.

22. $S = \frac{n}{2}(a + l)$ and $l = a + (n - 1)d$. If $a = -10$, $n = 8$, and $d = 3$, find l and then find S.

23. Simplify: $(x - 1)^2 - (2\,x - 3)(x + 4)$

24. Solve: $(x - 1)^2 = (x - 3)^2$

25. Solve: $(y + 5)(2\,y - 1) - y(2\,y + 4) = 0$

26. Solve: $\frac{b - 1}{2} + \frac{b - 2}{3} - \frac{b - 3}{4} = 6$

27. Find three consecutive integers whose sum is 297.

28. Find three consecutive integers whose sum is 282.

29. How can a five-dollar bill be changed into 29 coins consisting of nickels, dimes, quarters, and half-dollars if there are the same number of dimes and nickels, and the number of half dollars is one more than the number of quarters? (Let x = the number of quarters.)

30. A part of $6000 is invested at 4% and the remainder at 6%. If the 6% investment produces $10 more interest than the 4% investment, what is the amount invested at each rate?

31. $A = p + prt$. If $A = \$376.20$, $t = 1$ year, and $p = \$360$, what is r?

32. Solve for x: $(ax - 1)x + 2 = b + ax^2 - c$

33. Solve for y: $y + 2 + 4\,c = 3\,y + 2$

34. $4\,a + 4\,b - 5\,x - 5\,y = 4(?) - 5(?)$.

35. Frank is x years old and Chester is 6 years older. In 8 years, twice Chester's age increased by three times Frank's age will equal 97 years. How old is each?

36. Find the volume of a cylinder if its height is 18 inches and the radius of its base is 6 inches. $(V = \pi r^2 h)$.

CHAPTER TEST [A]

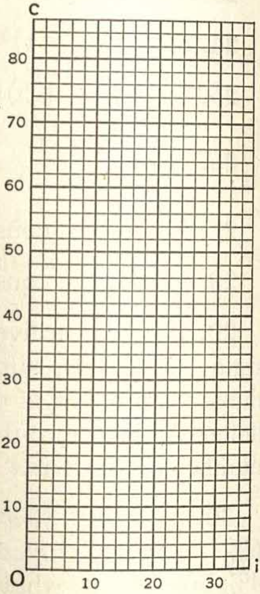
1. Round off 628 to the nearest hundred.

2. Round off 17460 to the nearest thousand.

3. Name three kinds of statistical graphs.

4. Why do newspapers and magazines often use graphs to represent statistical data?

5. The formula $c = 2.54\,i$ tells how to find the length in centimeters when the length in inches is known. Draw the graph for this formula, using values of i from 0 to 30. (Do not mark in your book.)



Ex. 5

6. On graph paper draw two perpendicular axes and plot the five points A, B, C, D, and E: $A\,(2, 3)$; $B\,(-3, -2)$; $C\,(4, -5)$; $D\,(-3, 6)$; and $E\,(7, 0)$.

7. Eileen Prescott uses 8 hours a day for sleep, 2 hours for eating, 6 hours for school, 2 hours for study, and the remainder of the day for recreation. Make a circle graph to represent these facts.

8. Graph the equation $2x - y = 6$.

ROCKET POWER

One of the earliest and most significant inventions of prehistoric man was that of the wheel. Perhaps no other invention, except that of fire, so greatly changed the course of human existence. Prior to the invention of the wheel, burdens could be carried only on the backs of men and animals. Afterward, they might be transported in carts or wagons. It was only natural that when fuel-burning engines were first developed, they were designed to give rotary motion to a wheel. The wheel has dominated transportation for many thousands of years. The modern automobile, train, and airplane, as well as the ancient cart and old-fashioned wagon, are based upon it.

However, there is a motor which does not operate on the principle of producing rotary motion in a shaft or wheel. This is a reaction motor, a motor that pushes. The idea of a reaction motor is very simple and very old. Hero of Alexandria, in the second century B.C., built a simple reaction engine which he called an aeolipile. Reaction motors of today are divided into two main classes — jet motors and rocket motors. A jet engine carries its own fuel, but obtains the necessary oxygen from the air through which it passes. A rocket engine carries not only its fuel but its own oxygen as well.

Men have experimented with rockets ever since the time of Hero. The mathematics necessary for producing the rocket engine was developed early, but the sciences of chemistry, physics, and metallurgy lagged far behind. Only in recent years have practical and useful rockets been developed. Earlier rockets were used chiefly in fireworks and as signals on the high seas. Prior to the Second World War there was speculation concerning possible intercontinental or even interplanetary travel by means of rocket space ships that would go up and beyond the earth's outer atmosphere, but no such ships were produced. It remained for the war to bring rocket transportation close to reality.

Scientists during the war found in the rocket a highly efficient engine of destruction. The most spectacular rocket weapon was the terrifying V–2 rocket, which could have changed the course of the war had it been perfected earlier. These huge bombs, powered by rocket engines, were capable of rising 65 and more miles above the earth's surface and flying at speeds in excess of 4,000 feet per second. They were, however, not the only rockets in use during the war. Both sides employed rocket-powered bombs which were launched from artillery and from airplanes.

The rocket experimenter needs a thorough understanding of scientific principles as well as skill in mechanical design and engineering.

Life photograph by Charles Steinheimer, © *Time Inc.*

A Plane Taking off Under Rocket Power

CHAPTER X

Systems of Linear Equations

Graphs of Pairs of Linear Equations[A]

If the graphs of two linear equations in two unknowns are made on the same set of axes, there are three possibilities, which will now be discussed.

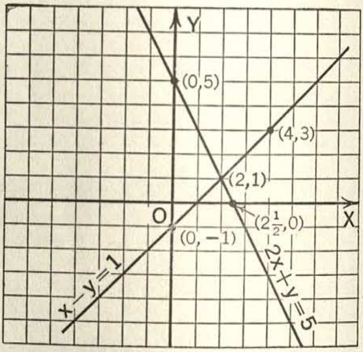
First,
the lines intersect.

$2x + y = 5$

x	0	$2\frac{1}{2}$
y	5	0

$x - y = 1$

x	0	4
y	−1	3



The graphs of these two equations intersect in the point (2, 1). Both equations have the common solution $x = 2$, $y = 1$. Although there is an infinite number of sets of values which satisfy each equation, $x = 2, y = 1$, is the only set which satisfies both equations. Two linear equations whose graphs intersect in a point are called *simultaneous* linear equations. Can you give a reason for calling them simultaneous?

Check. If we substitute $x = 2$, $y = 1$, in $2x + y = 5$, we have $4 + 1 = 5$. Substituting $x = 2$, $y = 1$, in $x - y = 1$, we have $2 - 1 = 1$. If the solution does not check, there is an error in the work. A mistake may be made either in finding one or more points or in plotting one or more points.

To solve two simultaneous linear equations by graphing, graph both equations on the same pair of axes. The co-ordinates of the intersection point represent the common solution of the two equations.

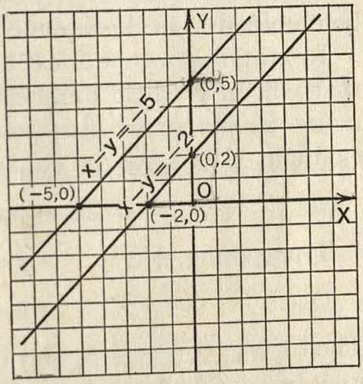
Second,
the lines are parallel.

$x - y = -2$

x	0	-2
y	2	0

$x - y = -5$

x	0	-5
y	5	0



The graphs of these two equations are parallel lines, that is, they never meet. No set of values of x and y which satisfies one equation will satisfy the other equation. In other words, they have no common solution. Two linear equations whose graphs are parallel are said to be *inconsistent*. You can see why they are so called. For one equation states that the difference of two numbers is -2 and the other equation states that the difference of the two numbers is -5. Two such numbers do not exist.

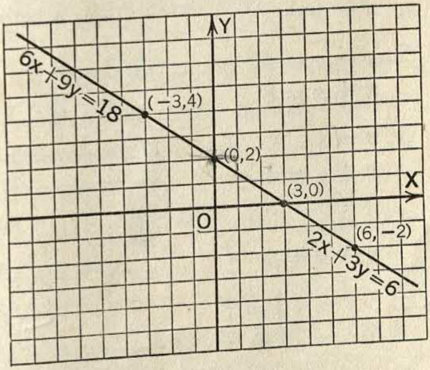
Third,
the lines coincide.

$2x + 3y = 6$

x	0	3
y	2	0

$6x + 9y = 18$

x	-3	6
y	4	-2



The graphs of these two equations just given coincide (one fits exactly on the other). Two linear equations whose graphs coincide are called *dependent equations*, because one is said to depend on the other. The equation $6x + 9y = 18$ may be obtained from the equation $2x + 3y = 6$ by multiplying both members of the latter by 3. How may the equation $2x + 3y = 6$ be obtained from the equation $6x + 9y = 18$?

In graphing sets of linear equations, try to choose values of x that will give whole numbers for y. In finding sets of points, many people prefer first to solve each equation for y. For example, if we solve the equation $2x - y = 5$ for y, we have

$$2x - y = 5$$

Transposing $+2x$, $\quad -y = -2x + 5$

D_{-1} $\quad y = 2x - 5$

Then if $x = 0$, $y = -5$, and if $x = 3$, $y = 1$. These two points may be used to draw the graph.

EXERCISES

A

Graph each of the following pairs of equations on the same axes. Tell whether each pair is simultaneous, dependent, or inconsistent. If a pair is simultaneous, give its solution, and check.

1. $x + y = 5$
$x - y = 1$

2. $x - y = 6$
$x + y = 8$

3. $y = x$
$x + y = 4$

4. $y = 3x$
$x + y = 8$

5. $y = 2x - 8$
$y = 4 - 2x$

6. $3x + 6y = 12$
$x + 2y = 4$

7. $x + y = 8$
$x - y = 0$

8. $3x + 5y = -6$
$x - 2y = -2$

9. $3x - y = 9$
$6x - 2y = 8$

10. $2x - y = -6$
$x + 3y = -10$

11. $y = -2x$
$x + y = 1$

12. $6x + 3y = 12$
$4x + 2y = 8$

13. $5x + 4y = 7$
$6x = 28 + 5y$

14. $2x - y = 6$
$y = 6 - x$

15. Copy and complete each table, based upon the equation above it.

$$2x - y = 5$$

x	-1	?	3	?	0	?	-2	?
y	?	0	?	3	?	-1	?	-11

$$x + 3y = 13$$

x	?	2	4	5	?	0	-2	?
y	4	?	?	?	$2\frac{1}{3}$	?	?	6

How many solutions have you found for each equation? What is their common solution?

B

Solve by graphing, and check:

16. $5x - 3y = 1$
$3x + 5y = 1$

17. $4s - 5 = 3t$
$2s - 1 = t$

18. $3x - 2y = -5$
$5x + 2y = -19$

19. $4x + y - 6 = 0$
$3x - y - 17 = 0$

20. $5x + 3y - 3 = 0$
$7x - 2y - 29 = 0$

21. $y = 6x + 1$
$2x = 3y - 11$

Historical Note[A]. The graphical method of solving sets of equations was discovered by the French mathematician and philosopher, DESCARTES, about 1637. Others had plotted points using positive numbers, but he was the first to plot negative numbers.

Solving Simultaneous Equations by Addition or Subtraction[A]

You have learned one method of solving a pair of simultaneous equations. You will now learn another method. A pair of simultaneous linear equations can be solved if they can be combined to form a third equation which has only one unknown.

Example 1. Solve

$$3x + y = 7 \tag{1}$$
$$x - y = 1 \tag{2}$$

Solution. In order to find the common solution for these two equations, it is necessary to derive a third equation, containing only

x or y. Since there is the same number of y's in

$$3x + y = 7 \qquad (1)$$

and

$$x - y = 1 \qquad (2)$$

we may add the left members and make the y's disappear. Since we add the left members, we must add the right members also. Performing these two additions, we get $4x = 8$. We have eliminated y. Dividing by 4, $x = 2$.

If we multiply both members of the first equation by 1 and both members of the second equation by 3, we have the same number of x's in each equation.

We then have

$$\begin{aligned} 3x + y &= 7 \\ 3x - 3y &= 3 \end{aligned}$$

If we add these two equations, we obtain $6x - 2y = 10$, which we cannot solve. But if we subtract the lower equation from the one above it, we obtain the equation $4y = 4$. Then $y = 1$.

We can also solve for y by substituting $x = 2$ in *either* of the two equations. If we substitute $x = 2$ in the first equation, we have $3(2) + y = 7$. Solving this equation, we get $y = 1$. The solution of the set of equations

$$\begin{aligned} 3x + y &= 7 \\ x - y &= 1 \end{aligned}$$

is $x = 2$, $y = 1$. *Both* of the given equations should be checked by substituting $x = 2$ and $y = 1$.

Directions for Solving Two Simultaneous Linear Equations by the Addition-Subtraction Method:

1. If necessary, multiply both members of the equations by such numbers as will make the coefficients of one of the letters numerically equal.
2. Add or subtract the two resulting equations to obtain an equation having only one letter.
3. Solve the equation containing one letter.
4. Solve for the other letter (*a*) by substituting the value of the letter found in step 3 in either of the two given equations; or (*b*) by using steps 1, 2, and 3.
5. Check by substituting in each of the given equations.

Example 2. Solve
$$\begin{aligned} 5x + 2y &= -5 \\ -3x + 4y &= 29 \end{aligned}$$

Solution.

To solve for y we must eliminate x. The smallest number that can be exactly divided by 5 and 3 is 15. We must therefore have $15x$ in each equation.

$$\begin{array}{l|rcl} M_3 & 5x + 2y &=& -5 \\ M_5 & -3x + 4y &=& 29 \\ \hline & 15x + 6y &=& -15 \\ & -15x + 20y &=& 145 \\ \hline \text{Adding} & 26y &=& 130 \\ D_{26} & y &=& 5 \end{array}$$

To solve for x we must eliminate y. The smallest number that can be exactly divided by 2 and 4 is 4. We must therefore have $4y$ in each equation.

$$\begin{array}{l|rcl} M_2 & 5x + 2y &=& -5 \\ M_1 & -3x + 4y &=& 29 \\ \hline & 10x + 4y &=& -10 \\ & -3x + 4y &=& 29 \\ \hline \text{Subtracting,} & 13x &=& -39 \\ D_{13} & x &=& -3 \end{array}$$

After the value of y has been found, we can find the value of x by substituting $y = 5$ in the equation $5x + 2y = -5$. We then have $5x + 10 = -5$. Solving this equation, we get $x = -3$, as we found in the right-hand column above.

In this method you may at first have trouble deciding whether the equations are to be added or subtracted. You should formulate your own rule. In the above solution was $-15x$ added to or subtracted from $+15x$ to make 0? Was $+4y$ added to $+4y$ or subtracted from $+4y$ to make 0?

Check. Substitute $x = -3$ and $y = 5$ in both equations.

Does $5(-3) + 2(5) = -5$?

Does $-15 + 10 = -5$?

Yes.

Does $-3(-3) + 4(5) = 29$?

Does $9 + 20 = 29$?

Yes.

When systems of equations are to be solved by the addition-subtraction method, they should be written in the standard form $ax + by = c$.

ORAL EXERCISES [A]

Read the equations in exercises A on page 304 and tell what multipliers can be used to eliminate the first letter. Tell what multipliers can be used to eliminate the second letter.

WRITTEN EXERCISES

A

Solve and check:

1. $x + y = 10$
$x - y = 2$

2. $x - y = 13$
$x + y = 15$

3. $2x - y = 14$
$x - y = 5$

4. $-2x + 5y = -3$
$2x - y = -1$

5. $3x - 2y = 25$
$5x - 2y = 39$

6. $a + 2b = 8$
$2a - b = 6$

7. $5m + 3n = -1$
$2m + 9n = 20$

8. $3x - y = 10$
$4x - 9y = 3$

9. $5x - 2y = 0$
$x - y = -3$

10. $3x - 2y = 11$
$2x + y = 12$

11. $3E - 4F = 1$
$5E - 4F = 23$

12. $3x + 2y = -9$
$5x + 3y = -19$

13. $7r - 4s = 56$
$3r - 5s = -45$

14. $6m - 2n = 1$
$8m + n = 5$

15. $3x + 2y = 118$
$x + 5y = 191$

16. $-p - 5q = 0$
$-3p - 10q = -10$

17. $3x - y = 6$
$y = 3x$

18. $\frac{1}{2}x + \frac{1}{3}y = 4$
$x - y = 3$

B

Solve and check:

19. $8h - 7k = 0$
$7h - 8k = 15$

20. $9x - 8y = -16$
$-10y + 7x = -126$

21. $8x + 17y = 536$
$12x - y = 592$

22. $7r - 3s = 23$
$21r + 4s = 30$

23. $5y - 3x + 8 = 4y + 2x + 7$
$4x - 2y = 3y + 2$

24. $6y - 5x - 4 = 5x + 10$
$2y - 2x - 4 = x - y + 2$

25. Using this method, solve equations 2–5 on page 306.

Elimination by Substitution [B]

The method of solving a set of linear equations by the substitution method will be illustrated by two examples.

Example 1. Solve

$$3x - 4y = 5 \qquad (1)$$
$$x + 7y = 10 \qquad (2)$$

Solution. Solving equation (2) for x, we get

$$x = 10 - 7y \qquad (3)$$

Substituting this value of x in equation (1), we get

$$\begin{aligned} 3(10 - 7y) - 4y &= 5 \\ 30 - 21y - 4y &= 5 \\ -21y - 4y &= 5 - 30 \\ -25y &= -25 \\ y &= 1 \end{aligned}$$

Substituting $y = 1$ in equation (3), we get

$$\begin{aligned} x &= 10 - 7 \\ x &= 3 \end{aligned}$$

The solution of these equations, then, is $x = 3, y = 1$.

Check. Does $3(3) - 4(1) = 5$? Does $9 - 4 = 5$? Yes. Does $3 + 7(1) = 10$? Does $3 + 7 = 10$? Yes.

Example 2. Solve

$$5x + 3y = -5 \qquad (1)$$
$$4x - 5y = 33 \qquad (2)$$

Solution. From (1)

$$5x = -3y - 5 \qquad (3)$$

D_5

$$x = \frac{-3y - 5}{5} \qquad (4)$$

Substituting this value of x in equation (2), we obtain

$$4\left(\frac{-3y - 5}{5}\right) - 5y = 33$$

M_5

$$\begin{aligned} 5(4)\left(\frac{-3y - 5}{5}\right) - 5(5y) &= 5(33) \\ -12y - 20 - 25y &= 165 \\ -12y - 25y &= 165 + 20 \\ -37y &= 185 \end{aligned}$$

D_{-37}

$$y = -5$$

Substituting in (4)

$$x = \frac{-3(-5) - 5}{5}$$
$$x = 2$$

The check is left to the pupil.

NOTE. Either of the given equations can be solved for x or for y in terms of the other letter. Then this value of the letter is substituted in the other equation. Often in sets of linear equations one equation is already solved for one letter. In that case it is easier to solve the set by the substitution method than by the addition-subtraction method.

EXERCISES [B]

1. Read Exercises 2–17 below, and tell in which exercises one equation is already solved for one letter.

Solve by the substitution method:

2. $3x - y = 14$
$x = 5y$

3. $y = x + 2$
$x + 2y = 1$

4. $3x - 2y = 1$
$x = y + 1$

5. $a = 2b$
$4a - 3b = 15$

6. $x + y = 1$
$3x + 4y = 4$

7. $1 - y = x$
$5y - x = 29$

8. $5x + y = 15$
$x - 3y = 25$

9. $5x = 1 - 2y$
$11x + 5y = 1$

10. $y + 3p = 10$
$4y + 5p = 12$

11. $m = 9n$
$13m - 12n = 35$

12. $2x = 5y$
$10x - 15y - 6 = 0$

13. $8B + A = -46$
$13A + B = -83$

14. $2p - 3q = 3$
$6p - 3q = 7$

15. $3y - 2x = 16$
$3x + 7y + 1 = 0$

16. $7x + 8y = 36$
$11x + 5y = -4$

17. $5x + 2y = 7$
$3x - 4y = 25$

Sets of Equations Containing Fractions and Parentheses [A]

Sets of equations which contain fractions and parentheses should be simplified and reduced to the standard form $ax + by = c$. Then either the addition-subtraction method or the substitution method can be used to complete the solution.

Example. Solve

$$\frac{5x}{6} = 7 - \frac{y}{4} \qquad (1)$$

$$\frac{2x}{3} + 1 = 4 + \frac{y}{8} \qquad (2)$$

Solution. Multiplying both members of (1) by 12, we get

$$10x = 84 - 3y$$

Transposing

$$10x + 3y = 84 \qquad (3)$$

Multiplying both members of (2) by 24, we get

$$16x + 24 = 96 + 3y$$

Transposing $$16x - 3y = 96 - 24$$

and $$16x - 3y = 72 \tag{4}$$

The two equations are now in standard form:

$$10x + 3y = 84 \tag{3}$$

$$\underline{16x - 3y} = \underline{72} \tag{4}$$

Adding $$26x = 156$$

D_{26} $$x = 6$$

Substituting $x = 6$ in equation (3), we obtain

$$60 + 3y = 84$$

$$3y = 84 - 60$$

$$3y = 24$$

$$y = 8$$

Then we must substitute $x = 6$ and $y = 8$ in the original equations (1) and (2) in order to check our solution. Substituting in equations (3) and (4) would not be a check.. Why? The check is left to the pupil.

EXERCISES

A

Solve:

1. $x + \frac{3y}{4} = 2$
$2x + \frac{5y}{4} = \frac{7}{2}$

2. $2x + 3(x + y) = 11$
$2x - 3y = -1$

3. $2x = 8(y + 1)$
$3(x - 3y) = 15$

4. $x + y = 2000$
$.04x + .05y = 85$

5. $\frac{5x}{2} - 5y = 20$
$x = \frac{1}{3}y$

6. $\frac{x}{3} - \frac{y}{2} = -\frac{8}{3}$
$\frac{x}{7} - \frac{y}{3} = -\frac{1}{21}$

7. $x - 2 = 3(x + y)$
$x + 6 = 3x + y$

8. $x + y = 4$
$2(x + y) + 3(3x + 5) = 12$

9. $\frac{1}{2}(x + y) - \frac{1}{3}(y + 4) = 3$
$2x - 3y = 4$

10. $x + y = 5000$
$\frac{4x}{100} + \frac{6y}{100} = 264$

B

Solve:

11. $\frac{7a}{4} + \frac{5b}{3} + 3 = 0$
$\frac{a}{2} - \frac{b}{3} = 4$

12. $x - 4y = 10$
$\frac{x+y}{5} - \frac{x-y}{7} = 2$

13. $2(5x - y) + 3(y - 2) = 0$
$5(3x + 1) = 2y$

14. $x - y + 2 = 5(3x + y)$
$3x - y = 9(1 - 2x)$

15. $4(x + y) + 5(x + 1) = -16$
$3(3x - 2y) - (2x - 3y) = 2$

16. $\frac{5x}{6} + \frac{7y - x}{5} = 16$
$5y + \frac{2x - 3y}{3} = 7$

17. $x + y = 8000$
$.04x + .06y = .0525(x + y)$

18. $12(x - y) + 11y = 14$
$18x + 5(y + 1) = 0$

19. $a - b = \frac{1}{7}(a + b)$
$2(a + 2b) = 5$

20. $(9a + 5b) + 7(1 - 2b) = 0$
$3b - 2(b + 1) = 1 + 4a$

Solving Problems by Systems of Equations [A]

So far in this course you have solved a verbal problem by letting one letter represent the unknown quantity; or if there was more than one unknown quantity, you let one of the letters represent one of the unknown quantities, and then represented the other unknown quantities in terms of that same letter. In either case you solved the problem by using one equation involving one letter.

Very often a problem involving more than one unknown may be solved more easily by representing each of the unknowns by a separate letter. If a problem has two unknowns, it may be solved by using two letters in two equations; and if it has three unknown quantities by using three letters in three equations. Compare the two solutions of the following example:

Example. The difference of two numbers is 18. The sum of twice the larger and three times the smaller is 151. Find the numbers.

Solution with One Letter

Let $s =$ the smaller no.

Then $s + 18 =$ the larger no.

$$2(s + 18) + 3s = 151$$
$$2s + 36 + 3s = 151$$
$$2s + 3s = 151 - 36$$
$$5s = 115$$

$s = 23$, the smaller

$s + 18 = 41$, the larger

Check. $41 - 23 = 18$

$2(41) + 3(23) = 151$

Solution with Two Letters

Let $s =$ the smaller no.

and $l =$ the larger no.

$$l - s = 18$$
$$2l + 3s = 151$$

$$\begin{array}{r|l} 2 & l - s = 18 \\ 1 & 2l + 3s = 151 \\ \hline \end{array}$$

$$\begin{array}{rcr} 2l - 2s & = & 36 \\ 2l + 3s & = & 151 \\ \hline -5s & = & -115 \end{array}$$

$s = 23$, the smaller

$l - 23 = 18$

$l = 41$, the larger

Check. $41 - 23 = 18$

$2(41) + 3(23) = 151$

To Solve a Problem with Two Letters:

1. Let one letter represent one unknown quantity and another letter represent the other.
2. Form two equations from the conditions of the problem or from facts you know.
3. Solve the two equations and check.

EXERCISES. PROBLEMS

A

1. Find two numbers if their sum is 33 and their difference is 9.

2. The sum of two numbers is 84 and the larger exceeds the smaller by 22. What are the numbers?

3. The difference of two numbers is 3. If twice the larger is subtracted from three times the smaller, the remainder is 9. Find the numbers.

4. Find two numbers such that four times the smaller plus five times the larger equals 73; and three times the smaller decreased by twice the larger equals 3.

5. One of two complementary angles is 4 times as large as the other. How many degrees are there in each angle?

6. Two angles are complementary. Find the number of degrees in each angle if twice the smaller exceeds the larger by 8°.

7. The difference of two supplementary angles is 44°. Find the angles.

8. Two angles are supplementary. Find the number of degrees in each angle if 5 times one angle equals 4 times the other.

9. The perimeter of a rectangle is 848 inches and the length exceeds the width by 200 inches. Find the dimensions of the rectangle.

10. The length of a rectangular flower bed is to be 50% greater than the width, and its perimeter must be 30 feet. How long and how wide must the bed be?

11. A woman wishes to invest \$10,000 in bonds and in some stocks so as to receive an annual income of \$470. How should she distribute the investments if the bonds pay 4% and the stocks pay 6%?

12. A man wishes to invest \$7500, part at 6% and the remainder at 4% so as to have an income of \$410. How much must he invest at each rate?

13. If 4 oranges and 6 grapefruit cost 37 cents, and 7 oranges and 5 grapefruit cost 40 cents, what is the cost of each?

14. The sum of the ages of a father and son is 44 years. Four years ago the father was 8 times as old as the son. What are their present ages?

15. A father is three times as old as his daughter. Ten years ago the father was seven times as old as his daughter. How old is each at present?

16. Find the capacity of two trucks if 6 trips of the smaller and 4 trips of the larger make a total haul of 39 tons, and 6 trips of the larger and 4 trips of the smaller make a total haul of 41 tons.

17. A weight of 16 pounds on one end of a lever balances a weight of 10 pounds on the other end. If the weights are 12 feet apart, how far is each from the fulcrum?

18. If Charles were two years older, he would be half as old as Fred. If Fred were six years older he would be three times as old as Charles. How old is each?

B

19. In a right triangle one angle contains 90°. Find the number of degrees in each of the other angles if the larger angle lacks 7° of being one half the other's supplement.

20. Two weights balance on a lever when one is 5 feet from the fulcrum and the other is 6 feet from the fulcrum. If 8 pounds are added to each weight, they balance when they are respectively 7 feet and 8 feet from the fulcrum. Find the number of pounds in each weight.

21. A butcher sold 4 pounds of round steak and 3 pounds of rib steak for $4.05. If he had sold 3 pounds of round steak and 4 pounds of rib steak, he would have received only $4.00. What was the selling price per pound of each kind of steak?

22. A wholesale grocer wishes to make 100 pounds of 27-cent coffee by mixing 30-cent coffee with 25-cent coffee. How many pounds of each kind must he use?

23. How many pounds of 45-cent coffee must be added to 20 pounds of 35-cent coffee to make a mixture of 38-cent coffee?

24. A seedsman mixes white-clover seed valued at 40 cents a pound with blue-grass seed valued at 25 cents a pound to make a mixture worth 30 cents a pound. How many pounds of clover seed are needed to make a mixture of 50 pounds?

25. A man uses 30 pounds of clover seed worth 15 cents a pound and 20 pounds of timothy seed worth 8 cents a pound to make a mixed seed. What is the value of a pound of the mixture?

WORD LIST

Can you spell and use each of the following words correctly?

abscissa (p. 287)
co-ordinates (p. 287)
dependent equations (p. 300)
eliminate (p. 302)
extremes (p. 228)
inconsistent equations (p. 299)
linear equation (p. 289)
ordinate (p. 287)
origin (p. 286)
proportion (p. 227)
ratio (p. 226)
simultaneous (p. 298)
substitution (p. 304)

CHAPTER REVIEW [A]

Solve graphically:

1. $\begin{aligned} x + y &= 6 \\ 2x - y &= 6 \end{aligned}$

2. $\begin{aligned} x + 5y &= 7 \\ 2y - x &= 1 \end{aligned}$

Solve algebraically:

3. $\begin{aligned} 2x - y &= 2 \\ 7x - y &= 12 \end{aligned}$

4. $\begin{aligned} 4x + 3 &= y \\ x + y &= 0 \end{aligned}$

5. $\begin{aligned} a - 4b &= -4 \\ a - 3b &= -2 \end{aligned}$

6. $\begin{aligned} 2r + 3s &= 13 \\ 4r + 3s &= 15 \end{aligned}$

Tell whether the following equations are dependent, simultaneous, or inconsistent:

7. $\begin{aligned} 2x - y &= 15 \\ 6x - 3y &= 45 \end{aligned}$

8. $\begin{aligned} 2x - 7y &= 11 \\ 5x - y &= 11 \end{aligned}$

9. $\begin{aligned} 4x - y &= 8 \\ 4x - y &= 21 \end{aligned}$

10. $\begin{aligned} 3a + 2b &= 2 \\ a - 3b &= 19 \end{aligned}$

Solve:

11. $\begin{aligned} y + 1 &= 4 - 2(x - 2) \\ x - y &= 7x - 3y - 16 \end{aligned}$

12. $\begin{aligned} \frac{x}{3} + \frac{y}{4} &= 4 \\ \frac{x}{6} - \frac{y}{2} &= -3 \end{aligned}$

13. Find two numbers if their sum is 36 and their difference is 6.

14. How many degrees are there in each of two complementary angles if their difference is 10°?

15. Two boys balance on the ends of a seesaw 12 feet long. One boy weighs 60 pounds and the other boy 40 pounds. How far does each boy sit from the fulcrum?

16. The perimeter of a rectangle is 54 feet. The length exceeds the width by 5 feet. Find the length and the width of the rectangle.

GENERAL REVIEW

A

1. Subtract $3x^2 + 11$ from $7x^2 - 3x + 2$.

2. Add $3a^2 + 7a - 4$, $11a + 7$, and $2a^2 - 12a$.

Multiply as indicated:

3. $x(x^5)$

4. $3a^2(5a^3)$

5. $3x^2(3xy)$

6. $-2b(-3b)$

7. $2n(n)(-2)$

8. $2n(n-2)$

9. $(x - 3y)(2x + 5y)$

10. $(3n^2 + 4)(3n^2 - 4)$

11. $-3x^2(x^4 - 5x^3 + 6x^2)$

12. $(x^2 - 3x + 1)(x + 2)$

13. $(a - b)(a + b)$

14. $5a(a^2 - a + 1)$

Divide as indicated:

15. $(-15x^2y) \div (-3y)$

16. $(18a^2b^2c) \div (3a^2b^2)$

17. $\dfrac{x^3 + 3x^2 - 6x}{x}$

18. $(10a^2b + 5ab^2) \div (5ab)$

19. $(3xy - 21x^2y^2) \div (3xy)$

20. $\dfrac{2a^2b - 4ab + 8ab^2}{2ab}$

21. $(x^2 - 7x + 12) \div (x - 3)$

22. $(10x^3 - 11x^2 - 4x - 3) \div (2x - 3)$

Find the value of each of the following expressions when $x = 2$, $y = -3$, and $z = 0$:

23. x^2yz

24. xy^3

25. $x^2y - z$

26. $xy^2 + 6z$

27. $x^2 + y^2 + z^2$

28. $x^3 - y^3 - z^3$

29. What per cent of 60 is 6?

30. What per cent of 6 is 60?

B

Solve the following sets of equations:

31. $5 - 2(x + y) = 3$
$x + 2(x - y) = 18$

32. $x + y = 6000$
$.06\,x + .03\,y = .04(x + y)$

Solve:

33. $\frac{6y-5}{8} - \frac{4y-1}{3} = \frac{1}{3} - y$

34. $y - \frac{3y+4}{6} = \frac{1}{2}$

35. $y - 1 = \frac{5y+1}{4} - \frac{4y+6}{9}$

Simplify:

36. $3\,x(x - 2) + 4\,x(x + 5)$

37. $2\,y^2 + 3\,y(y - 1) - 6\,y$

38. $3(a^2 + b^2 + c^2) - 2\,b^2$

39. $2\,x(x + y) - 3\,x(x - y)$

40. Multiply $(a^2 + 2\,ab + b^2)(a - b)$.

41. Divide $x^8 - y^8$ by $x^2 - y^2$.

42. John can do a certain piece of work in 12 days and Ben in 15 days. After John has worked alone for 5 days he hires Ben to help him finish the work. How many days does Ben work?

43. What principal will amount to $470 in 5 years at $3\frac{1}{2}\%$? ($A = p + prt$)

44. Sam is 12 years older than Henry, and Sam's age 10 years ago was $\frac{3}{4}$ of what Henry's age will be 6 years from now. Find the age of each.

45. At the beginning of a school year a mother made the following agreement with her daughter Mary. Each month that Mary's algebra grade was 90 or above Mary was to receive a quarter. Each month that her grade was below 90 she was to give her mother a dime. At the end of the school year (10 months) Mary had $1.45. How many months did Mary fail to receive a grade of 90 or above?

CHAPTER TEST [A]

1. Graph the equation $x + y = 6$.

2. Solve graphically: $2\,x - y = 4$
$x + 2\,y = 7$

Solve the following sets of equations by the addition-subtraction method:

3. $x + y = 8$
$x - y = 2$

4. $2x - y = 14$
$3x + y = 16$

5. $.5x - .3y = 1.7$
$.4x - .1y = 1.5$

6. $8x - 3y = 2$
$3x - 5y = -7$

Tell whether the following equations are dependent, simultaneous, or inconsistent:

7. $3x - y = 16$
$2x + 7y = 26$

8. $x + 3y = 5$
$4x + 12y = 20$

9. Find two numbers if their sum is 30 and five times the larger number decreased by three times the smaller number is 54.

10. If 4 grapefruit and 2 pineapples cost 74 cents, and 5 grapefruit and 3 pineapples cost $1.05, what is the cost of one pineapple?

CHAPTER TEST [B]

1. Solve graphically: $3x + 2y = 14$
$y = \frac{1}{2}x$

Solve by the substitution method:

2. $9x - 2y = 19$
$3y = -2x$

3. $x + 5y = 23$
$3x - y = -11$

Solve by any method:

4. $.07x + .5y = 1.06$
$.2x - .07y = 1.53$

5. $2(x - y) + 11 = 5x$
$3x - 2(x + y) = 2x - 1$

6. A grocer mixes coffee worth 40 cents a pound with coffee worth 48 cents a pound to make a mixture of 400 pounds to sell at 45 cents a pound. How many pounds of the coffee worth 48 cents a pound are there in the mixture?

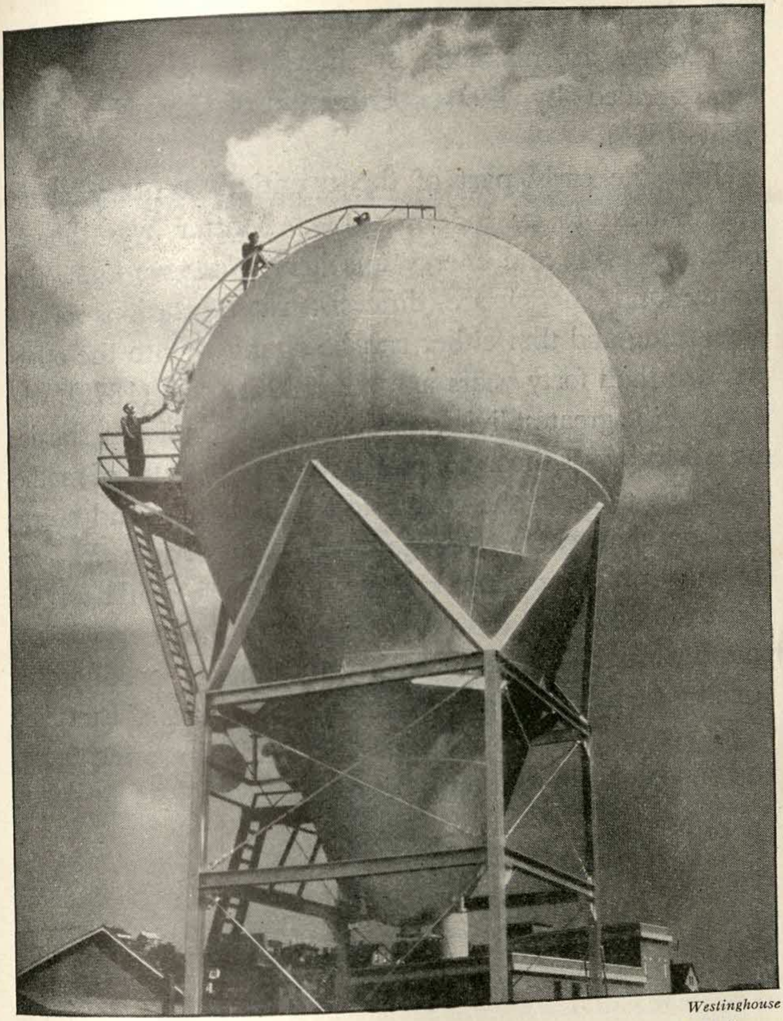
7. Two weights balance on a lever when one is 4 feet from the fulcrum and the other is 5 feet from the fulcrum. If 10 pounds are added to each weight, they balance when they are respectively 6 feet and 7 feet from the fulcrum. How many pounds are there in each weight?

MATHEMATICS IN RESEARCH

By the use of equations scientists are able to think more clearly and express their thoughts more concisely than they otherwise could. As the scientist has learned more about the laws that govern the universe, he has depended more upon mathematics to express these laws. The physicist and the astronomer have used mathematics in their work for centuries. In more recent years mathematics has been applied to various fields. For example, it is now used in the study of economics.

Let us examine a few cases that show how mathematics is associated with scientific research.

Near the close of the sixteenth century Kepler, a German astronomer, after years of observation proclaimed that the planets move around the sun in elliptical paths. Sir Isaac Newton (1642–1727), the great English physicist and mathematician, used mathematics to show that the path of a planet is an ellipse only if the force between the sun and planet varies as the square of the distance between them. Michael Faraday (1791–1867), one of the greatest scientific investigators of the nineteenth century, reasoned that if electricity can produce magnetism, then magnetism should produce electricity. By experimenting he discovered that moving a bar magnet into or out of a coil of wire causes a flow of electricity in the wire of the coil. This is the underlying principle of the electric generator. James Clerk-Maxwell (1831–1879) extended the discovery of Faraday. By the use of mathematics he came to the conclusion that magnetic action travels through space



Westinghouse

The 4,000,000-volt Atom Smasher of the Westinghouse Research Laboratories Is One of the Largest Van de Graaff Generators Ever Built · One advantage of this type of atom smasher over the cyclotron is that the atomic bullets can be more accurately controlled; another is that the particles are more consistent and can be more easily focused on the target. These factors are important in research, where exact, critical experiments are carried on. However, the cyclotron produces higher voltages and stronger beams of particles and is better suited to smashing atoms on a large scale. Like the cyclotron, the Van de Graaff generator uses charged particles, or ions, as its atomic ammunition. Protons, deuterons, and alpha particles are the bullets usually employed in the form of waves and with the speed of light, and that light waves are magnetic in character. His views were later verified by the experiments of Heinrich Hertz (1857–1894).

Until the early part of the twentieth century matter was thought to be indestructible. Matter was thought to be one thing and energy another. Now we know that matter and energy are different manifestations of the same thing and that either can be changed into the other.

More than forty years ago Albert Einstein (1879), who is one of the greatest living mathematical physicists, startled the world by his theory of relativity. By the use of mathematics he came to the conclusion that matter and energy are interchangeable. He gave us the formula $E = mv^2$ for converting mass into energy. In this formula v is the speed of light. The formula tells us that the energy of one pound of matter is equivalent to the energy produced by burning $1\frac{1}{2}$ million tons of coal. This formula of Einstein's was a great aid to the scientists who studied atomic energy and succeeded in smashing the atom.

CHAPTER XI

Special Products and Factoring

Products and Factors[A]

When two or more numbers are multiplied together, they are called the *factors* of the result, and the result is called their *product*. Since $3 \times 7 = 21$, the number 21 is the product of 3 and 7, and the numbers 3 and 7 are the factors of 21. Likewise, since $x(x + 3) = x^2 + 3\,x$, the expression $x^2 + 3\,x$ is the product of x and $x + 3$, and the numbers x and $x + 3$ are the factors of $x^2 + 3\,x$.

Factoring[A]

Study the three columns below and then answer the questions relating to them:

Multiplication	*Division*	*Factoring*
$2 \times 3 = ?$	$6 \div 2 = ?$	$6 = ? \times ?$
$5 \times 8 = ?$	$40 \div 5 = ?$	$40 = ? \times ?$
$x(x + 1) = ?$	$(x^2 + x) \div x = ?$	$x^2 + x = ? \times ?$

1. How many factors are given in each multiplication problem above? in each division problem? in each factoring problem?

2. How many products are given in each multiplication problem? in each division problem? in each factoring problem?

3. In which of the three operations, multiplication, division, and factoring, are the products given?

4. How is a factoring problem like a division problem? How is it different from a division problem?

5. Tell what is needed to complete this statement: Factoring an expression is finding its __?__. It is finding the numbers whose __?__ is given.

Multiplying and Dividing by Monomials [A]

Since factoring is a special kind of division in which both the divisor and quotient are to be found, you can see how important it is for you to be able to multiply and divide with speed and accuracy. The following exercises offer the drill which many of you need at this time.

ORAL EXERCISES [A]

Multiply:

1. $3(x + y)$

2. $2\,m(m^2 + 4)$

3. $x^2(x - 1)$

4. $7\,x(x^2 - 2\,x + 3)$

5. $-\,4\,a(a^2 - 2\,ab + b^2)$

6. $xy(x^2 - 2\,xy + y^2)$

7. $x^2y(3\,x^2 - 8\,xy - y^2)$

8. $-\,2\,x^3(x^4 - x^2 - 1)$

9. $-\,x^3(1 - xy - y^2)$

10. $-\,3\,c^3(c^2 - 4\,c + 1)$

11. $2\,\pi r(r + h)$

12. $a(x - y + z)$

13. $a^2b^2(a^2 + b^2 + 1)$

14. $x^4y^3(x - y - z)$

15. $-\,5\,a^3(a^2 - 6\,a + 7)$

16. $-\,2\,x^{10}(x^3 - 10\,x - 2)$

17. $\pi(R^2 - r^2)$

18. $\pi h(r^2 + R^2 + rR)$

Divide:

19. $(5\,m - 10) \div 5$

20. $(7\,x - 14\,y) \div 7$

21. $(20\,a^3 - 15\,a) \div a$

22. $(20\,a^3 - 15\,a) \div (5\,a)$

23. $(mc - md) \div m$

24. $(12\,x^2 - 18\,xy) \div (2\,x)$

25. $(8\,x^2 - 12\,x + 4) \div 4$

26. $(P + Prt) \div P$

27. $(\pi R^2 + \pi r^2) \div \pi$

28. $(\frac{1}{3}\,\pi r^2h + \frac{1}{3}\,\pi R^2h) \div (\frac{1}{3}\,\pi h)$

Name the missing factors:

29. $15\,b^2 - 10\,b^5 = 5\,b^2(\ ?\ )$

30. $8\,xy - 4\,x^3 = 4\,x(\ ?\ )$

31. $my + ny = ?(m + n)$

32. $m^4 - 5\,m^3 - m^2 = m^2(\ ?\ )$

33. $ab + bc + b = b(\ ?\ )$

34. $6\,x^3 - 4\,x^2 - 2\,x = ?(3\,x^2 - 2\,x - 1)$

Factoring a Polynomial Which Has a Monomial Factor [A]

When you factor a number, you not only find the quotient but you must find the divisor. So in factoring you first find a divisor and then find the quotient. The divisor and quotient are the factors of the number. Let us look at some examples.

Example 1. Factor $4x - 12$.

Solution. Each term of $4x - 12$ is exactly divisible by 2 and by 4. Then either 2 or 4 may be used as a divisor of $4x - 12$. In factoring polynomials it is customary to use the *largest monomial divisor* as one of the factors. So 4 is one factor of $4x - 12$. Since $(4x - 12) \div 4 = x - 3$, the other factor is $x - 3$. Although 4 is not a prime factor (one that cannot be factored), we do not separate it into its factors 2×2. The solution is set down as follows: $4x - 12 = 4(x - 3)$.

To check the solution, multiply $x - 3$ by 4.

Example 2. Factor $5x^4 - 15x^3 - 20x^2$.

Solution. By inspecting the three terms of $5x^4 - 15x^3 - 20x^2$ we see that 5 is the largest numerical factor that will exactly divide 5, -15, and -20; and that x^2 is the highest power of x that will exactly divide x^4, x^3, and x^2. Then $5x^2$ is the largest monomial factor. In factoring one should be sure to find both numerical and literal parts of the monomial divisor. The solution is set down as $5x^4 - 15x^3 - 20x^2 = 5x^2(x^2 - 3x - 4)$.

EXERCISES

A

Factor when possible:

1. $3n + 3$
2. $4a - 8$
3. $5x - 25$
4. $ax + ay$
5. $6x - 6y$
6. $2a + 6$
7. $3ab + 3ac$
8. $15x - 20$
9. $5x - 11y$
10. $a^2 + 4a$
11. $c^2 - c$
12. $bc - b$
13. $ax + a$
14. $y^3 - y$
15. $\pi R - \pi r$

16. $m^2x + m^2y$

17. $20\,c - 30\,d$

18. $9 - 27\,x$

19. $7\,xy - 14\,y$

20. $10 - 3\,c$

21. $P + Prt$

22. $h^2 - 9\,h$

23. $5\,x^2 - 5\,x - 10$

24. $6\,x^2 + 48\,x + 72$

25. $5\,a - 7\,b + c$

26. $x^3y^3 + x^2y^2 + xy$

27. $x^3 - x^2 - 42\,x$

28. $a^4 + 7\,a^3 + 12\,a^2$

29. $5\,k^3 + 5\,k^2 - 60\,k$

30. $7\,p^5 + 70\,p^2 - 112\,p$

31. $22\,m^2 - 33\,m + 66$

32. $3\,a^2b^2 - 6\,a^2b + 9\,ab$

33. $\frac{1}{2}\,x + \frac{1}{2}\,y - \frac{1}{2}\,z$

B

34. $180\,n - 360$

35. $\pi r^2 + \pi rl$

36. $2\,\pi r^2 + 2\,\pi rh$

37. $x^2y^3 - x^4y^5$

38. $1.5\,a^2 + 2.5\,b^2$

39. $\frac{1}{8}\,tx - \frac{3}{8}\,ty$

40. $42\,x^2y^3 - 35\,x^3y^4 - 14\,x^5y^6$

41. $3\,y^5 - 6\,y^4z - 9\,y^5z$

42. $x^4 - x^3 + x^2 - x$

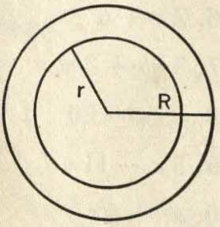
43. $abcx - ab^2cx + b^2x$

44. $27\,a^2b^3 - 18\,a^3b^2 + 9\,a^2b^3$

45. $24\,a^7b^2 + 40\,a^{10}b^5 - 48\,a^{12}$

46. The area of a rectangle is $4\,ax + 4\,bx$. What are the sides of the rectangle if they are factors of the area?

47. What is the area of the larger circle shown here? of the smaller circle? Show that the area of the circular ring may be expressed by $A = \pi R^2 - \pi r^2$. Show that the formula $A = \pi(R + r)(R - r)$ will give the area of the ring.



C

48. Show that $T = 2(lw + lh + wh)$ is a formula for finding the total area of the rectangular solid in Fig. 1.

49. $S = \frac{1}{2}na + \frac{1}{2}nl$. Change this formula by factoring its right member.

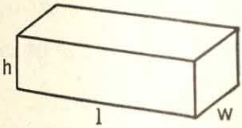


Fig. 1

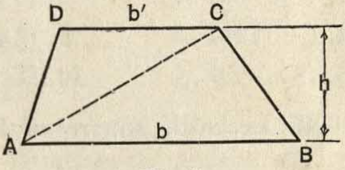


Fig. 2

50. The figure $ABCD$ in Fig. 2 is a trapezoid having a height h and bases b and b'.

a. Write a formula for the area of $\triangle ABC$.

b. Write a formula for the area of $\triangle ADC$.

c. Write a formula for the area of $ABCD$.

d. Rewrite the formula for the area of $ABCD$ again, having the right member of the formula factored.

Squaring Monomials [A]

In finding certain kinds of products (called *special products*), it is necessary to square monomials quickly. Squaring a monomial is using it twice as a factor, or multiplying it by itself.

Example 1. Square $4x^3y$.

Solution. $(4x^3y)^2 = 4x^3y \cdot 4x^3y = 16x^6y^2$

Can you give the answer by thinking "4 squared is 16, x^3 squared is x^6, and y squared is y^2"?

Example 2. $(-6a^3b^4)^2 = ?$

Solution. The even powers of negative numbers are positive; 6 squared is 36, a^3 squared is a^6, and b^4 squared is b^8.

Then $(-6a^3b^4)^2 = 36a^6b^8$

ORAL EXERCISES [A]

Square as indicated:

1. 3^2
2. $(-4)^2$
3. -4^2
4. $(-1)^2$
5. $(8\,m)^2$
6. $(7\,a^2)^2$
7. $(-2\,r^2s)^2$
8. $(5\,s^2t^4)^2$
9. $(\frac{1}{2}\,gt)^2$
10. $(-ab^2c^3)^2$
11. $(.4\,x^2)^2$
12. $(\frac{1}{3}\,abc^3)^2$
13. $(1.2\,c^7)^2$
14. $(20\,x^5)^2$
15. $(.01\,a^{10})^2$

16. Learn the squares of the numbers 1 to 20 inclusive.

Finding the Square Roots of Monomials [A]

When a number has two equal factors, either of the factors is called the *square root* of that number. Since $16 = 4 \times 4$, the number 4 is a square root of 16. Since 5 squared is 25, the 5 is a square root of 25. Since the square of a^3b^2 is a^6b^4, then a^3b^2 is a square root of a^6b^4.

We know that $(-4)^2 = (-4)(-4) = 16$. Then -4 is also a square root of 16. Likewise -5 is a square root of 25 and $-a^3b^2$ is a square root of a^6b^4. These three cases illustrate the fact that any number has two square roots which are equal numerically but opposite in sign. In this chapter we shall consider only the positive, or plus, square roots.

The sign $\surd$, which indicates that the positive square root is to be taken, is called the *radical sign*. It is usually combined with the bar, or vinculum, which is written above the number. Thus $\sqrt{9}$ means "the positive square root of 9," which is $+3$. Likewise, $\sqrt{x^6} = x^3$, $\sqrt{4\,y^6} = 2\,y^3$, and $\sqrt{25\,m^{10}} = 5\,m^5$.

Remember, if no sign is written before the radical, the plus sign is understood.

Example 1. $\sqrt{25\,p^8} = 5\,p^4$. **Check.** $(5\,p^4)^2 = 25\,p^8$

Example 2. $\sqrt{x^4y^{12}} = x^2y^6$. **Check.** $(x^2y^6)^2 = x^4y^{12}$

Example 3. $\sqrt{\frac{25\,k^6}{36}} = \frac{5\,k^3}{6}$. **Check.** $\left(\frac{5\,k^3}{6}\right)^2 = \frac{25\,k^6}{36}$

EXERCISES [A]

1. Show that x^7 is a square root of x^{14}. Show that $-x^7$ is a square root of x^{14}.

2. Find the positive square roots of

a. 16
b. 1
c. 64
d. 36
e. 81
f. 25
g. 49
h. 9
i. 225
j. 324
k. 289
l. 400

3. Which of the following are *perfect squares* (numbers whose exact square roots can be taken)? What are the positive square roots of these perfect squares? Check your answers.

a. $64\,p^6$
b. ab^2
c. $125\,x^4$
d. $-9\,x^{10}$
e. $4\,x^5$
f. $16\,a^2$
g. $\frac{1}{4}\,x^{10}$
h. $81\,c^6$

Find the following indicated roots:

4. $\sqrt{49}$
5. $\sqrt{h^2k^4}$
6. $\sqrt{49\,p^6}$
7. $\sqrt{121\,p^2}$
8. $\sqrt{144\,a^4}$
9. $\sqrt{225\,a^8}$
10. $\sqrt{a^2c^4}$
11. $\sqrt{9\,m^2x^6}$
12. $\sqrt{(a+b)^2}$
13. $\sqrt{(x-y)^2}$
14. $\sqrt{.04}$
15. $\sqrt{2.25}$
16. $\sqrt{100\,a^{10}}$
17. $\sqrt{25\,a^2b^4c^6}$
18. $\sqrt{\dfrac{49\,a^2}{64\,b^2}}$

19. Think of some number and square it. Take the square root of the result. What is your answer?

Find the value of

20. $\sqrt{a} + \sqrt{b}$ if $a = 16$ and $b = 25$.

21. $\sqrt{c} - \sqrt{d}$ if $c = 64$ and $d = 81$.

22. $\sqrt{2\,x} + \sqrt{5\,y}$ if $x = 50$ and $y = 45$.

How to Find the Product of Two Binomials Mentally [A]

In Chapter VIII you learned that one polynomial is multiplied by another by multiplying *each* term of the multiplicand by *each* term of the multiplier and then combining the like terms of the products. Let us see how we can shorten the process of multiplying when both multiplicand and multiplier are certain binomials. Suppose we are to multiply $3x - 4$ by $2x + 5$.

The Long Method

$$\begin{array}{rl}
3x - 4 & \text{Multiplicand} \\
\nwarrow\!\nearrow & \\
\swarrow\!\searrow & \\
\underline{2x + 5} & \text{Multiplier} \\
6x^2 - 8x \phantom{{}-20} & \\
\underline{+15x - 20} & \\
6x^2 + 7x - 20 & \text{Product}
\end{array}$$

The Short Method

$$\underbrace{(3x \underbrace{-4)\;(2x}_{-8x} + 5)}_{+15x} = 6x^2 + 7x - 20$$

ORAL EXERCISES [A]

Refer to the examples above in answering the following questions:

1. How many times is $6x^2$ written in the long method? in the short method? How many times is -20 written in each method?

2. What is the first term of $3x - 4$? of $2x + 5$? of $6x^2 + 7x - 20$? How is the first term of the product obtained?

3. What is the last term of $3x - 4$? of $2x + 5$? of $6x^2 + 7x - 20$? How is the last term of the product found?

4. Why do you think $-8x$ and $+15x$ are called *cross products?*

5. Study the two methods and tell how the middle term $+7x$ is found in each method.

The three steps in the short, or inspection, method are given in the rectangle below.

1. The *first* term of the product is the product of the *first* terms of the binomials.
2. The middle term of the product is the sum of the cross products.
3. The *last* term of the product is the product of the *last* terms of the binomials.

Example 1. Find by inspection the product $(2x - 3)(4x + 7)$.

Solution. $(2x - 3)(4x + 7)$. The first terms of the binomials are $2x$ and $4x$. Their product is $8x^2$, which is the first term of the answer. The last terms of the binomials are -3 and $+7$. Their product is -21, which is the last term of the answer.

Then $(2x - 3)(4x + 7) = 8x^2 \cdots - 21$. We shall now find the middle term. After you have learned the short method, you will follow the order given in the rectangle. One cross product is $-12x$ and the other is $+14x$. Their sum is $+2x$, which is the middle term of the answer. Now we can write the solution:

$$(2x - 3)(4x + 7) = 8x^2 + 2x - 21.$$

Example 2. By inspection find the product $(3a + 1)(a - 2)$.

Solution. 1. The first term of the answer $= (3a)(a) = 3a^2$. 2. $(3a)(-2) = -6a$ and $(+1)(a) = a$. Adding, $-6a + a = -5a$. 3. The last term of the answer $= (+1)(-2) = -2$.
The written work is $(3a + 1)(a - 2) = 3a^2 - 5a - 2$.

ORAL EXERCISES [A]

What are the missing terms in the following statements?

1. $(x + 1)(x + 2) = x^2 + 3x + (\ ?\ )$

2. $(a - 3)(2a + 1) = (\ ?\ ) - 5a - 3$

3. $(m + 3)(4m + 1) = 4m^2 + (\ ?\ ) + 3$

4. $(2x + 4)(x - 1) = 2x^2 + (\ ?\ ) - 4$

5. $(3k - 1)(2k + 3) = (\ ?\ ) + 7k - (\ ?\ )$

6. $(7c - 1)(c + 1) = 7c^2 \quad ? \quad -1$

7. What is the first term in the answer of each of the first twenty exercises on page 328?

8. What is the middle term in the answer of each of these exercises?

9. What is the last term in the answer of each of these exercises?

EXERCISES

A

Find the following products mentally and check each solution by using the long method of multiplying:

1. $(a + 1)(a + 2)$
2. $(c + 2)(c + 3)$
3. $(c + 3)(c + 4)$
4. $(m - 1)(m - 1)$
5. $(c + 3)(c - 4)$
6. $(a - 2)(a - 2)$
7. $(a - 2)(a + 2)$
8. $(x + 5)(x + 8)$
9. $(1 + x)(1 + 2x)$
10. $(k + 3)(k + 7)$
11. $(2 - x)(7 + x)$
12. $(m - 7)(m - 7)$
13. $(x + 2)(x - 5)$
14. $(y - 6)(-y + 3)$
15. $(a - 20)(a - 1)$
16. $(x + 4)(x - 15)$
17. $(y + 10)(y - 8)$
18. $(7c - 1)(2c + 3)$
19. $(7c - 1)(7c + 1)$
20. $(3p - 1)(2p + 3)$

Find the following products mentally:

21. $(4c + 1)(5c - 4)$
22. $(5 + a)(6 - a)$
23. $(x^2 - 3)(x^2 - 4)$
24. $(a + b)^2 = (a + b)(a + b)$
$= ?$
25. $(x + 2y)^2$
26. $(x - y)^2$
27. $(2 - 3y)(3 - 4y)$
28. $(x^2 - 3y)(x^2 + 4y)$
29. $(a^2 - 4)(3a^2 - 1)$
30. $(10a - 1)(9a - 2)$
31. $(a^2 - 1)(a^2 - 4)$
32. $(x^2 - 1)(x^2 - 9)$
33. $(11x + 1)(x - 2)$
34. $(xy + 3)^2$
35. $(m + 8n)(m - 8n)$
36. $(2y - 5)(2y + 5)$
37. $(xy - 12)(xy + 1)$
38. $(x - 1)(-x - 1)$
39. $(3a - 4b)(3a + 4b)$
40. $(x^3 + 1)(x^3 - 2)$
41. $(x^2 + y^2)(x^2 + 2y^2)$
42. $(c^2 - 3)(3c^2 - 7)$
43. $(x + 0.3)(x + 0.4)$
44. $(a - \frac{1}{2})(a - \frac{1}{3})$
45. $(b - \frac{1}{4})(b + \frac{1}{3})$

B

Multiply:

46. $(4n - 7p)(6n + 8p)$

47. $(x^2y - 3)(x^2y - 4)$

48. $(7 - 3y)(4 - y)$

49. $(\frac{1}{2}x - p)(\frac{1}{3}x + p)$

50. $(9a - \frac{1}{2}b)(9a + \frac{1}{2}b)$

51. $(6c + ab)(\frac{1}{5}c + \frac{1}{2}ab)$

52. $(\frac{3}{4}x + 8y)(\frac{1}{3}x - 8y)$

53. $(\frac{1}{8}x + \frac{2}{5})(5x - 8)$

54. $(\frac{1}{2}x - \frac{1}{3}y)(3x - 4y)$

55. $(x + 0.1)(x + 0.1)$

56. $(2c + 0.3)(2c - 0.3)$

57. $(x + \frac{1}{2})(x + \frac{1}{4})$

58. $(x - \frac{1}{2}y)(x - \frac{1}{2}y)$

59. $(x - 4c)(\frac{1}{4}x - c)$

60. $(a^3 - \frac{1}{5})(a^3 + \frac{1}{5})$

61. $(x + \frac{2}{3}y)(x - \frac{1}{3}y)$

62. $(m - 0.4n)(m - 0.1n)$

63. $(c^2 - 0.3b)(c^2 - 0.8b)$

64. Find the area of a rectangle whose base is $3c - 4$ and altitude $c - 6$.

65. Find the area of a triangle if its base is $2c + 6$ and its altitude is $c - 1$.

How to Factor a Trinomial of Form $ax^2 + bx + c$ [A]

When two binomials are multiplied together, their product is a binomial, a trinomial, or a four-term polynomial.

For example,

$(x + 2)(x - 2) = x^2 - 4$, a binomial
$(x + 2)(x - 3) = x^2 - x - 6$, a trinomial
$(x + a)(x + b) = x^2 + ax + bx + ab$, a polynomial of 4 terms

When the product of two binomials is a trinomial, it is of the form $ax^2 + bx + c$. When such a trinomial is the product of two binomials, these binomials can be found by reversing the method used in finding their product. You will now learn how to factor such a trinomial. No general rule can be given for factoring trinomials of this kind, so you should study the solutions of the examples very carefully.

Example 1. Factor $x^2 + 6x + 8$.

Solution. To factor this trinomial we must find two binomials whose product is $x^2 + 6x + 8$. We know at once that the first terms of the two binomials are x and x. So we can write $x^2 + 6x + 8 = (x \quad)(x \quad)$, leaving space for the signs and last terms in the parentheses. Since there are no minus signs in the trinomial, there are no minus signs in the factors. Now we can write this much of the solution: $x^2 + 6x + 8 = (x + \quad)(x + \quad)$. The last terms of the binomials are either 8 and 1, or 4 and 2. Why? (We are not considering fractions.) Let us try 8 and 1 as the last terms. Then does $x^2 + 6x + 8 = (x+1)(x+8)$? We know that the answer is "no" because $(x + 1)(x + 8) = x^2 + 9x + 8$. Now let us try 2 and 4 for the last terms. Does $x^2 + 6x + 8 = (x + 2)(x + 4)$? The answer is "yes." The solution is set down as $x^2 + 6x + 8 = (x + 2)(x + 4)$.

$$\boxed{\begin{array}{lr} ? & ? \\ \underline{?} & \underline{?} \\ x^2 & \\ & +8 \\ \hline x^2 + 6x + 8 & \end{array}}$$

EXERCISES [A]

Factor each of the following trinomials by the method shown in Example 1 above.

1. $a^2 + 6a + 8$
2. $c^2 + 5c + 4$
3. $m^2 + 3m + 2$
4. $x^2 + 3x + 2$
5. $a^2 + 2a + 1$
6. $y^2 + 4y + 3$
7. $y^2 + 9y + 8$
8. $y^2 + 7y + 10$
9. $x^2 + 11x + 24$
10. $b^2 + 9b + 14$
11. $x^2 + 7x + 6$
12. $h^2 + 7h + 12$
13. $k^2 + 8k + 15$
14. $m^2 + 8m + 12$

Example 2. Factor $x^2 - 2x - 15$.

Solution. The first terms of the two binomial factors are x and x. Then we have $x^2 - 2x - 15 = (x \quad)(x \quad)$. The -15 tells us that one of the last terms of the factors is plus and one is minus. Why? Then we have $x^2 - 2x - 15 = (x + \quad)(x - \quad)$. The last terms of the factors may be $+15$ and -1, -15 and $+1$, -5 and $+3$, or $+5$ and -3. Let us try $+5$ and -3. Does $x^2 - 2x - 15 = (x + 5)(x - 3)$? No, because $(x + 5)(x - 3) =$

$x^2 + 2x - 15$. The sign of the middle term is wrong. In such a case we try changing the middle signs of the factors. Let us do this. Does $x^2 - 2x - 15 = (x - 5)(x + 3)$? Yes.

NOTE. It so happened in Example 2 that we made a good guess in choosing the numbers 5 and 3. If these had not been the correct numbers, we should have had to try $+15$ and -1, or -15 and $+1$. This method of factoring is called the "*guess*" *method* or the "*try*" *method.* Which name do you prefer?

EXERCISES [A]

Factor:

1. $c^2 - c - 2$
2. $m^2 + m - 30$
3. $m^2 - m - 30$
4. $c^2 - c - 20$
5. $c^2 + c - 20$
6. $a^2 + a - 12$
7. $a^2 + a - 42$
8. $x^2 - 2x - 3$
9. $x^2 + 2x - 3$
10. $m^2 - 2m - 63$
11. $x^2 - 7x - 18$
12. $x^2 - 5x - 14$
13. $y^2 - 6y - 40$
14. $p^2 + 3p - 4$

Example 3. Factor $x^2 - 9x + 18$.

Solution. The first terms of the two factors are x and x. Then $x^2 - 9x + 18 = (x \quad)(x \quad)$. The $+18$ tells us that the signs of the last terms of the factors are alike. Why? Since the sign before $9x$ is minus, the signs of the last terms of the factors are minus. Then $x^2 - 9x + 18 = (x - \quad)(x - \quad)$. For the last terms of the factors we may try -3 and -6, -2 and -9, or -1 and -18. Let us try -2 and -9. Does $x^2 - 9x + 18 = (x - 9)(x - 2)$? The answer is "no." Now let us try -3 and -6. Does $x^2 - 9x + 18 = (x - 3)(x - 6)$? The answer is "yes." Then $x^2 - 9x + 18 = (x - 3)(x - 6)$.

EXERCISES [A]

Factor:

1. $x^2 - 5x + 4$
2. $x^2 - 8x + 15$
3. $x^2 - 4x + 3$
4. $c^2 - 7c + 12$
5. $x^2 - 11x + 18$
6. $y^2 - 6y + 8$
7. $a^2 - 9a + 20$
8. $a^2 - 10a + 24$

Example 4. Factor $8x^2 - 22x + 15$.

Solution. In factoring a trinomial such as $x^2 - 2x - 24$ we do not need to guess what the first terms are. We know immediately that they are x and x. But in this example we are forced to guess both the first terms and last terms of the factors. The first terms of the factors are x and $8x$, or $2x$ and $4x$. The plus sign before the 15 tells us that the signs of the last terms of the factors are alike and the minus sign before the $22x$ tells us that they are minus. The last terms of the factors are -1 and -15, or -3 and -5. The eight possible solutions are as follows:

1. $(x - 1)(8x - 15)$
2. $(x - 15)(8x - 1)$
3. $(x - 3)(8x - 5)$
4. $(x - 5)(8x - 3)$
5. $(2x - 1)(4x - 15)$
6. $(2x - 15)(4x - 1)$
7. $(2x - 3)(4x - 5)$
8. $(2x - 5)(4x - 3)$

Any one of these eight combinations will give $8x^2$ as the first term of the product and $+15$ as the last term of the product. Only one of the combinations will give $-22x$ as the middle term of the product. By trial we find that combination 7 is the correct one, for the cross products are $-10x$ and $-12x$ and their sum is $-22x$.

Then $8x^2 - 22x + 15 = (2x - 3)(4x - 5)$.

When factoring trinomials of this kind, it is sometimes necessary to make several guesses. If you do not guess right the first time, keep on trying other combinations until you find the correct one. As you gain experience by factoring many trinomials, you will be able to factor with fewer trials. Always keep in mind that the sign of the third term of the trinomial tells whether the signs of last terms of the factors are like or unlike.

EXERCISES

A

Factor when possible:

1. $3x^2 - 14x - 5 = (3x + ?)(x\ ?\ ?)$
2. $6x^2 - 13x - 5 = (? - 5)(? + 1)$
3. $6x^2 + x - 5 = (6x - ?)(x\ ?\ ?)$

4. $5x^2 - 17x + 14 = (5x - 7)(?)$

5. $9x^2 + 3x - 2 = (3x + ?)(3x - ?)$

6. $5y^2 + 17y + 14$

7. $x^2 + 3x - 28$

8. $x^2 + 4x - 12$

9. $15x^2 + 22x + 8$

10. $x^2 - 4x + 4$

11. $9x^2 - 6x + 1$

12. $2m^2 - m - 3$

13. $3c^2 - 2c - 5$

14. $2m^2 + 3$

15. $3c^2 + 8c + 5$

16. $2p^2 - 9p + 4$

17. $2p^2 + 7p - 14$

18. $x^2 - 6x + 9$

19. $y^2 - 10y + 25$

20. $3x^2 - 13x - 20$

21. $2y^2 + 5y + 1$

22. $4r^2 - 4r + 1$

23. $x^2 + x + 1$

24. $b^2 + 4b - 60$

25. $7x^2 + 9x + 2$

26. $2r^2 + r - 15$

27. $8c^2 - 26c - 7$

28. $x^2 + 8xy + 16y^2 = (? + 4y)(? + 4y)$

29. $x^2 - xy - 12y^2 = (x + 3y)(?)$

30. $3a^2 - 2ab - 5b^2 = (? - 5b)(? + b)$

31. $x^2 + 3xy + 2y^2$

32. $c^2 - 6cd + 5d^2$

33. $x^2 - 12xy + 36y^2$

34. $2m^2 + 5mn - 12n^2$

35. $h^2 - 9hk + 18k^2$

36. $15m^2 - 22mn - 5n^2$

37. $2x^2 - 3xy + y^2$

38. $2y^2 + y - 21$

39. $3a^2 + 11ab - 20b^2$

40. $3m^2 - 29mn + 20n^2$

41. $h^2 - hk - 20k^2$

42. $6x^2 - 17xy + 12y^2$

43. $4p^2 - 4pq - 3q^2$

44. $21c^2 - 58cd + 21d^2$

45. $12R^2 + 7RS - 12S^2$

46. $a^2 - 9ab + 8b^2$

B

47. $1 - 2x + x^2 = (1 - ?)(1 - ?)$

48. $x^2 - 2x + 1 = (x - ?)(x - ?)$

Compare the answers of exercises 47 and 48.

Factor when possible:

49. $13y + 10y^2 - 3 = 10y^2 + 13y - 3 = (\ ?\ )(\ ?\ )$

50. $-mn + m^2 - 42n^2 = m^2 - mn - 42n^2 = (\ ?\ )(\ ?\ )$

51. $5xy - 3y^2 + 2x^2 = 2x^2 + 5xy - 3y^2 = (\ ?\ )(\ ?\ )$

52. $7ab + 6a^2 - 3b^2$

53. $4b^2 + 4bc - 3c^2$

54. $8x^2 + 2xy - 3y^2$

55. $3 - 10k - 8k^2$

56. $2 - 11c + 12c^2$

57. $3x^2 - 5xy - 12y^2$

58. $-15 + r + 2r^2$

59. $m^2 + 2mn - 8n^2$

60. $x^2 + xy + y^2$

61. $10y^2 - 6 - 11y$

62. $2 + a^2 - 36a^4$

63. $2x^2 - 7x - 15$

C

Factor:

64. $x^4 + 2x^2 - 35 = (x^2\ ?\ ?)(x^2\ ?\ ?)$

65. $a^2b^2 + ab - 6 = (ab\ ?\ ?)(ab\ ?\ ?)$

66. $x^6 - x^3 - 20 = (x^3\ ?\ ?)(x^3\ ?\ ?)$

67. $x^6 - 3x^3 - 18$

68. $y^4 + 2y^2 - 15$

69. $k^6 - 5k^3 - 14$

70. $x^2y^2 - 2xy - 24$

71. $2x^4 + 5x^2y^2 - 12y^4$

72. $A^2 + 4AB + 4B^2$

73. $1 - 7x^2 + 12x^4$

74. $10m^2 + 9m - 91$

75. $1 - 21x + 110x^2$

76. $8c^2 + 5c - 3$

77. $(a - b)^2 + 2(a - b) - 8 = (a - b + 4)(a - b - ?)$

78. $(x - y)^2 - 5(x - y) - 50 = (x - y\ ?\ )(\ ?\ )$

79. $x^4 - 10x^2 + 25$

80. $a^6 - 13a^3b + 32b^2$

81. $x^2y^2 - 2xy - 48$

82. $r^2s^2 + rs - 12$

83. $x^4 + 5x^2 - 36$

84. $x^4y^2 - x^2y - 30$

85. $25a^2 + 18a - 12$

86. $42x^2 - 79x + 35$

87. The area of a rectangle is $2x^2 + x - 190$. What may its base and altitude be?

88. A dividend is $b^4 - 10b^2 + 24$. Name a binomial divisor and the resulting quotient.

Multiplying the Sum of Two Numbers by Their Difference[A]

A special product is obtained when the sum of two numbers is multiplied by their difference. Study the two multiplications below.

$$\begin{array}{lll} a & +b & \\ a & -b & \\ \hline a^2 & +ab & \\ & -ab & -b^2 \\ \hline a^2 & & -b^2 \end{array}$$

$$\underbrace{(a+\underbrace{b)\cdot(a}_{+ab}-b)}_{-ab}=a^2-b^2$$

ORAL EXERCISES[A]

The following exercises refer to the two multiplications above.

1. What words are omitted in the following?

a. $a+b$ is the --?-- of a and b.

b. $a-b$ is the --?-- of a and b.

c. a^2-b^2 is the --?-- of $a+b$ and $a-b$.

d. a^2-b^2 is the difference of the --?-- of a and b.

2. Why is $+ab$ called a cross product?

3. What is the other cross product?

4. Is the product of the two binomials a binomial or a trinomial?

Is the following rule clear to you?

> **The product of the sum and difference of two numbers is equal to the square of the first number minus the square of the second number.**

Example 1. $(x+4)(x-4)=?$

Solution. The square of x is x^2 and the square of 4 is 16.

Then $$(x+4)(x-4)=x^2-16$$

EXERCISES

A

Multiply by inspection:

1. $(m + n)(m - n)$

2. $(c - d)(c + d)$

3. $(x + y)(x - y)$

4. $(c - 3)(c + 3)$

5. $(a + 4)(a - 4)$

6. $(x + 7)(x - 7)$

7. $(y - 5)(y + 5)$

8. $(k + h)(k - h)$

9. $(x^2 - 1)(x^2 + 1)$

10. $(3c - 1)(3c + 1)$

11. $(5x - 3)(5x + 3)$

12. $(2 + k)(2 - k)$

13. $(2x + n)(2x - n)$

14. $(b - 2a)(b + 2a)$

15. $(ab - 1)(ab + 1)$

16. $(1 - x)(1 + x)$

17. $(3x^2 - y)(3x^2 + y)$

18. $(10x + 9)(10x - 9)$

19. $(4s + 3t)(4s - 3t)$

20. $(x + \frac{1}{2})(x - \frac{1}{2})$

21. $(0.5c - 1)(0.5c + 1)$

22. $(\frac{3}{4}x - \frac{1}{2}y)(\frac{3}{4}x + \frac{1}{2}y)$

23. $(\frac{1}{7}x - \frac{1}{5}y)(\frac{1}{7}x + \frac{1}{5}y)$

24. $(0.3a - 0.5b)(0.3a + 0.5b)$

25. $(abc - \frac{1}{3})(abc + \frac{1}{3})$

26. $(0.1c + 6)(0.1c - 6)$

Example 2. $(10 + 6)(10 - 6) = 100 - 36 = 64.$

B

Find the products:

27. $(8 + 3)(8 - 3)$

28. $(11 - 4)(11 + 4)$

29. $(1 + 10)(1 - 10)$

30. $(50 + 1)(50 - 1)$

31. $(100 - 3)(100 + 3)$

32. $(70 + 2)(70 - 2)$

33. $19 \times 21 = (20 - 1)(20 + 1) = 400 - 1 = ?$

34. $42 \times 38 = (40 + 2)(\ ?\ ) = ?$

35. $37 \times 43 = (40 - ?)(40 + ?) = ?$

36. 101×99

37. 49×51

38. 23×17

39. 24×26

40. 52×48

41. 18×22

42. 97×103

43. 76×84

44. 105×95

Factoring the Difference of Two Squares[A]

You have just learned how to multiply the sum of two numbers by their difference without using pencil and paper. You know that the product of the sum and difference of two numbers is equal to the difference of the squares of the numbers. Now we shall reverse the process by finding the two factors when their product, which is the difference of two squares, is given.

Example 1. Factor $x^2 - y^2$.

Solution. x^2 is the square of x and y^2 is the square of y. Then the square root of x^2 is x and the square root of y^2 is y. The sum of the square roots is $x + y$ and the difference of the square roots is $x - y$.

Then $$x^2 - y^2 = (x + y)(x - y)$$

Remember that the sum of the cross products of $(x + y)(x - y)$ is zero.

Example 2. Factor $4\,c^4 - 25$.

Solution. The square root of $4\,c^4$ is $2\,c^2$ and the square root of 25 is 5. One factor is $2\,c^2 + 5$ and the other is $2\,c^2 - 5$. Then $4\,c^4 - 25 = (2\,c^2 + 5)(2\,c^2 - 5)$.

Example 3. Factor $1 - 81\,m^2$.

Solution. $\sqrt{1} = 1$ and $\sqrt{81\,m^2} = 9\,m$

Then $1 - 81\,m^2 = (1 + 9\,m)(1 - 9\,m)$

Example 4. Factor $a^2b^4 - .01$.

Solution. $a^2b^4 - .01 = (ab^2 + .1)(ab^2 - .1)$

To Factor the Difference of Squares of Two Numbers:

1. Find the positive square root of each of the squares.
2. Write the sum of these square roots as one factor and the difference of the square roots (in the order given) as the other factor.

EXERCISES [A]

Factor the following when possible:

1. $x^2 - m^2$
2. $c^2 - b^2$
3. $x^2 - 1$
4. $c^2 - 4$
5. $4 - c^2$
6. $c^2 - d^2$
7. $x^2 - 9$
8. $y^2 - 16$
9. $a^2 - 25$
10. $m^2 - 36$
11. $a^2 - 45$
12. $64 - x^2$
13. $1 + c^2$
14. $4x^2 - 1$
15. $9x^2 - 1$
16. $1 - 9a^2$
17. $c^2 - 9y^2$
18. $x^2 - 25$
19. $R^2 - r^2$
20. $36h^2 - 25$
21. $x^6 - y^6$
22. $x^6 - 1$
23. $9x^2 - 100$
24. $49 - m^4$
25. $64x^2 - 49y^2$
26. $25r^2 - 1$
27. $x^2 - \frac{1}{4}$
28. $m^2 - \frac{1}{9}$
29. $c^2 - \frac{1}{16}$
30. $p^2 - \frac{1}{25}$
31. $a^2b^2 - 4$
32. $c^2d^2 - 9$
33. $m^2n^2 - p^2$
34. $c^2 - a^2b^2$
35. $x^6 - y^2$
36. $x^2 - y^4$
37. $c^2d^2 - a^2b^2$
38. $1 - a^2b^2c^2$
39. $c^2d^2 + 16$

$(c+x)(c-x)$

The Three Types of Factoring [A]

So far you have learned how to factor three types of polynomials. You will need to recognize each type when you meet it.

A polynomial can be factored:

1. If it contains a monomial factor.
2. If it is a binomial that is the difference of two squares.

*3. If it is a trinomial that can be factored by the "guess" method.

When factoring an expression, you should first determine whether it contains a monomial factor. If it does, be sure that you find the *largest* monomial factor. If there is no monomial factor, see if the expression is the difference of squares. If there is no monomial factor and the expression is a trinomial, see if the guess method will factor it.

*Teachers who wish to take up the special case of a perfect square trinomial at this time may easily adapt pages 455–457 to this purpose.

EXERCISES [A]

Some of the following polynomials have monomial factors, some are differences of squares, some can be factored by the guess method, and some are prime, that is, they cannot be factored. Factor those that can be factored.

1. $5x + 10$
2. $x^2 - 1$
3. $c^2 + 2c + 1$
4. $3x^2 - 18$
5. $m^2 - 25$
6. $x^2 + 4x + 4$
7. $x^2 + 4$
8. $a^2 - a$
9. $a^2 - a - 42$
10. $6a^2 - 13a - 5$
11. $h^2 - 6h + 8$
12. $2x^2 - 6x$
13. $m^2 - 10m + 25$
14. $1 - z^2$
15. $a^2 - 11ac + 18c^2$
16. $r^2 - 7r - 60$
17. $b^2 - a^2$
18. $\pi R + \pi r$
19. $abx + a$
20. $12x^8 - 10x^6$
21. $a^2 + 64$
22. $c^2 - 29$
23. $c^2 - d^6$
24. $c^2 - 9c$
25. $c^2 + 9c$
26. $c^2 - 9$
27. $6x^4 - 9x^2$
28. $a^2b^2 - 2ab - 35$
29. $100 - x^2$
30. $9a^2 - 16b^2$
31. $a^2 + 4a + 4$
32. $25x^2 - 36$
33. $x^3 - x^2 + x$

EXERCISES [A]

$(c-3)(c+3)$

Factor the polynomials which are not prime:

1. $p^2 - 6p + 9$
   $= (p-3)(p-3)$
   $= (\ ?\ )^2$
2. $y^2 - 4y + 4$
3. $x^2 - 3x - 10$
4. $c^2 - 6c - 9$
5. $6x^3y - 15x^2y - 3xy$
6. $x^2 + 7x + 10$
7. $p^2 - p - 56$
8. $9 - 4x^2$
9. $r^2 - 7r - 60$
10. $-2a^3 + 4ab^2 - 6ac^2$
11. $a^2 - 64b^2$
12. $m^2 - 14m + 49$
13. $-12 - x + x^2$
14. $2c^2 - 17c + 35$
15. $x^2 + 8x + 12$
16. $12a^2 - 35ab$
17. $a^2 - 6ab^2 + 9b^4$
18. $16x^2 + 8xy + y^2$
19. $p^2 + p - 72$
20. $1 + x^2$
21. $x^2 + 16xy - 16x^2$
22. $25x^2 - 1$

23. $x^2 + 5\,xy - 36\,y^2$
24. $c^2d^2 - 7\,cd - 35$
25. $2\,x^2 + 3\,x + 1$
26. $3\,x^2 - 14\,x + 8$
27. $a^2b^2 - d^2$
28. $6\,x^3y - 15\,x^2y - 24\,xy$
29. $y^2 - 20\,b + 100$
30. $a^2 - 4\,ab^2$
31. $10\,x - 810$
32. $4\,x^2 - 20\,x + 25$
33. $4\,x^5 - 3\,x^3$
34. $5\,x + 45$
35. $a^2 - 1$
36. $2\,a^2 + a - 10$
37. $x^2 - 5\,xy - 36\,y^2$
38. $a^2b^2 - 16$
39. $x^4 - 9\,x^2 + 8$
40. $6\,y^2 - 8\,y - 14$
41. $2\,c^2 + 5\,c + 2$
42. $12\,c^2 - 5\,c - 2$
43. $k^{10} + 2\,k^5 + 1$
44. $2\,b^2 + 5\,b - 3$
45. $a^2b^2 - 6\,abc + 9\,c^2$
46. $w^4x^4 - 81$

Complete Factoring [A]

Often one or more factors of an algebraic expression can be factored. In factoring any expression, factoring should be continued until the factors are prime numbers (numbers that cannot be factored).

Example 1. Factor $5\,x^2 - 5\,x - 60$.

Solution. The *first* operation is to remove the largest monomial factor, if there is one. Then $5\,x^2 - 5\,x - 60 = 5(x^2 - x - 12)$. The factor $x^2 - x - 12$ is a trinomial. Using the guess method, $x^2 - x - 12 = (x - 4)(x + 3)$. The solution should be written:

$$\begin{aligned} 5\,x^2 - 5\,x - 60 &= 5(x^2 - x - 12) \\ &= 5(x - 4)(x + 3) \end{aligned}$$

Example 2. Factor $a^4 - 1$.

Solution. There is no monomial factor; $a^4 - 1$ is the difference of two squares.

Then $$a^4 - 1 = (a^2 + 1)(a^2 - 1)$$

$a^2 + 1$ cannot be factored. $a^2 - 1$ is the difference of two squares. Then $a^2 - 1 = (a + 1)(a - 1)$. The solution is written

$$\begin{aligned} a^4 - 1 &= (a^2 + 1)(a^2 - 1) \\ &= (a^2 + 1)(a + 1)(a - 1) \end{aligned}$$

Directions for Complete Factoring:

1. Remove the largest monomial factor (if any).
2. Factor (if possible) any binomial or trinomial into prime factors.

From now on *factoring* means "factoring into prime factors."

EXERCISES [A]

Factor the following expressions. If any expression is prime, write "prime" after it on your paper.

1. $6a + 6$
2. $x^2 - 4x$
3. $4c^2 - 4$
4. $x^4 - a^4$
5. $x^4 + 7x^3 + 12x^2$
6. $4a^2 - 12a + 9$
7. $16a^2 - 4ab^2$
8. $5x^3 - 20xy^2$
9. $4c^2 - 9c$
10. $m^2 + 8m + 16$
11. $bx^2 - 9b$
12. $x^2 - 16x$
13. $3cx^2 + 7cx + 2c$
14. $a^2 - 25$
15. $a^2 - 25a$
16. $9x^3 - xy^2$
17. $\pi R^2 - \pi r^2$
18. $w^4 - 1$
19. $2\pi r^2 + 2\pi rh$
20. $a^4 - 16$
21. $a^4 - 16a$
22. $3x^2 - 6$
23. $x^3 - x$
24. $x^3 - x^2$
25. $x^5 - x^3$
26. $a^5 - a^4$
27. $a^4 + 1$
28. $a^4 - 16b^4$
29. $ax^2 - 4ay^2$
30. $5h^2 + 40h + 60$
31. $a^4 - 4a^2 + 3$
32. $3t^2 + 30t + 48$
33. $10x^2 + 35x + 15$
34. $6a^2y^2 + 5ay^2 - 6y^2$
35. $3a^3 + 8a^2 + 5a$
36. $12x^2 - 22x + 6$
37. $8a^2 + 16a + 8$
38. $8x^2 - 80x + 168$
39. $10ac^2 + 13ac - 3a$
40. $m^2 - 12m + 35$
41. $x^3a^2 - x$
42. $1 - x^2$
43. $x^4 - 2x^2y^2 + y^4$
44. $6c^2 - 54cd + 84d^2$
45. $4x^2 + 4x - 48$
46. $\frac{1}{3}\pi R^2h + \frac{1}{3}\pi r^2h$
47. $2\pi R - 2\pi r$
48. $p + pqt$
49. $9x^3 - 81xy^2$
50. $4a^2 - 36a$

51. $6n^2 - 12np + 6p^2$

52. $x^4 - 9x^2 + 20$

53. $by^4 - 10by^2 + 9b$

54. $a^3 - 3a^2 - 4a$

55. $z^3 - 16z$

56. $5d^2 - 10d + 5$

57. $25 - 60a + 36a^2$

58. $6k^2 + 15k + 6$

59. $abx^2 - ab$

60. $A^2B^2 - 5AB^3 + 6B^2$

61. $a^2b^2 + a^2b - 12a^2$

62. $x^2 - 22x + 121$

63. $x^2 + 28xy + 196y^2$

64. $9a^2b^2 - 16c^2$

WORD LIST

Be sure that you can spell and use correctly each of these words.

abscissa (p. 287)
binomial (p. 325)
co-ordinate (p. 287)
extremes (p. 228)
factor (p. 319)
linear equation (p. 289)
means (p. 228)
monomial (p. 321)
ordinate (p. 287)
origin (p. 286)
product (p. 319)
ratio (p. 226)
square root (p. 324)
trinomial (p. 329)

CHAPTER REVIEW [A]

Multiply by inspection:

1. $\pi(a - b)$

2. $2\pi(m - n)$

3. $3a(a^2 - 4a + 1)$

4. $5c^2(c^2 - c + 3)$

5. $ab(a^2 - ab + b^2)$

6. $-3(m^2 - n^2)$

7. $2a^2(b^2 - 3b + 4)$

8. $(a - 1)(a + 1)$

9. $(a - 1)(a - 1)$

10. $(2x + 1)(x - 3)$

11. $(3x - 4)(x + 2)$

12. $(b - 4)^2$ $(b-4)$

13. $(x - 3y)(x - 3y)$

14. $(x - 3y)(x + 3y)$

15. $(k - y)(k + 2y)$

16. $(2m - 1)(m + 5)$

17. $(x^3 - 1)(x^3 + 2)$

18. $(ab + 3)(ab - 2)$

19. $(3y + 5)(2y - 4)$

20. $(pq - 5)(pq + 4)$

Factor:

21. $6x - 6$
22. $x^2 - 9$
23. $a^2 - 36$
24. $y^2 - 10y + 25$
25. $m^2 - 3m + 2$
26. $2p^2 + p - 3$
27. $z^2 - z - 20$
28. $25 - y^2$
29. $bx^2 - dx^2$
30. $10d^3 - 30cd$
31. $a^2 + 4a + 4$
32. $a - a^3$
33. $1 - 6a + 9a^2$
34. $9a^2y^2 - 16$
35. $9a^2 - 9ay^2$
36. $x^3 - x^2 - 72x$
37. $6x^2 - 6a^2$
38. $x^4 - 6x^2 + 5$
39. $dm^2 - dn^2$
40. $bx - by$

GENERAL REVIEW [A]

What are the missing words and numbers in the following sentences?

1. The value of the unknown letter in an equation is called the --?-- of the equation.

2. The expression $3c^2 - 4c + 1$ has three --?--.

3. The result obtained by multiplying one number by another is their --?--.

4. In the expression $4a^2b^5$ the number 5 is called the --?-- of b.

5. In the expression $2x^3y^4$ the number 2 is the --?-- of x^3y^4.

6. 5 and 7 are the --?-- of 35.

7. The formula for the area of a circle is $A = \pi$ --?--.

8. The equation $\frac{x}{3} = \frac{5}{6}$ is called a --?--.

9. The growing cost of producing an acre of tomatoes was \$36, the harvesting cost was \$24, and the marketing cost was \$12. Make a circle graph using these figures.

10. Solve:

a. $3x - 4y = -17$
$2x - 5y = -16$

b. $y = x + 2$
$3x - y = 4$

11. Draw the graph for $y = x + 3$.

12. Draw the graph for $2x - y = 10$.

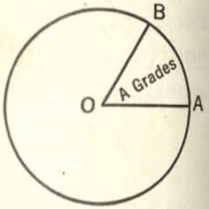
13. One factor of $x^2 - 8x + 12$ is $x - 6$. What is the other factor?

14. One factor of $x^3 - 3x^2 + 3x - 2$ is $x^2 - x + 1$. Find the other factor.

15. $l = a + (n - 1)d$. Find the value of n when $l = 47$, $a = 5$, and $d = 3$.

16. $x = 3y$. If y decreases, how does x change?

17. The teacher wished to make a circle graph to show the distribution of marks in an algebra class. The marks consisted of 5 A's, 6 B's, 10 C's, 8 D's, and 1 E. How many degrees should there be in $\angle AOB$?



Solve for x:

18. $2x + 3 = 7x$

19. $6x - 9x = 0$

20. $\frac{x}{2} - \frac{x}{3} = 1$

21. $2x + 5 - 5(2 + x) = 3(x + 6)$

22. $4x - (x - 2) = 10 - x$

23. $\frac{4x + 1}{2} - \frac{2x - 3}{3} = \frac{5x - 1}{4}$

24. The larger of two numbers exceeds the smaller by 4. Five times the larger, decreased by 6 times the smaller, gives 8 as the result. What are the two numbers?

25. At one basketball game 1845 tickets were sold. Pupils paid 25 cents each for their tickets and outsiders paid 40 cents each. If the total receipts for the game were \$553.35, how many tickets of each kind were sold?

$(a^2+4)(a\pm 2)$

CHAPTER TEST [A]

Write the indicated products:

1. $3a(a^2 + 2a - 1)$

2. $(a + b)(a - b)$

3. $(x - 3)^2$

4. $(x - 5)(x - 3)$

5. $(2x + 1)(3x + 5)$

6. $(3x + 7)^2$

Square as indicated:

7. $(5a)^2$

8. $(-3)^2$

9. $(8a^3)^2$

10. $(\frac{1}{2}gt)^2$

Find the prime factors:

11. $4a + 4b$

12. $x^2 + 7x + 12$

13. $c^2 - 5c + 6$

14. $x^2 - 9$

15. $x^2 - x - 2$

16. $2a^2 - 2b^2$

17. $5y^2 + 17y + 6$

18. $a^2 - 4b^2$

19. $2x^3 - 4x^2 - 2x$

20. $1 - 25a^2$

21. $3y^2 - y - 4$

22. $3x^2 - 21x + 18$

Write the indicated roots:

23. $\sqrt{36}$

24. $\sqrt{a^2b^2}$

25. $\sqrt{(x+y)^2}$

CHAPTER TEST [B]

Write the indicated products:

1. $3ab(a^2 + 5a - 6)$

2. $(2x - 5y)^2$

3. $(3c + 1)(3c - 1)$

4. $(5x + 3y)(3x - 4y)$

5. $(x - \frac{1}{2})^2$

6. $(6ab + 3c)(2ab - 3c)$

Tell which of the following expressions are prime and which are not prime:

7. $x + 6$

8. $x^2 - x - 12$

9. $x^2 + 9$

Find the prime factors:

10. $2a^4 - 4a^3 - 4a^2$

11. $x^2y^2 - 1$

12. $x^2 - 6x - 16$

13. $4a^2 - \frac{1}{9}$

14. $12x^2 + x - 8$

15. $2\pi a^2 - 2\pi b^2$

16. $3 - 19c - 14c^2$

17. $ax^6 - ay^6$

18. $2x^3 - 4x^2 + 2x$

19. $3m^2 + 21m - 3$

20. $c^3 - c$

21. $x^3y^3 - 2x^2y^2$

22. $a^4 - 16$

23. $a^4 - 5a^2 + 4$

24. $x^3 - 3x^2 - 10x$

25. $x^4 - 9x^2 + 20$

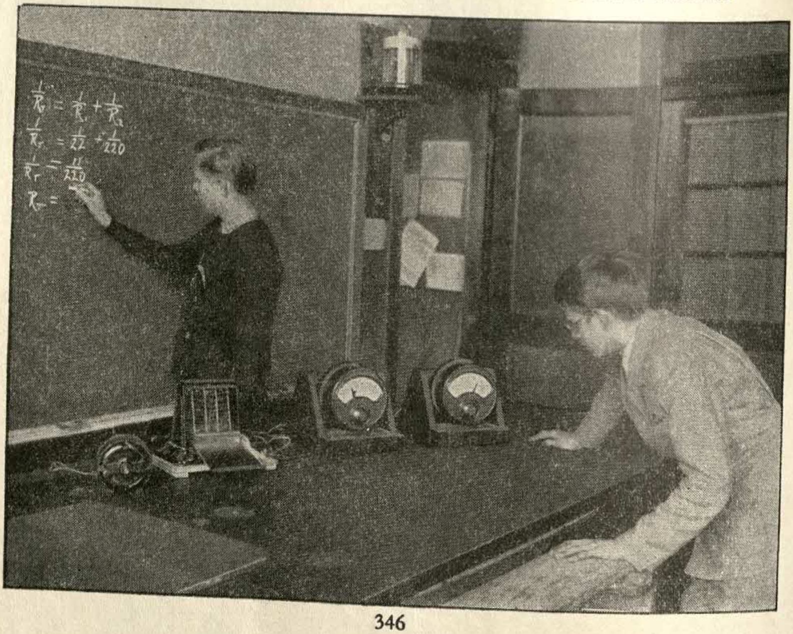
p 349
has ans-test

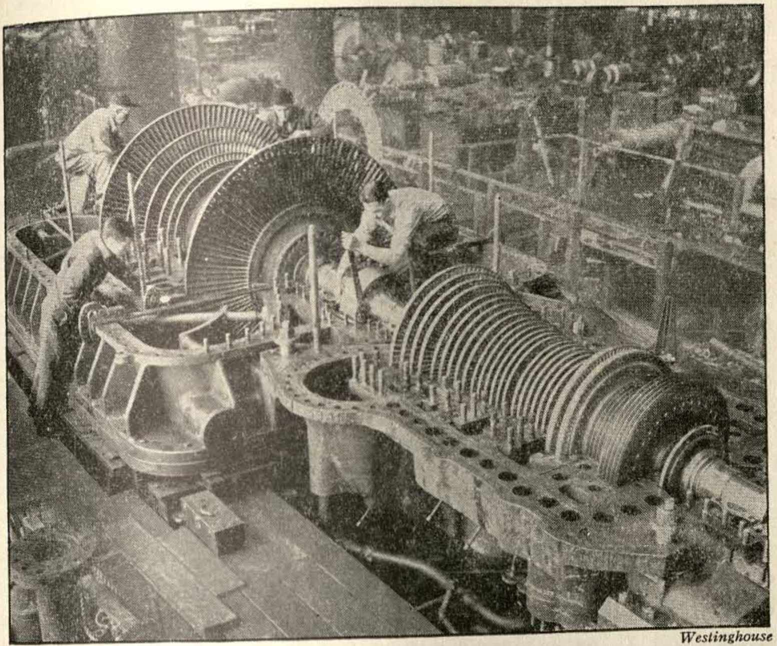
MATHEMATICS AND ELECTRICITY

When you watch an electrician installing the wiring system in an average dwelling, you do not associate electricity with mathematics. Yet the electrical contractor used arithmetic in computing the cost of the wiring before he submitted his bid on the work. He figured the cost of the wire, conduit, fuse box, outlets, switches, labor, and other items. To this he added his desired profit to find the amount of his bid.

A real electrician must understand the nature of electricity. He must make a study of magnetism, know the effects of direct and alternating currents, know how and why motors, generators, and electric meters work.

Boys Determining the Combined Resistance of Two Circuits





Westinghouse

The Tips of the Longest Blades of This 35,000-k.w. Steam Turbine Will Rotate at a Speed of 1178 Feet per Second

To understand electricity and its applications one must know more mathematics than ordinary arithmetic. For example, one cannot understand the difference between a detector tube and an amplifying tube unless he can interpret curve graphs. Also, one cannot understand the alternating current unless he has a knowledge of algebra, geometry, and trigonometry. The electrical engineer in designing electrical equipment often makes use of calculus and mathematics beyond calculus. Some formulas that are used in electricity follow:

$$I = \frac{E}{R} \qquad \text{H.P.} = \frac{EI}{746} \qquad R = \frac{KL}{d^2}$$

$$W = \frac{E^2}{R} \qquad \text{K.W.} = \frac{EI}{1000} \qquad I = \frac{E}{R + r}$$

CHAPTER XII

Equations

The Degree of an Equation [A]. (Review)

The degree of an equation in one letter is the same as the highest power of the letter in the equation after it has been cleared of fractions and parentheses. Since the graph of a first-degree equation is a straight line, it is often called a *linear equation.* Also a first-degree equation is sometimes called a *simple equation.* A *quadratic equation* is one of the second degree and a *cubic equation* is one of the third degree. Compare the three kinds of equations below:

SIMPLE EQUATIONS	QUADRATIC EQUATIONS	CUBIC EQUATIONS
$4x = 36$	$4x^2 = 36$	$x^3 = 8$
$7x + 2 = 23$	$2x^2 + 3x = 0$	$x^3 - 4x^2 + 8 = 0$
	$x^2 - 6x + 18 = 0$	$2x^3 + 5x = 7$

Solving Equations by Factoring [A]

Any equation whose right member is zero and whose left member can be completely factored may be solved by the *factoring method.* Solving an equation by factoring depends upon the following principle:

If the product of two or more factors is zero, at least one of the factors is zero.

ORAL EXERCISES [A]

1. What is the value of 4×0? of $b \times 0$? of 0×0?

2. If the product of two numbers is zero, are both numbers necessarily zero? Explain.

3. What is the value of $(x-1)(x+2)$ if $x-1=0$? if $x+2=0$? What is the value of x if $x-1=0$? What is the value of x if $x+2=0$?

4. If $x^2-x-6=0$, does $(x-3)(x+2)=0$? If $x=3$, does $x-3=0$? If $x=3$, does $(x-3)(x+2)=0$? If $x=-2$, does $(x-3)(x+2)=0$? What values of x make $x^2-x-6=0$? What are the roots of $x^2-x-6=0$?

Example 1. Solve $x^2-3x-18=0$.

Solution. The right member is zero. We next factor the left member and have $(x-6)(x+3)=0$. There are two ways of making this product equal zero. One way is setting $x-6=0$ and the other way is setting $x+3=0$. If $x-6=0$, $x=6$; and if $x+3=0$, $x=-3$. Then the solutions are $x=6$ and $x=-3$.

To solve an equation by factoring:

1. If necessary change the equation so that the right member is zero.
2. Factor the left member.
3. Set each factor containing the unknown letter equal to zero.
4. Solve the resulting equations.
5. Check each solution.

Example 2. Solve $4x^2=25$.

Solution. The right member must equal zero.

Transposing $4x^2-25=0$

Factoring the left member $(2x+5)(2x-5)=0$

Either $2x+5=0$ or $2x-5=0$

Either $2x=-5$ or $2x=5$

Either $x=-\frac{5}{2}$ or $x=\frac{5}{2}$

Check. Does $4(-\frac{5}{2})^2=25$ and does $4(\frac{5}{2})^2=25$?

Does $25=25$ and does $25=25$? Yes.

Example 3. Solve $5\,x^2 = 2\,x$.

Solution.

$$\begin{aligned} 5\,x^2 &= 2\,x \\ 5\,x^2 - 2\,x &= 0 \\ x(5\,x - 2) &= 0 \end{aligned}$$

Either $x = 0$ or $5\,x - 2 = 0$

$$\begin{aligned} 5\,x &= 2 \\ x &= \tfrac{2}{5} \end{aligned}$$

Check. Does $5(0)^2 = 2(0)$ and does $5(\frac{2}{5})^2 = 2(\frac{2}{5})$?

Does $0 = 0$ and does $\frac{4}{5} = \frac{4}{5}$? Yes.

EXERCISES

A

Solve by factoring:

1. $x^2 + 2\,x = 15$
2. $x^2 + 3\,x + 2 = 0$
3. $m^2 - 4 = 0$
4. $x^2 - 1 = 0$
5. $x^2 - 3\,x = 0$
6. $x^2 + 5\,x = 0$
7. $x^2 + 6\,x = -9$
8. $x^2 - 35 = 2\,x$
9. $x^2 - 4\,x + 4 = 0$
10. $x^2 = 9$
11. $x^2 - 6\,x = 0$
12. $x^2 - 2\,x = 24$
13. $x^2 - 12\,x = -32$
14. $5\,y^2 - 17\,y = 0$
15. $4\,n^2 = 4$
16. $y^2 - 4\,y = 21$
17. $c^2 + 2 = 3\,c$
18. $4\,x^2 + 16\,x - 20 = 0$
19. $4\,x^2 = 9$
20. $y^2 - 8\,y = 0$
21. $2\,y^2 = 9\,y - 9$
22. $6\,x^2 + 34\,x = 12$
23. $\pi r^2 - 2\,\pi r = 0$
24. $d^2 + 8\,d = 0$

Change each equation to standard form and solve:

25. $y(y + 2) = 15$
26. $\dfrac{x^2}{6} + \dfrac{x}{3} = \dfrac{1}{2}$
27. $x^2 = \dfrac{5}{2} - \dfrac{3\,x}{2}$
28. $6\,x^2 + \dfrac{13\,x}{2} = 7$
29. $x^2 + 25 = 1 - 10\,x$
30. $(6\,x + 13)x = 15$
31. $\dfrac{x^2}{2} = 6\,x$
32. $\dfrac{x^2}{12} + x + \dfrac{9}{4} = 0$
33. $\dfrac{y^2}{15} = \dfrac{y}{5} + \dfrac{2}{3}$
34. $(y - 3)(y - 5) = y - 5$

B

Solve for x:

35. $x^2 - a^2 = 0$

36. $6x^2 + 7cx - 5c^2 = 0$

37. $\frac{x^2}{8} + yx = \frac{5y^2}{2}$

38. $x^2 - 3ax = 10a^2$

39. $\frac{x^2}{3} - \frac{c^2}{2} = \frac{cx}{6}$

40. $3x^2 - 10m^2 = \frac{7mx}{2}$

41. $(x-1)(x^2 - 3x + 2) = 0$

42. $(x-3)(x^2 - cx - 2c^2) = 0$

Special Products in Equations [A]

The solutions of many equations which contain indicated multiplications are shortened when one can find the products mentally.

Example 1. Solve $(x-6)^2 - (x+3)(x-1) = 11$.

Solution.

$$\begin{aligned}
(x-6)^2 - (x+3)(x-1) &= 11\\
(x^2 - 12x + 36) - (x^2 + 2x - 3) &= 11\\
x^2 - 12x + 36 - x^2 - 2x + 3 &= 11\\
x^2 - 12x - x^2 - 2x &= 11 - 36 - 3\\
-14x &= -28\\
x &= 2
\end{aligned}$$

Check. Does $(2-6)^2 - (2+3)(2-1) = 11$?

Does $16 - 5 = 11$? Yes.

Caution. You should be sure to place the products of binomials in parentheses to avoid errors. This was done in the example above. In the exercises that follow find the products mentally.

EXERCISES

A

Solve and check:

1. $(x+1)^2 - x^2 = 7$

2. $(x-5)^2 = x^2 + 5$

3. $(y-3)^2 = y^2 + 6$

4. $(m-1)(m+1) = (m+1)^2$

5. $(n+2)(2n+1)-(n-1)^2=19$

6. $(3x-1)(3x+1)-(2x-1)(2x+1)=45$

7. $4y^2+7y=10+(2y-3)^2$

8. $4r^2-5(r-1)=(2r+5)(2r-5)$

9. $6x(3x-5)-2x(9x+1)=-128$

10. $(5x+7)(6x-2)+5(3x-9)=30x^2+35$

11. $(y-6)^2-(y+4)^2+55=0$

B

Solve and check:

12. $3(c-5)^2+2(c-4)^2-5c^2-15=0$

13. $(y+2)(4y-3)-4(y+3)(y-3)=35$

14. $(2-3x)^2-(3x-1)(3x+1)=2$

15. $x^2+(x-1)^2-(2x-1)(x-8)=38$

16. $(2x-1)(3x+4)-3(x-1)(x+2)=18$

Literal Equations [A]

A *literal equation* is one in which the unknown number and at least one of the known numbers are represented by letters. In solving some literal equations it is necessary to factor one or more expressions. Example 1 illustrates the method of solving an equation of this kind.

Example 1. Solve $ax-a^2=bx-b^2$ for x.

Solution. $ax-a^2=bx-b^2$

Transposing $ax-bx=a^2-b^2$

Factoring $x(a-b)=(a-b)(a+b)$

$\mathbf{D}_{a-b}$ $\frac{x(a-b)}{a-b}=\frac{(a-b)(a+b)}{a-b}$

or $x=a+b$

Note. You should be sure to transpose so that all terms containing the unknown letter are in the left member, and all other terms are in the right member.

1359 9 9 2

EXERCISES

A

Solve for x:

1. $cx = d$
2. $mx - n = 0$
3. $(a - b)x = 2(a - b)$
4. $ax - bx = 2a - 2b$
5. $ax - 2a = bx - 2b$
6. $(c + d)x = m(c + d)$
7. $(b - 3)x = b^2 - 6b + 9$
8. $ax - d = bx + c$
9. $ax + 9 = 3x + a^2$
10. $bx + 6b + b^2 = 0$
11. $a - 2x = 3x - 4a$
12. $mx - m^2 = nx - mn$
13. $2b^2 - bx = 0$
14. $mx - m^2 = x + m - 2$
15. $2bx + 4b = 2b + 4b^2 + x$
16. $2bx = 9d - 5cx$
17. $b - ax = a + ax$
18. $a(bx + 2) = c$
19. $x - c^2 + c = -2 - cx$
20. $mx - 1 = nx$
21. Solve $A = P + Prt$ for P.
22. Solve $cst = r - s$ for s.
23. Solve $mf = mg - t$ for m.
24. Solve $c = Ra + Rc$ for c.

B

Solve for x:

25. $cx + 3b^2 = c^3 - 3bx$.
26. $m^2x + n^2x = 2$
27. $d(a - x) = x(c - d)$
28. $\frac{2acx}{3} = b$
29. $5(x - b) = 4(x - a)$
30. $r(x - s) - r^2 = 2sr$
31. $2m^2x + 2m^2 = 2m^4 + 2mx$
32. $bcx + 5c = c^2 + 5bx$
33. $cx + c = 3x + c^2 - 6$
34. $adx + bd = cd + bcx + c^2$
35. $acdx - bcdk = bc^2x - bdhx$
36. $\frac{a + b}{2} = \frac{x}{3}$
37. $a^2x - a^2 - ax = 6x - 2a - 3$
38. $a(x - 1) = x + 2a^2 - 3$
39. $c - \frac{x}{2} - \frac{x}{3} = \frac{c}{6}(c - x) + \frac{x}{6}$
40. $a(3x + b) + b(x + b) = 6a^2$
41. Solve $A = \frac{h}{2}(b + b')$ for b.
42. Solve $C(R + nr) = en$ for n.

43. Solve $A = \pi r^2 + 2\ \pi rh$ for h.

44. Solve $C = \frac{5}{9}(F - 32)$ for F.

Solving Verbal Problems [A]

Any verbal problem you have had so far could be solved by a first-degree equation containing one letter, or by a set of simultaneous first-degree equations.

From now on some of the verbal problems lead to quadratic equations, or equations of the second degree. In this course use only one letter in solving problems whose equations are quadratics.

Each number to be found in a problem whose equation is a quadratic has two values. Thus if you are asked in the problem to find two numbers, you are required to find four answers, two for each of the two numbers.

Example. The difference of two numbers is 3 and the sum of their squares is 29. Find the numbers.

Solution. Let $x =$ the smaller number

Then $x + 3 =$ the larger number

$$\begin{aligned}(x + 3)^2 + x^2 &= 29\\ (x^2 + 6\,x + 9) + x^2 &= 29\\ x^2 + 6\,x + 9 + x^2 &= 29\\ x^2 + x^2 + 6\,x + 9 - 29 &= 0\\ 2\,x^2 + 6\,x - 20 &= 0\\ 2(x^2 + 3\,x - 10) &= 0\\ 2(x + 5)(x - 2) &= 0\end{aligned}$$

If $x + 5 = 0$	If $x - 2 = 0$
Then $x = -5$, the smaller no.	Then $x = 2$, the smaller no.
$x + 3 = -2$, the larger no.	$x + 3 = 5$, the larger no.

Notice that the pair of numbers may be -5 and -2, or $+2$ and $+5$.

Check. $(-2) - (-5) = 3$	$5 - 2 = 3$
$(-2)^2 + (-5)^2 = 29$	$5^2 + 2^2 = 29$

PROBLEMS

A

1. The difference of the squares of two consecutive integers is 21. What are the integers? (Let $x =$ the smaller integer. Then $x + 1 =$ the larger integer.)

2. The difference of two numbers is 4 and the difference of their squares is 40. Find the numbers.

3. The area of a rectangle is 176 square inches. What are its dimensions if the length exceeds the width by 5 inches?

4. The square of a certain number is 56 more than the number itself. What is the number? (Two answers)

5. What is the number whose square exceeds 10 times itself by 96?

6. One of two complementary angles equals the square of the other. How many degrees are there in each angle?

7. The sum of two numbers is 21 and their product is 104. What are the numbers?

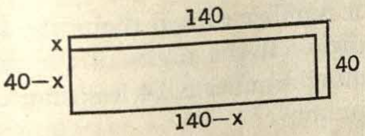
8. The sum of two numbers is 16 and the difference of their squares is 32. What are the numbers?

9. Three times a number increased by twice the square of the number equals 44. Find the number.

10. Mary said to Priscilla, "If six times my age in ten years is subtracted from the square of my present age, the result is 52 years." How old is Mary?

B

11. A city lot is 40 feet wide and 140 feet long. How wide a strip must be cut off one end and one side to make the area of the lot 4725 square feet?



12. A rectangular flower bed is 10 feet wide and 25 feet long. If the length is increased by x feet and the width is decreased by x feet, the area is 216 square feet. Find x.

13. The perimeter of a rectangle is 80 feet and its area is 384 square feet. Find the length and width.

14. A farmer wishes to build a granary for wheat that will hold 400 cubic feet. The bin is to be 8 feet high and twice as long as it is wide. Find its length and width.

15. A farmer planted 360 cherry trees, having 9 more trees in a row than the number of rows. How many rows did he have?

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C

16. A rectangular plot of ground is 20 feet long and 15 feet wide. The plot is surrounded by a walk whose area is two thirds the area of the plot. What is the width of the walk?

17. One number exceeds another by d and the sum of the numbers is s. What are the numbers?

18. The perimeter of a rectangle is p and the length exceeds the width by d. Find the dimensions of the rectangle.

Digit Problems [B]

The word *digit* comes from the Latin word for finger. Since early peoples counted with their fingers, the figures 1, 2, 3, 4, 5, 6, 7, 8, and 9, which stand for numbers, are also called *digits*.

In our way of writing numbers each digit has a place value. Thus in the number 358, the digit 3 tells the number of hundreds, the digit 5 tells the number of tens, and the digit 8 tells the number of units. In solving a problem dealing with the digits of a number you should remember that a number such as 358 means $3 \times 100 + 5 \times 10 + 8$.

Example. In a number of two digits the tens' digit is four times the units' digit. If the digits of the number are interchanged, the resulting number is 54 less than the original number. What is the number?

Solution 1.

Let $x =$ the units' digit

Then $4x =$ the tens' digit

$40x + x =$ the number

$10x + 4x =$ the new number

$$10x + 4x = 40x + x - 54$$

$$10x + 4x - 40x - x = -54$$

$$-27x = -54$$

$x = 2$, the units' digit

$4x = 8$, the tens' digit

The number is 82.

Solution 2.

Let $t =$ the tens' digit

and $u =$ the units' digit

$10t + u =$ the number

$10u + t =$ the new number

$$\begin{cases} t = 4u \\ 10u + t = 10t + u - 54 \end{cases}$$

Solve this set of simultaneous equations and complete the solution.

EXERCISES[B]. DIGIT PROBLEMS

1. The sum of the two digits of a number is 11. If the tens' digit is 3 more than the units' digit, what is the number?

2. In a two-digit number the sum of the digits is 8. If 18 is added to the number, the resulting number has the same digits as the original number. What is the original number?

3. The value of a two-digit number is 3 more than 4 times the sum of its digits. If the sum of its digits is 8, what is the number?

4. The sum of the two digits of a number is 9. If the number formed by interchanging the digits is subtracted from the number, the result is 27. Find the number.

5. The difference of the two digits of a number is 5. If the value of the number is equal to three times the sum of the digits, what is the number?

6. Two numbers, each having two digits, differ by 27. The same digits are found in each number. If the smaller number is increased by its tens' digit and the larger number is increased by its units' digit, the smaller number is half the larger number. What are the two numbers?

WORD LIST

At this time you should be able to use and spell each of the following words correctly:

co-ordinates (p. 287)
cubic equation (p. 348)
degree (p. 348)
digit (p. 356)
factor (p. 319)
linear equation (p. 348)
literal equation (p. 352)
monomial (p. 13)
ordinate (p. 287)
origin (p. 286)
proportion (p. 227)
ratio (p. 226)
simultaneous (p. 298)
vary (p. 46)

CHAPTER REVIEW [A]

$x^2 + 5x - 24 = 0$
$(x - 3)(x + 8)$
$x = 3 \quad x = -8$

Solve the following quadratic equations:

1. $x^2 - 6x + 5 = 0$

2. $x^2 - x - 30 = 0$

3. $x^2 = 10x$

4. $x^2 = 36$

5. $y^2 + 3y = 40$

6. $2x^2 - 5x = 3$

7. $x^2 - 64 = 0$

8. $2x^2 + 3x = 0$

Solve the following literal equations for x:

9. $abx = c$

10. $dx + e = 0$

11. $mx - m^2 = nx - n^2$

12. $ax - bd = ad - bx$

13. $a(x - a) = b(x - b)$

14. $c^2(x - 1) = cx - 1$

15. $2mx + 6n^2 = 3nx + 4mn$

16. $a(x - 1) = a^2 + 3(x - 4)$

Solve:

17. $(y + 5)^2 - y(y - 2) + 11 = 0$

18. $(x - 3)(3x + 1) - (2x - 3)(2x + 3) = 18$

19. The length of a room is 3 feet more than the width. If the length and width are each increased by 2 feet, the area is increased by 58 square feet. Find the original dimensions of the room.

20. The area of a triangular pennant is 90 square inches. If the altitude is 3 inches more than the base, what are the lengths of the base and altitude?

21. The square of a certain number is 40 more than 6 times the number. What is the number?

22. The sum of the square of a certain number and 5 times the number is 24. Find the number.

23. Separate 18 into two parts such that the sum of their squares is 170.

GENERAL REVIEW

A

1. Combine: $2x^3 - 3x^2 - 8x + 6 + x^2 + 2x - 1$

2. From $5a^2 - 3a + 4$ take $a^3 - a^2 + 6$.

3. Multiply $2x^2 - 3xy + y^2$ by $x + 3y$.

4. Divide $x^3 + 4x^2 + 7x + 6$ by $x + 2$.

5. Factor:

a. $x^2 - x - 12$

b. $c^2 - 11c + 18$

c. $6x^2 - xy - y^2$

d. $2m^2 - 11mn - n^2$

e. $4x^2 - 25$

f. $pq + q$

g. $x^4 + 14x^2 + 49$

h. $9x^2 - y^2z^2$

i. $m^2n^2 + 4m^2n$

j. $2x^2 + 2x$

Solve:

6. $8(y - 1) = 17 - (27 - 18y)$

7. $\frac{p}{4} = \frac{3}{8} - \frac{3p - 1}{2}$

8. $16x^2 - 81 = 0$

9. $b^2 + 12b + 27 = 0$

10. $(x - 1)^2 + 3 = 3x$

11. $6x^2 + 15x + 6 = 0$

12. $\frac{x^2}{2} - \frac{x^2}{3} = \frac{3}{2}$

13. $3x = 5x^2$

Solve the following sets of equations:

14. $2x + 3y = 9$
$5x = -3y$

15. $x + y = 6$
$3x - 2y = 23$

16. $3r - 2s = -10$
$2r + 3s = -11$

17. $x - \frac{y}{3} = \frac{1}{6}$
$2x + \frac{y}{4} = \frac{5}{4}$

18. $2\frac{1}{3}x - \frac{1}{2}y = 0$
$\frac{x + y}{5} = 2$

19. Find the value of $3x^2 + 2$ when x is 6 less than $3a$.

20. Solve, using one letter and also using two letters. The difference of two numbers is 5. Find the numbers if 3 times the smaller is 2 more than twice the larger.

21. Separate 44 into two parts such that if twice the smaller is subtracted from the larger, the remainder is 5.

22. If the area of a rectangle is $a^3 - b^3$ and the length is $a^2 + ab + b^2$, what is the width?

23. Elsie is 4 years older than Sylvester. In 15 years one fifth of Elsie's age added to one seventh of Sylvester's age will equal 8 years. How old is each?

24. At what price must a real-estate dealer purchase a lot in order that he may sell it for $1312.50 and make a 5% profit on the cost?

B

25. Solve $x(x - 4) = (x - 2)(x + 2) - 4$.

26. Solve $(x^2 - 1)(x^2 + 1) - (x^2 + 2)(x^2 - 3) - (x + 7) = 0$.

27. Solve for x: $5x^2 + 4ax = a^2$

28. Solve for x: $x^2 + \frac{4bx + b^2}{3} = 0$

Factor:

29. $16 - 36m^2$

30. $16a^2 - 16$

31. $a^4 - 1$

32. $x^4 - 13x^2 + 36$

33. $y^6 - 15y^3 + 36$

34. $5x^4 - 5x^3 - 150x^2$

35. Divide $x^5 + x^4 - x^3 - x^2$ by $x + 1$.

CHAPTER TEST [A]

Solve the following quadratic equations:

1. $x^2 - 9x + 20 = 0$

2. $x^2 = 6x$

3. $\frac{x^2}{4} = 4$

4. $2x^2 + x - 1 = 0$

5. $6x^2 - 19x = -10$

6. $\frac{x^2}{3} - 2x + \frac{9}{4} = 0$

Solve the following literal equations for x:

7. $ax = b$
8. $x + a = b$
9. $cx - 2c^2 = 0$
10. $bx - 2b = 2c - cx$
11. $mx - m^2 = 2x - m - 2$
12. $cxt = a - x$

Solve:

13. $x(x + 3) = x - 1$
14. $(x + 2)(2x + 3) = (x - 1)^2 - 9$

15. Three times a certain number increased by twice the square of the number equals 65. Find two numbers satisfying this condition.

CHAPTER TEST [B]

Solve the following literal equations for x:

1. $x^2 = b^2$
2. $x^2 + 3c^2 = 4cx$
3. $\frac{x^2}{6} + \frac{a^2}{3} = \frac{ax}{2}$
4. $a(x - b) = a(a - 2b)$
5. $c(1 + x) = bx$
6. $\frac{a - b}{3} = \frac{x}{6} + \frac{x}{2}$

Solve:

7. $\frac{x^2}{2} = 3x$
8. $\frac{x^2}{3} - \frac{1}{2} = \frac{23x}{12}$
9. $3(x - 3)^2 + 2(x + 4)(x - 4) = 3$
10. $(y + 2)(4y - 3) - 4(y + 2)(y - 2) = 21$

11. The sum of the squares of two consecutive integers is 85. Find the integers.

12. A rectangular garden is 12 feet wide and 30 feet long. If the length is decreased by x feet and the width is increased by x feet, the area is 416 square feet. Find x.

ALGEBRA IN BUSINESS

Some pupils study algebra and know why they are studying it. Others study it because they are advised to do so, but they do not see the need of algebra and think that they are wasting time. Often they ask if algebra will help them in their life's work, such as being a mechanic, a farmer, a housewife, or a business man or woman.

Anyone who has studied algebra should have a clearer understanding of the rules and processes of arithmetic. Algebra is an extension of arithmetic and may be called general arithmetic. In algebra you learn to use negative numbers and literal numbers. Your thinking with concrete numbers should then be easier. For example, when you know how to express $a + \frac{m}{n}$ as a fraction, the process of changing $6 + \frac{1}{7}$, or $6\frac{1}{7}$, to a fraction should be clearer. Moreover the algebraic solution of problems should help the pupil in problem-solving in arithmetic.

Anyone engaged in business continually uses arithmetic. In buying and selling one deals with items such as the cost, selling price, margin, overhead, and profit or loss. The algebraic method of finding the base and rate should be useful in finding some of these items.

A man or woman in business should be able to read graphs, as much valuable information about business is presented graphically in trade journals. These graphs are prepared by trained men and women who study imports, exports, costs of production, supply and demand, rise and fall of prices, and hundreds of other items.

Harold M. Lambert

The Selling Price of an Article Is Determined by the Formula

$$S = C + O + P$$

CHAPTER XIII

Fractions

Algebraic Fractions [A]

Fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{5}$ are arithmetic (ăr′-ĭth-mĕt′-ĭk) fractions while fractions such as

$$\frac{a}{b},\quad \frac{4}{c},\quad \text{and}\quad \frac{x+3}{5}$$

are algebraic fractions. In this chapter you will add, subtract, divide, and multiply fractions which, in the main, are algebraic.

The fraction $\frac{a}{b}$ is read "*a* divided by *b*." As in arithmetic, *a* is the numerator and *b* is the denominator. The numerator and denominator are the *terms* of the fraction.

While studying fractions you should always bear in mind that a fraction is an indicated division. The fraction bar which separates the numerator and denominator serves as a sign of grouping. For example,

$$\frac{2x-4}{3}$$

means that both terms of $2x - 4$ are to be divided by 3.

An Important Principle of Fractions [A]

You all know that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equal, or equivalent, fractions. The fraction $\frac{2}{4}$ can be obtained from the fraction $\frac{1}{2}$ by multiplying both terms of $\frac{1}{2}$ by 2, and the fraction $\frac{3}{6}$ can be obtained from the fraction $\frac{1}{2}$ by multiplying both of its terms by 3. The fraction $\frac{1}{2}$ can be obtained from the fraction $\frac{3}{6}$ by dividing both terms of $\frac{3}{6}$ by 3, and from the fraction $\frac{2}{4}$ by dividing both terms of $\frac{2}{4}$ by 2. These examples illustrate the most important principle of fractions:

Both terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.*

ORAL EXERCISES [A]

Tell how the second fraction is obtained from the first:

1. $\frac{2}{3} = \frac{4}{6}$

2. $\frac{1}{2} = \frac{5}{10}$

3. $\frac{m}{n} = \frac{am}{an}$

4. $\frac{x}{y} = \frac{3x}{3y}$

5. $\frac{12}{42} = \frac{2}{7}$

6. $\frac{24}{36} = \frac{2}{3}$

7. $\frac{x^2}{xy} = \frac{x}{y}$

8. $\frac{ab}{c} = \frac{abd}{cd}$

9. $\frac{-3}{-4} = \frac{3}{4}$

10. $\frac{4a - 4b}{8} = \frac{a - b}{2}$

11. $\frac{(x+y)(x-y)}{x+y} = \frac{x-y}{1}$

12. $\frac{a^2 - b^2}{a + b} = \frac{a - b}{1}$

Find the missing numerator or denominator:

13. $\frac{5}{8} = \frac{15}{?}$

14. $\frac{8}{3} = \frac{?}{6}$

15. $\frac{?}{9} = \frac{4}{18}$

16. $\frac{5}{3} = \frac{?}{12}$

17. $\frac{4}{6a} = \frac{8}{?}$

18. $\frac{3a}{?} = \frac{a}{b}$

19. $\frac{?}{12x} = \frac{5}{4x}$

20. $\frac{bc}{c^2} = \frac{b}{?}$ c

21. $\frac{a}{x} = \frac{?}{x^2y}$

22. $\frac{8bc}{24b^2} = \frac{c}{?}$ 30

23. $\frac{7}{x+y} = \frac{?}{2x+2y}$ 14

24. $\frac{3m}{m+n} = \frac{?}{5(m+n)}$

25. $\frac{x+4}{x-1} = \frac{?}{(x-1)(x+2)}$

26. $\frac{2c+1}{c-3} = \frac{?}{c^2 - 3c}$ × c

27. $\frac{3b+5}{b-4} = \frac{?}{b^2 - 7b + 12}$

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* We cannot multiply or divide both terms by zero.

Reducing Fractions to Lowest Terms [A]

A fraction is in its simplest form when the numerator and denominator have no common factor. Changing a fraction to its simplest form is called *reducing the fraction to lowest terms.* A fraction is reduced to lowest terms by dividing both numerator and denominator by their greatest common divisor (G.C.D.).

Example 1. Reduce $\frac{14\,ax}{21\,a^3}$ to lowest terms.

Solution 1. The G.C.D. of 14 and 21 is 7, and the G.C.D. of ax and a^3 is a. Then the G.C.D. of $14\,ax$ and $21\,a^3$ is $7\,a$.

$$\frac{14\,ax}{21\,a^3} = \frac{(14\,ax) \div (7\,a)}{(21\,a^3) \div (7\,a)} = \frac{2\,x}{3\,a^2}$$

Solution 2. To make division easier, both numerator and denominator may be factored.

$$\frac{14\,ax}{21\,a^3} = \frac{2 \times \overset{1}{\cancel{7}}\,\overset{1}{\cancel{a}}x}{3 \times \underset{1}{\cancel{7}}\,\underset{a^2}{\cancel{a^3}}} = \frac{2\,x}{3\,a^2}$$

EXERCISES [A]

Reduce each fraction to lowest terms:

1. $\frac{6}{18}$

2. $\frac{5}{20}$

3. $\frac{10}{25}$

4. $\frac{14}{35}$

5. $\frac{15}{18}$

6. $\frac{12}{20}$

7. $\frac{x}{x^2}$

8. $\frac{ax}{x^3}$

9. $\frac{3\,a}{6\,a}$

10. $\frac{9\,m}{12\,m}$

11. $\frac{4\,x}{3\,x}$

12. $\frac{2\,a^3}{7\,a}$

13. $\frac{b^2c}{bc^2}$

14. $\frac{mn}{m^2n^2}$

15. $\frac{r^3s^3}{rs}$

16. $\frac{6\,ac}{15\,c^3}$

17. $\frac{xy}{x^2y^3}$

18. $\frac{11\,a^2b}{33\,ac}$

19. $\frac{a^7}{a^3}$

20. $\frac{a^9}{a^7}$

21. $\frac{26\,x^4y^6}{39\,x^3y}$

22. $-\frac{90\,b^2c^3}{36\,a^3b^2}$

23. $\frac{-rst^4}{r^2st^4}$

24. $\frac{180\,a}{100\,a}$

Example 2. Reduce $\frac{a^2-2a-24}{5a-30}$ to lowest terms.

Solution. When fractions contain polynomials, the polynomials should be factored to make division easier.

$$\frac{a^2-2a-24}{5a-30}=\frac{\overset{1}{\cancel{(a-6)}}(a+4)}{5\underset{1}{\cancel{(a-6)}}}=\frac{a+4}{5}$$

The following is a wrong solution:

$$\frac{\overset{a}{\cancel{a^2}}-2a-\overset{-4}{\cancel{24}}}{\underset{5}{\cancel{5a}}-\underset{-5}{\cancel{30}}}=\frac{a-2a-4}{5-5}=\frac{a-2a-4}{0}$$

The reason this solution is incorrect is that only parts of the numerator and denominator are divided by a and 6. In the correct solution, the whole numerator and denominator are divided by $a-6$. Remember that one cannot cancel parts of expressions.

EXERCISES

A

1. Add 2 to each term of the fraction $\frac{8}{10}$. Reduce the resulting fraction to lowest terms. Does it equal $\frac{8}{10}$? Can one add a number to each term of a fraction without changing the value of the fraction?

2. Subtract some number from each term of the fraction $\frac{8}{10}$. Does the resulting fraction equal $\frac{8}{10}$? Can one subtract the same number from each term of a fraction without changing the value of the fraction?

3. Tell whether the following solutions are correct, and give reasons for your answers:

a. $\frac{2a\overset{1}{\cancel{(a+5)}}}{\underset{1}{\cancel{a+5}}}=2a$

b. $\frac{2a+\overset{1}{\cancel{a+5}}}{\underset{1}{\cancel{a+5}}}=2a$

c. $\frac{\overset{2}{\cancel{2a}}+\overset{1}{\cancel{5}}}{\underset{1}{\cancel{a}}+\underset{1}{\cancel{5}}}=\frac{3}{2}$

d. $\frac{\overset{1}{\cancel{a}}\overset{1}{\cancel{(a+5)}}}{\underset{2}{\cancel{2a}}\underset{1}{\cancel{(a+5)}}}=2$

Simplify each of the following fractions:

4. $\dfrac{5x+5}{6x+6}$

5. $\dfrac{4x-28}{6x-42}$

6. $\dfrac{7x-21}{8x-24}$

7. $\dfrac{5b^2+15b}{b^2+3b}$

8. $\dfrac{m}{m^2+m}$

9. $\dfrac{c^2-2c-15}{c^2-c-20}$

10. $\dfrac{2m^3-6m^2+2m}{m^2-3m+1}$

11. $\dfrac{x^2-1}{x^2-2x+1}$

12. $\dfrac{6x^2+11x+4}{2x^2-9x-5}$

13. $\dfrac{a^2-6ab+9b^2}{a-3b}$

14. $\dfrac{R^2-r^2}{Rr+R^2}$

15. $\dfrac{m^2-mn}{m^2+mn}$

B

Simplify:

16. $\dfrac{4ab^3c}{4ab+4bc}$

17. $\dfrac{2x^2+xy-3y^2}{6x^2+5xy-6y^2}$

18. $\dfrac{12x^2+2x-24}{12x^2-31x+20}$

19. $\dfrac{xy^2-13xy+22x}{y^2-9y-22}$

20. $\dfrac{(a+b)^2(a-b)}{a^2-b^2}$

21. $\dfrac{3x^3+3x^2y-6xy^2}{x^3-3x^2y+2xy^2}$

22. $\dfrac{k^4-8k^3+12k^2}{k^3-4k}$

23. $\dfrac{5xyz+15z}{x^2y^2-xy-12}$

24. $\dfrac{2a^2-10a+12}{a^3-3a^2+2a}$

Multiplication of Fractions [A]

In algebra, as in arithmetic, the product of two or more fractions is equal to the product of the numerators divided by the product of the denominators. The resulting fraction should be reduced to lowest terms.

Example 1. $\frac{3}{4} \times \frac{4}{9} = \frac{12}{36} = \frac{1}{3}$. This multiplication should be shortened as follows:

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{1}{3}$$

Example 2. Multiply 12 by $1\frac{1}{3}$.

Solution. The denominator of 12 is 1 and $1\frac{1}{3} = \frac{4}{3}$.

Then $$\frac{\overset{4}{\cancel{12}}}{1} \times \frac{4}{\underset{1}{\cancel{3}}} = \frac{16}{1} \text{ or } 16$$

Example 3. Multiply $\frac{7\,ab^2}{3}$ by $\frac{15}{14\,a^2}$.

Solution. $$\frac{\overset{b^2}{\cancel{7\,ab^2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2\,a}{\cancel{14\,a^2}}} = \frac{5\,b^2}{2\,a}$$

EXERCISES [A]

Multiply:

1. $\frac{2}{3} \times \frac{1}{2}$

2. $\frac{3}{5} \times \frac{5}{9}$

3. $\frac{a}{b} \times \frac{b}{a}$

4. $\frac{1}{5}$ of 20

5. $15 \times \frac{2}{3}$

6. $3\frac{1}{3} \times \frac{3}{10}$

7. $\frac{2}{3} \times 4$

8. $\frac{7}{8} \times \frac{7}{8}$

9. $\frac{8}{9} \times \frac{9}{8}$

10. $\frac{4\,b}{7} \times \frac{1}{2\,b^2}$

11. $\frac{c^2}{d} \times \frac{1}{c}$

12. $\frac{3\,ab}{4} \times \frac{12}{b^4}$

13. $\frac{4}{x} \cdot 3$

14. $\left(\frac{a}{b}\right) b^2$

15. $x^3 \cdot \frac{1}{x^4}$

16. $\frac{\pi r}{1} \cdot \frac{1}{R^2}$

17. $\frac{a}{3} \cdot \frac{b}{2}$

18. $\frac{3\,c}{2\,ab} \cdot \frac{5\,a}{6\,c}$

19. $\frac{6\,m}{15\,mn^2} \cdot \frac{n}{8\,m^2}$

20. $\frac{a}{b} \cdot 20$

21. $\frac{3}{4}\,(12\,a^2c)$

22. $\frac{a}{6} \cdot \frac{3\,b^7}{2}$

23. $\frac{20\,a^2}{15\,b^3} \cdot 10\,b^2$

24. $\frac{x^2y^2}{m^3n} \cdot \frac{m}{x^3}$

25. $\left(\frac{2\,c}{7}\right)\left(\frac{7}{6\,c}\right)$

26. $\frac{1}{a} \cdot \frac{1}{c} \cdot \frac{1}{e}$

27. $\frac{a^2}{b^3} \cdot \frac{a^2}{b} \cdot \frac{b}{a}$

28. $\frac{2\,c^3}{15\,d^6} \cdot 2\,cd$

29. $\frac{2\,c^5}{5\,b^4} \cdot \frac{10\,b^4}{8\,c^5}$

30. $\frac{r^2}{c} \cdot \frac{c}{rst}$

$X^2 =$

Example 4. $\dfrac{4x^2 - 25y^2}{3x - 2y} \cdot \dfrac{9x^2 - 12xy + 4y^2}{2x^2 - 5xy}$

Solution. Factoring the numerators and denominators,

$$\frac{\overset{1}{\cancel{(2x-5y)}}(2x+5y)}{\underset{1}{\cancel{3x-2y}}} \cdot \frac{\overset{1}{\cancel{(3x-2y)}}(3x-2y)}{x\underset{1}{\cancel{(2x-5y)}}}$$

$$= \frac{(2x+5y)(3x-2y)}{x}$$

EXERCISES

A

Multiply as indicated:

1. $\dfrac{x+y}{8} \cdot \dfrac{10}{x^2-y^2}$

2. $\dfrac{a+3}{a-5} \cdot \dfrac{2a-10}{3a+9}$

3. $\dfrac{2a}{3b} \cdot \dfrac{b^2-b}{2ab-2a}$

4. $\dfrac{x^2-1}{x^2-2x} \cdot \dfrac{3x-6}{4x+4}$

5. $\dfrac{a^2-6a+5}{a+1} \cdot \dfrac{a+1}{a-5}$

6. $\dfrac{m^2-64}{8} \cdot \dfrac{24}{m^2-15m+56}$

7. $\dfrac{1-c^2}{b^3} \cdot \dfrac{b^2}{1-2c+c^2}$

8. $\dfrac{3x+3b}{4x^2} \cdot \dfrac{2x^2}{x^2-b^2}$

9. $\dfrac{x^2-4}{(x-2)^2} \cdot \dfrac{7x-14}{3x+6}$

10. $\dfrac{b^2+6b+9}{3} \cdot \dfrac{4b+8}{b^2+7b+12}$

11. $\dfrac{(x-y)^2}{3} \cdot \dfrac{6}{(x-y)^3}$

12. $\dfrac{3x-12}{4x+20} \cdot \dfrac{x^2+5x}{x^2-4x}$

13. $\left(\dfrac{2m^2+6}{15y^2}\right)\left(\dfrac{21y^4}{3m^2+9}\right)$

14. $\dfrac{2r^2-7r-15}{r^2-10r+25} \cdot \dfrac{r^2-25}{2r^2+13r+15}$

15. $\dfrac{x^2+4}{x^2-4} \cdot \dfrac{(x^2-4)^2}{x^4-16}$

16. $\dfrac{1}{3x+2}(6x^2-11x-10)$

B

Multiply:

17. $\dfrac{m-5}{2m-1} \cdot \dfrac{2m-1}{2m+1} \cdot \dfrac{4m^2-9}{2m^2-13m+15}$

18. $\dfrac{3a^2-27a+60}{2a^2-72} \cdot \dfrac{3a^2-17a-6}{a^3-25a}$

19. $\dfrac{2b^2+bx-3x^2}{b^2-3bx-18x^2} \cdot \dfrac{2bx^2+6x^3}{3b^2-2bx-x^2}$

20. $\dfrac{3a^2-2ax-x^2}{4a^2-5ax-x^2} \cdot \dfrac{4a^2+11ax-3x^2}{3a^2+10ax+3x^2}$

Division of Fractions [A]

To divide one fraction by another, we invert the divisor and proceed as in multiplication.

Example 1. $\dfrac{4ab}{3c} \div \dfrac{2b^2}{9c}$

$$= \frac{\overset{2a}{\cancel{4ab}}}{\underset{1}{\cancel{3c}}} \times \frac{\overset{3}{\cancel{9c}}}{\underset{b}{\cancel{2b^2}}} = \frac{6a}{b}$$

Example 2. Divide $\dfrac{b^2-c^2}{c^2-3c-4}$ by $\dfrac{b-c}{c^2+c}$.

Solution. $\dfrac{b^2-c^2}{c^2-3c-4} \div \dfrac{b-c}{c^2+c}$

$$= \frac{\overset{1}{\cancel{(b-c)}}(b+c)}{(c-4)\underset{1}{\cancel{(c+1)}}} \cdot \frac{c\overset{1}{\cancel{(c+1)}}}{\cancel{b-c}}$$

$$= \frac{c(b+c)}{c-4}$$

In order to save time the factoring and the inversion of the divisor were performed in one operation.

EXERCISES

A

Perform the indicated divisions:

1. $\frac{7}{8} \div \frac{1}{2}$

2. $\frac{6}{9} \div 3$

3. $\frac{2}{3} \div \frac{5}{6}$

4. $1 \div \frac{b}{c}$

5. $\frac{a}{b} \div c$

6. $\frac{m^2}{n^2} \div \frac{2\,mn}{n^2}$

7. $\frac{2\,a}{3\,b} \div \frac{6\,a^2}{b^2}$

8. $3\frac{1}{3} \div \frac{2\,a}{3}$

9. $\frac{a}{b} \div (2\,c)$

10. $\frac{1}{x^2} \div \frac{1}{x^3}$

11. $\frac{3\,x-6}{8} \div \frac{1}{24}$

12. $\frac{a-4}{5} \div \frac{10}{3\,a-12}$

13. $\frac{(m-1)^2}{4} \div (m-1)$

14. $\frac{ab}{(a-b)^2} \div \frac{1}{a-b}$

15. $\frac{9-y^2}{15} \div \frac{3-y}{5}$

16. $\frac{c^2-9}{c^2+3\,c} \div \frac{c-3}{4}$

17. $\frac{a-x}{y} \div \frac{a^2-x^2}{xy}$

18. $1 \div \frac{x-y}{x+y}$

19. $\frac{x^2+7\,x+6}{x^2+6\,x+5} \div \frac{x^3+6\,x^2}{x^2+5\,x}$

20. $\frac{x^2-7\,x+10}{x^2+3\,x} \div \frac{x^2-9\,x+20}{4\,x+12}$

21. $\frac{a^2-9\,b^2}{(a-3\,b)^2} \div \frac{a+3\,b}{a-3\,b}$

22. $\frac{m^2-6\,m+5}{m-1} \div \frac{m-5}{m-1}$

23. $\frac{x^2+2\,xy+y^2}{x^2-2\,xy-3\,y^2} \div \frac{x+y}{x-3\,y}$

24. $\frac{r^2-9\,r+14}{r^2+7\,r+12} \div \frac{3\,r^2-21\,r}{4\,r^3+16\,r^2}$

25. $\frac{3(a+b)}{4\,c^2} \div \frac{6(a^2-b^2)}{cd}$

B

Divide:

26. $\frac{x^4-1}{3} \div \frac{x^2-1}{6}$

27. $\frac{x^4-y^4}{x+y} \div \frac{x^2-y^2}{4\,x+4\,y}$

28. $\frac{bx+by}{x-2\,y} \div \frac{bx^2-by^2}{x^2-3\,xy-2\,y^2}$

29. $\dfrac{2x^3 - 2x^2 - 4x}{3x^2 - 3x - 18} \div \dfrac{4x^3 - 4x^2}{6x - 18}$

30. $\dfrac{x^3 + 4x - 4x^2}{x^3 - 2x^2} \div \dfrac{5x^2 + 20x + 20}{x^3 - 4x}$

31. $\dfrac{a+b}{a-b} \cdot \dfrac{a-2b}{a+2b} \div \dfrac{a+b}{a+2b}$

32. $\dfrac{a+b}{a-b} \div \dfrac{a-2b}{a+2b} \cdot \dfrac{3a+6b}{4a+4b}$

NOTE. Only the second fraction should be inverted.

Addition and Subtraction of Fractions[A]

Some pupils have difficulty with fractions because they do not understand what fractions mean. Let us make a study of a very bad error which is due to not thinking properly.

Example 1. Add: $\frac{1}{2} + \frac{2}{3}$

One Pupil's Solution. $\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$

In this solution the pupil forgot to think that he was adding halves and thirds. According to this solution halves added to thirds give fifths. He was combining unlike terms.

Another Pupil's Solution. $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

In this solution unlike terms are not combined. The unlike terms are changed to like terms before they are added. This pupil thought "1 half + 2 thirds = 3 sixths + 4 sixths. And 3 sixths + 4 sixths = 7 sixths." This example is similar to "3 pounds + 4 ounces = 48 ounces + 4 ounces = 52 ounces." From what has been said, the following rule should not be difficult to follow:

1. To add (or subtract) fractions, change them, if necessary, to equal fractions having the same denominator.
2. Then place the sum (or difference) of the numerators over the common denominator.
3. Reduce the result to lowest terms.

Example 2. Combine: $\frac{3}{5} + \frac{7}{5}$

Solution. The fractions have the same denominator. Then

$$\frac{3}{5} + \frac{7}{5} = \frac{10}{5} \text{ or } 2.$$

Example 3. $\frac{4}{b} + \frac{3}{b} = \frac{7}{b}$

EXERCISES [A]

Add or subtract as indicated. Be sure your answer is in its simplest form.

1. $\frac{2}{3} + \frac{4}{3}$

2. $\frac{2}{15} + \frac{13}{15}$

3. $\frac{7}{12} + \frac{1}{12}$

4. $\frac{14}{15} + \frac{16}{15}$

5. $\frac{7}{9} - \frac{1}{9}$

6. $\frac{2}{a} + \frac{5}{a}$

7. $\frac{5}{x} - \frac{3}{x}$

8. $\frac{1}{2a} + \frac{5}{2a}$

9. $\frac{a}{c} + \frac{b}{c}$

10. $\frac{2}{5y} - \frac{6}{5y}$

11. $\frac{3}{x+1} - \frac{2}{x+1}$

12. $\frac{7}{x-y} - \frac{3}{x-y}$

13. $\frac{a}{a+b} + \frac{b}{a+b}$

14. $\frac{2x}{x-1} - \frac{2}{x-1}$

15. $\frac{4x^2}{2x-1} - \frac{1}{2x-1}$

Example 4. Simplify $\frac{x}{6} + \frac{2x}{9}$.

Solution. Before these two fractions can be added, they must be changed to equal fractions which have the same denominator. It saves time to change them to equal fractions which have the *lowest* common denominator. The smallest number that can be exactly divided by 6 and 9 is 18. Since the denominator 6 must be multiplied by 3 to give 18, the numerator x must also be multiplied by 3. Then $\frac{x}{6} = \frac{3x}{18}$. Since the denominator 9 must be multiplied by 2 to give 18, the numerator $2x$ must also be multiplied by 2. Then $\frac{2x}{9} = \frac{4x}{18}$. The solution is written as follows:

$$\frac{x}{6} + \frac{2x}{9} = \frac{3x}{18} + \frac{4x}{18} = \frac{7x}{18}$$

Example 5. Simplify $4 + \frac{a}{b}$.

Solution. The denominator of 4 is 1. The L.C.D. is b. Then

$$\frac{4}{1} + \frac{a}{b} = \frac{4\,b}{b} + \frac{a}{b} = \frac{4\,b + a}{b}$$

EXERCISES [A]

Simplify:

1. $1 + \frac{c}{d}$
2. $\frac{1}{2} + \frac{5}{6}$
3. $8 + \frac{1}{3}$
4. $\frac{1}{a} + \frac{1}{b}$
5. $\frac{1}{x} + 6$
6. $\frac{m}{n} + \frac{1}{n}$
7. $\frac{a}{b} + 1$
8. $m + \frac{1}{n}$
9. $\frac{2\,a}{3} - \frac{a}{5}$
10. $\frac{3\,x}{4} - \frac{5\,x}{6}$
11. $\frac{x}{y} + \frac{y}{x}$
12. $\frac{x}{2\,y} - \frac{4}{y^2}$
13. $\frac{2\,a}{3} - \frac{a}{2} + \frac{a}{4}$
14. $\frac{5\,m}{6} - \frac{5\,m}{12} - \frac{m}{3}$
15. $\frac{1}{m} + \frac{1}{n} + \frac{1}{p}$
16. $\frac{4}{a^2} + \frac{3}{a} + \frac{5}{a^3}$

Example 6. Combine: $\frac{a-3}{4} + \frac{2\,a-1}{5} - \frac{2\,a+15}{3}$

Solution. The L.C.D. $= 4 \times 5 \times 3 = 60$. When the denominators have no common factor, the L.C.D. is their product.

$$\begin{aligned}
&\frac{a-3}{4} + \frac{2\,a-1}{5} - \frac{2\,a+15}{3} \\
&= \frac{15(a-3)}{60} + \frac{12(2\,a-1)}{60} - \frac{20(2\,a+15)}{60} \\
&= \frac{15(a-3) + 12(2\,a-1) - 20(2\,a+15)}{60} \\
&= \frac{15\,a - 45 + 24\,a - 12 - 40\,a - 300}{60} \\
&= \frac{-a-357}{60} \text{ or } -\frac{a+357}{60}
\end{aligned}$$

EXERCISES [A]

Combine:

1. $\frac{a-1}{3}+\frac{2a+3}{6}$
2. $\frac{3a+5}{5}-\frac{a-3}{3}$
3. $\frac{5x+2y}{10}-\frac{3x-7y}{15}$
4. $\frac{4x-5}{12}-\frac{3x-8}{18}$
5. $\frac{x+y}{2}+\frac{x-y}{3}$
6. $\frac{a+3b}{4}-\frac{2a+b}{5}$
7. $\frac{x^2+xy}{10}-\frac{x^2-xy}{12}$
8. $\frac{2b+3c}{18}-\frac{5b-6c}{36}$
9. $\frac{2x-1}{3x}+\frac{4x-10}{2x}$

   The L.C.D. is $6x$.
10. $\frac{3a-2b}{ab}-\frac{3b-a}{b^2}$
11. $\frac{a+b-1}{a}-\frac{2a-b+1}{2a}$
12. $\frac{3x^2-4x+1}{6x^2}-\frac{x+2}{3x}$
13. $\frac{2a-5}{14a^2}-\frac{4-a}{7a}+\frac{3a-2}{2a}$
14. $\frac{2p+1}{3p}-\frac{p-5}{2p}+\frac{p+4}{30p^2}$
15. $\frac{3x+1}{5x^2}-\frac{4x-3}{2x}$

Example 7. Combine: $\frac{3}{a-b}+\frac{4}{a+2b}$

Solution. Since $a-b$ and $a+2b$ have no common factor, the L.C.D. is $(a-b)(a+2b)$.

$$\begin{aligned}
&\frac{3}{a-b}+\frac{4}{a+2b}\\
&=\frac{3(a+2b)}{(a-b)(a+2b)}+\frac{4(a-b)}{(a-b)(a+2b)}\\
&=\frac{3(a+2b)+4(a-b)}{(a-b)(a+2b)}\\
&=\frac{3a+6b+4a-4b}{(a-b)(a+2b)}\\
&=\frac{7a+2b}{(a-b)(a+2b)}
\end{aligned}$$

EXERCISES [A]

Combine:

1. $\dfrac{a+3}{a+2}+\dfrac{2a-1}{a+4}$

2. $\dfrac{x-2}{3}+\dfrac{3}{x-2}$

3. $\dfrac{1}{x+2}-\dfrac{2}{x-3}$

4. $\dfrac{b}{a+b}-\dfrac{a}{a-b}$

5. $\dfrac{2}{x-1}-\dfrac{3}{x+1}$

6. $\dfrac{a}{a+2b}-\dfrac{3a}{a-b}$

7. $\dfrac{x}{x-5}+\dfrac{2x}{x+5}$

8. $\dfrac{c}{c-6}-\dfrac{c}{c+6}$

Example 8. Combine: $\dfrac{x^2-5}{x^2-1}-\dfrac{2x-7}{5x+5}$

Solution.

$$\frac{x^2-5}{x^2-1}-\frac{2x-7}{5x+5}$$

$$=\frac{x^2-5}{(x-1)(x+1)}-\frac{2x-7}{5(x+1)}$$

The L.C.D. $= 5(x-1)(x+1)$

Then

$$\frac{5(x^2-5)}{5(x-1)(x+1)}-\frac{(x-1)(2x-7)}{5(x-1)(x+1)}$$

$$=\frac{(5x^2-25)-(2x^2-9x+7)}{5(x-1)(x+1)}$$

$$=\frac{5x^2-25-2x^2+9x-7}{5(x-1)(x+1)}$$

$$=\frac{3x^2+9x-32}{5(x-1)(x+1)}$$

Why do we factor the denominators?
Why is $2x^2-9x+7$ placed in parentheses?

EXERCISES

B

Combine:

1. $\dfrac{a}{a+1}+\dfrac{a^2}{a^2-1}$

2. $\dfrac{3a}{a^2-4}+\dfrac{4}{a-2}$

3. $\dfrac{4x}{x+1}-\dfrac{3x^2}{x^2-1}$

4. $\dfrac{1}{6a-12}-\dfrac{4}{a-2}$

5. $\frac{5}{a^2-1}+\frac{a+2}{3}$

6. $\frac{2h}{h^2-3h-4}+\frac{1}{h-4}$

7. $\frac{3a}{a+3}+\frac{a^2+4a-5}{a^2-a-12}$

8. $\frac{y+2}{y^2+3y-54}-\frac{5}{y+9}$

9. $\frac{a+b}{a-b}+\frac{a^2+b^2}{a^2-b^2}$

10. $\frac{3c-5}{c^2+3c-18}+\frac{2}{c-3}$

C

Combine:

11. $\frac{2m-4}{9m^2-9m}-\frac{5}{6m-6}$

12. $\frac{5x}{x^2+x}+\frac{2x-4}{x^2-1}$

13. $\frac{x}{x^2-6x+8}-\frac{x}{x^2-16}$

14. $\frac{4}{x^2-16}-\frac{2}{x^2-8x+16}$

15. $\frac{y}{x^2+xy}-\frac{x}{xy+y^2}$

16. $\frac{a-4b}{a^2-7ab+12b^2}-\frac{a-3b}{a^2-ab-12b^2}$

17. $\frac{a-1}{a+1}+\frac{a+1}{a-1}-\frac{2a^2+2}{a^2-1}$

18. $\frac{4}{x-2}+\frac{5}{x-3}-\frac{7}{x^2-5x+6}$

EXERCISES. ADDITIONAL DRILL

A

Simplify:

1. $\frac{3x}{5}+\frac{x}{4}$

2. $\frac{3y}{7}-\frac{y}{7}$

3. $\frac{4x}{5}-\frac{x}{10}$

4. $\frac{3y}{8}-\frac{y}{12}$

5. $\frac{a}{6}+\frac{b}{3}$

6. $\frac{3c}{10}-\frac{4c}{12}$

7. $\frac{4x}{7}-\frac{9x}{10}$

8. $\frac{2n}{5}-\frac{3n}{20}$

9. $\frac{x}{4}-\frac{x}{3}$

10. $\dfrac{2a-1}{4}+\dfrac{3a+2}{6}$

11. $\dfrac{3x-1}{3}-\dfrac{x}{4}$

12. $\dfrac{3c+2}{9}-\dfrac{c-5}{6}$

13. $\dfrac{3x-1}{4x^2}+\dfrac{2}{5x}$

14. $\dfrac{a}{c}-\dfrac{a+n}{c+n}$

15. $\dfrac{c^2}{c+3}-\dfrac{3c}{c-3}$

B

Simplify:

16. $\dfrac{a^2+b^2}{a^2-b^2}+\dfrac{2a}{a+b}$

17. $\dfrac{x^2+3}{x^2-x-6}-\dfrac{x+1}{x-3}$

18. $\dfrac{c^2}{c^2-3c-18}+\dfrac{6}{c-6}$

19. $\dfrac{x}{1-x}-\dfrac{x}{1+x}$

20. $\dfrac{m}{3m+2}-\dfrac{m+1}{6m^2+13m+6}$

21. $\dfrac{c-2}{5c-5}-\dfrac{c+1}{6c+18}$

C

Simplify:

22. $\dfrac{5}{2a-4}-\dfrac{3}{6a-12}-\dfrac{2}{3a+6}$

23. $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}-\dfrac{4ab}{a^2-b^2}$

24. $\dfrac{m+3}{2m^2+5m-3}+\dfrac{m-2}{4m^2-1}$

25. $\dfrac{c}{c^2+2c-3}-\dfrac{5c-1}{5c^2+17c+6}$

Changing Common Fractions to Decimal Fractions[A]

We often find it necessary to change common fractions to decimal fractions.

Example. Change $\frac{3}{8}$ to a decimal fraction.

Solution. The fraction $\frac{3}{8}$ means that 3 is to be divided by 8. So we divide 3 by 8 and get 0.375.

$$\boxed{\begin{array}{r} 0.375 \\ 8\overline{)3.000} \end{array}}$$

EXERCISES [A]

Change the following to decimal fractions:

1. $\frac{3}{5}$
2. $\frac{3}{4}$
3. $\frac{5}{4}$
4. $\frac{13}{4}$
5. $\frac{9}{100}$
6. $\frac{32}{10}$
7. $\frac{9}{50}$
8. $\frac{5}{8}$
9. $\frac{7}{32}$
10. $\frac{9}{64}$
11. $1\frac{5}{8}$
12. $1\frac{7}{8}$

Changing Decimal Fractions to Common Fractions and Mixed Numbers [A]

Example. Change 3.625 to a mixed number.

Solution. $3.625 = 3 + \frac{625}{1000} = 3 + \frac{5}{8} = 3\frac{5}{8}$

EXERCISES [A]

Change to common fractions or mixed numbers:

1. 0.5
2. 0.75
3. 0.45
4. 0.8
5. 0.12
6. .375
7. .125
8. 1.25
9. .625
10. .425
11. .3125
12. $1.33\frac{1}{3}$

The Three Signs of a Fraction [B]

Every fraction has three signs associated with it: the sign of the numerator, the sign of the denominator, and the sign of the fraction proper. When these signs are omitted, they are understood to be plus. Thus the fraction $\frac{3}{4}$ means $+\frac{+3}{+4}$. Consider the four fractions below.

$$+\frac{+8}{+2} = +(+4) = 4 \qquad -\frac{-8}{+2} = -(-4) = 4$$

$$+\frac{-8}{-2} = +(+4) = 4 \qquad -\frac{+8}{-2} = -(-4) = 4$$

The value of each fraction is 4. Each of the four fractions may be obtained from any one of the others by changing two of the three signs. These fractions illustrate the following rule:

The value of a fraction is not changed

1. If the sign of the numerator and the sign of the denominator are changed (if both terms are multiplied by -1). Or,
2. If the sign of the fraction and the sign of either the numerator or the denominator are changed.

Briefly, this rule may be stated as follows:

Any two of the three signs of a fraction may be changed without changing the value of the fraction.

EXERCISES [B]

Tell which of the following statements are true and which are false:

1. $+\frac{-8}{2} = -\frac{+8}{+2}$
2. $\frac{-a}{-b} = \frac{a}{b}$
3. $-\frac{-a}{-b} = \frac{-a}{-b}$
4. $-\frac{m}{-n} = \frac{-m}{n}$
5. $\frac{c}{d} = -\frac{-c}{d}$
6. $\frac{5}{-6} = \frac{-5}{6}$
7. $\frac{-3}{4} = \frac{3}{4}$
8. $\frac{6}{-7} = -\frac{-6}{7}$

Example 1. Change $\frac{a-b}{c-d}$ to an equal fraction whose denominator is $d-c$.

Solution. The denominator $c-d$ is to have the signs of its terms changed, or to be multiplied by -1. In this way we change the sign of the denominator. Then either the numerator or the fraction must have its sign changed. If we change the sign of the numerator, we must change the signs of all its terms.

Then $$\frac{a-b}{c-d} = +\frac{b-a}{d-c}, \quad \text{or} \quad -\frac{a-b}{d-c}$$

Notice that the terms were rearranged after their signs were changed.

If either the numerator or the denominator of a fraction is a polynomial, you should remember that the sign of a polynomial is changed by changing all of its signs, or by multiplying it by -1.

EXERCISES [B]

Find the missing terms:

1. $\frac{a-3}{b-2} = +\frac{?}{2-b}$
2. $\frac{c-d}{x-y} = -\frac{c-d}{?}$
3. $\frac{4-a}{x-y} = +\frac{?}{y-x}$
4. $\frac{4}{a-b} = +\frac{?}{b-a}$
5. $\frac{x-y}{b} = -\frac{?}{b}$
6. $\frac{6}{2-y} = \frac{?}{y-2}$
7. $-\frac{a^2-2b}{a^2-b^2} = +\frac{?}{a^2-b^2}$
8. $\frac{a^2+b^2}{a^2-4} = -\frac{?}{4-a^2}$

It is often convenient to arrange the numerator and denominator of a fraction in descending (or ascending) powers of one letter or to make the first terms of the numerator and denominator positive.

Example 2. Reduce $\frac{x^2-6x}{24+2x-x^2}$ to lowest terms.

Solution. We shall arrange the denominator in descending powers of x to agree with the numerator.

Then $$\frac{x^2-6x}{24+2x-x^2} = \frac{x^2-6x}{-x^2+2x+24}$$

To make the solution easier we should have the first term of the denominator positive. Then, changing the sign of the denominator by multiplying each of its terms by -1, the denominator becomes $x^2-2x-24$. We do not wish to change the sign of the numerator, so we shall change the sign of the fraction proper. Then

$$\frac{x^2-6x}{24+2x-x^2} = -\frac{x^2-6x}{x^2-2x-24} = -\frac{x\cancel{(x-6)}}{\cancel{(x-6)}(x+4)} = -\frac{x}{x+4}$$

EXERCISES [B]

Following the plan in Example 2, simplify the fractions below:

1. $\dfrac{a^2 - 4a + 3}{3 - a}$
2. $\dfrac{x^2 + x - 12}{4 - 47x - 12x^2}$
3. $\dfrac{1 - x^2}{3x^2 - 6x + 3}$
4. $\dfrac{(n - m)^2}{m^2 + 2mn - 3n^2}$
5. $\dfrac{4a - 4b}{b^2 - a^2}$
6. $\dfrac{x^2 - 6x + 9}{9 - x^2}$

Example 3. Combine: $\dfrac{x+3}{x+1} + \dfrac{x+2}{1-x} + \dfrac{x^2+3}{x^2-1}$

Solution. The denominator of the second fraction should be rearranged.

$$\frac{x+2}{1-x} = \frac{x+2}{-x+1} = -\frac{x+2}{x-1}$$

Then $\dfrac{x+3}{x+1} - \dfrac{x+2}{x-1} + \dfrac{x^2+3}{(x+1)(x-1)}$

$$= \frac{(x+3)(x-1)}{(x+1)(x-1)} - \frac{(x+1)(x+2)}{(x+1)(x-1)} + \frac{x^2+3}{(x+1)(x-1)}$$

$$= \frac{(x+3)(x-1) - (x+1)(x+2) + (x^2+3)}{(x+1)(x-1)}$$

$$= \frac{(x^2+2x-3) - (x^2+3x+2) + (x^2+3)}{(x+1)(x-1)}$$

$$= \frac{x^2+2x-3-x^2-3x-2+x^2+3}{(x+1)(x-1)}$$

$$= \frac{x^2-x-2}{(x+1)(x-1)} = \frac{(x-2)(x+1)}{(x+1)(x-1)} = \frac{x-2}{x-1}$$

EXERCISES [C]

Combine:

1. $\dfrac{x+y}{x-y} + \dfrac{1}{x+y} - \dfrac{x^2+y^2}{y^2-x^2}$
2. $\dfrac{a^2+5a+1}{a^2-a-20} + \dfrac{a+3}{5-a}$
3. $\dfrac{4a}{a-1} + \dfrac{3a}{4-4a} - \dfrac{a^2-6}{a^2-1}$
4. $\dfrac{x-3}{x^2-25} + \dfrac{6}{x+5} + \dfrac{3}{5-x}$

Simplify:

5. $\dfrac{x^2 - 2x - 8}{x^2 - x^4} \times \dfrac{x^2 - 4x + 4}{5x^2 - 30x + 40}$

6. $\dfrac{abx^2 - aby^2}{x^2 + 2xy + y^2} \div \dfrac{x^2 - 2xy + y^2}{y^2 - xy}$

PRACTICE TEST

See how many of these twenty exercises you can do correctly in 35 minutes:

1. $(-5a)(-3a^4) = ?$
2. $(-2x)^3 = ?$
3. $(5x - 1)(x^2 + 3x - 3) = ?$

Factor:

4. $x^2 - 7xy + 12y^2$
5. $8x^2 - 2x - 15$
6. $8a^2 - 12ab$
7. $25c^2 - d^2$
8. $2ay^2 + 14ay + 24a$

9. Solve $\frac{3}{5}c - \frac{1}{4}c = 1$

10. $F = \frac{9}{5}C + 32$. Find F when $C = -40$.

Simplify:

11. $4a(a - b) - a^2$
12. $2a - 3(a + 1)$
13. $\dfrac{4a}{8a^3}$
14. $\dfrac{3x^2 - 3x}{x^2 - 1}$
15. $\dfrac{c^2 + c - 2}{c^2 + 2c} \cdot \dfrac{c^2}{c - 1}$
16. $\dfrac{y^2 - 5y + 6}{y + 4} \div (y - 3)$
17. $\dfrac{2a + 3b}{4b} - \dfrac{a + 5b}{6b}$
18. $\dfrac{7x}{9} - \dfrac{5x - 2}{6}$
19. $\dfrac{y + 2}{2y - 18} - \dfrac{3y - 1}{y - 9}$
20. $\dfrac{x + 2}{x - 2} + \dfrac{x - 2}{x + 2} - \dfrac{2x^2 + 2y^2}{x^2 - 4}$

Changing Mixed Numbers to Fractions [A]

A mixed number consists of a whole number and a fraction. For example, $3\frac{2}{5}$, $1\frac{2}{3}$, and $a + \dfrac{1}{b}$ are mixed numbers. Notice that the plus sign is omitted in the arithmetic mixed numbers, but is not omitted in the algebraic mixed number. A mixed number is changed into a simple fraction by adding the whole number and the fraction.

Example 1. $4 + \frac{3}{5} = \frac{4}{1} + \frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$

Example 2. $a + \frac{b}{c} = \frac{a}{1} + \frac{b}{c} = \frac{ac}{c} + \frac{b}{c} = \frac{ac + b}{c}$

Example 3.

$$\begin{aligned} x + 3 - \frac{2x}{x-1} &= \frac{x+3}{1} - \frac{2x}{x-1} = \frac{(x+3)(x-1)}{x-1} - \frac{2x}{x-1} \\ &= \frac{(x+3)(x-1) - 2x}{x-1} = \frac{x^2 + 2x - 3 - 2x}{x-1} \\ &= \frac{x^2 - 3}{x-1} \end{aligned}$$

Some of the steps in the examples above can be done mentally.

EXERCISES

A

Change the following mixed expressions into common fractions:

1. $7 + \frac{1}{10}$

2. $8\frac{3}{4}$

3. $\frac{a}{b} + 1$

4. $\frac{c}{d} - 1$

5. $4x - \frac{5}{x}$

6. $2x - 3 + \frac{1}{x}$

7. $2a - 3 + \frac{3}{a}$

8. $c + 1 - \frac{1}{c^2}$

9. $2p + 4 - \frac{1}{3p}$

10. $h^2 + h + \frac{1}{h}$

11. $4m^2 - 2m + 3 + \frac{1}{3m}$

12. $7x^2 + 3x + 4 - \frac{1}{2x}$

13. $y^2 + y - 3 + \frac{3}{4y}$

14. $x^2 - xy + y^2 + \frac{2}{xy}$

15. $c + d + e - \frac{1}{cd}$

HUGHES

B

16. $a - b + \frac{b^2}{a + b}$

19. $x^2 + xy + y^2 + \frac{y^3}{2x - y}$

17. $2x - y - \frac{-y^2}{x + y}$

20. $a^2 - ab + b^2 - \frac{b^3}{a + b}$

18. $2a^2 + 3a + 1 + \frac{a^2 + 1}{2a - 1}$

21. $x^2 + xy - y^2 - \frac{x^3 + 2y^3}{x - 2y}$

Changing Fractions into Mixed Expressions [A]

The fraction $\frac{14}{3}$ can be changed into the mixed number $4\frac{2}{3}$ by dividing the numerator 14 by the denominator 3. The fraction $\frac{12x^2 - 8x + 3}{4x}$ can be changed into the mixed expression $3x - 2 + \frac{3}{4x}$ by dividing $12x^2 - 8x + 3$ by $4x$.

Example 1. Change $\frac{9y^2 + 6y - 2}{3y}$ into a mixed expression.

Solution. $\frac{9y^2 + 6y - 2}{3y} = 3y + 2 - \frac{2}{3y}$

EXERCISES [A]

Change into mixed expressions:

1. $\frac{x^3 + x^2 + 1}{x}$

2. $\frac{15x^2 - 20x + 2}{5x}$

3. $\frac{12c^2 - 8c - 3}{2}$

4. $\frac{m^2n^2 + 6mn + 4}{mn}$

5. $\frac{4x^2y^2 - 6xy - 5}{2xy}$

6. $\frac{7a^3b - 8a^2b + 7}{a^2b}$

7. $\frac{10y^4 - 8y^3 + 6}{2y^3}$

8. $\frac{24x^3 - 16x^2 + 8}{8x}$

Example 2. Change $\frac{2x^2 + 2x - 7}{2x - 4}$ into a mixed expression.

Solution.

$$\begin{array}{r|l} & \quad\;\; x + 3 \\ \cline{2-2} 2x - 4 & 2x^2 + 2x - \;\;7 \\ & \underline{2x^2 - 4x} \\ & \qquad\;\; 6x - \;\;7 \\ & \qquad\;\; \underline{6x - 12} \\ & \qquad\qquad\quad 5 \end{array}$$

$$\frac{2x^2 + 2x - 7}{2x - 4} = x + 3 + \frac{5}{2x - 4}$$

EXERCISES [A]

Change into mixed or integral expressions:

1. $\frac{9y^4 + 15y^3 + y^2}{3y^2}$
2. $\frac{2x^2 + 3x + 2}{x + 2}$
3. $\frac{5a^2 - 7a + 1}{a - 1}$
4. $\frac{x^3 + 1}{x - 1}$
5. $\frac{x^3 - 1}{x - 1}$
6. $\frac{3x^2 + 4xy - 4y^2 + 2}{x + 2y}$

Complex Fractions [B]

A *complex fraction* is one which has one or more fractions in its numerator, in its denominator, or in both its numerator and denominator. Examples of complex fractions are

$$\frac{\frac{1}{2}}{\frac{3}{4}}, \quad \frac{\frac{a+b}{3}}{7}, \quad \text{and} \quad \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$$

There are two methods used in changing complex fractions to simple fractions: 1. Multiply both numerator and denominator of the fraction by the lowest common denominator of all fractions appearing in the numerator and denominator. 2. Divide the numerator by the denominator. If the second method is used, the numerator and denominator must be simple fractions.

Example. Simplify: $\dfrac{\dfrac{a^2-b^2}{6}}{\dfrac{a+b}{3}}$

Solution 1. The L.C.D. is 6.

$$\frac{6\left(\frac{a^2-b^2}{6}\right)}{6\left(\frac{a+b}{3}\right)} = \frac{a^2-b^2}{2(a+b)} = \frac{\overset{1}{\cancel{(a+b)}}(a-b)}{2\underset{1}{\cancel{(a+b)}}} = \frac{a-b}{2}$$

Solution 2.

$$\frac{\frac{a^2-b^2}{6}}{\frac{a+b}{3}} = \frac{a^2-b^2}{6} \div \frac{a+b}{3} = \frac{\overset{1}{\cancel{(a+b)}}(a-b)}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{a+b}}} = \frac{a-b}{2}$$

EXERCISES [B]

Change to simple fractions:

1. $\dfrac{2\frac{2}{3}}{\frac{4}{5}}$

2. $\dfrac{3}{3\frac{1}{3}}$

3. $\dfrac{6-\frac{4}{5}}{1+\frac{3}{10}}$

4. $\dfrac{10-\frac{1}{2}}{1+\frac{1}{5}}$

5. $\dfrac{\frac{a}{b}+1}{\frac{a}{b}-1}$

6. $\dfrac{\frac{x}{y}-1}{\frac{x}{y}+1}$

7. $\dfrac{a}{1-\frac{1}{a}}$

8. $\dfrac{\frac{1}{x+y}}{\frac{x}{y}}$

9. $\dfrac{1-\frac{x}{y}}{1-\frac{x^2}{y^2}}$

10. $\dfrac{\frac{x^2-y^2}{3}}{\frac{x+y}{5}}$

11. $\dfrac{4}{\frac{a+b}{a-b}}$

12. $\dfrac{m+\frac{m}{n}}{\frac{1}{n}+\frac{1}{n^2}}$

13. $\dfrac{\frac{a+b}{a-b}}{\frac{a-b}{a+b}}$

14. $\dfrac{\dfrac{x^2}{3}-3}{\dfrac{x}{6}-\dfrac{1}{2}}$

15. $\dfrac{a-\dfrac{25}{a}}{a+5}$

16. $\dfrac{a-2+\dfrac{3}{a}}{1+\dfrac{1}{a}}$

17. $\dfrac{1-\dfrac{2}{c}-\dfrac{3}{c^2}}{1+\dfrac{1}{c}}$

18. $\dfrac{\dfrac{2\,x}{a-3\,b}}{\dfrac{2\,xy}{2\,a-6\,b}}$

19. $\dfrac{\dfrac{1}{x}+1}{\dfrac{1}{x^2}-1}$

WORD LIST

Can you spell all these words?

abscissa (p. 287)
binomial (p. 13)
coefficient (p. 10)
digit (p. 356)
extremes (p. 228)
integer (p. 135)
ordinate (p. 287)
origin (p. 286)
polynomial (p. 13)
quotient (p. 256)
ratio (p. 226)
trinomial (p. 13)

CHAPTER REVIEW

A

1. Name two errors pupils may make in combining fractions.

2. Why do we factor the denominators when combining fractions?

Reduce to lowest terms:

3. $\dfrac{4\,abc}{6\,a}$

4. $\dfrac{4\,a-4\,b}{a^2-b^2}$

5. $\dfrac{8\,x^2y^3}{12\,xy^5}$

6. $\dfrac{mn}{m^2}$

7. $\dfrac{x^2-5\,x}{x^2-25}$

8. $\dfrac{12\,a^3b}{20\,ab^2}$

9. $\dfrac{2\,x^2+x-15}{3\,x^2+8\,x-3}$

10. $\dfrac{3\,a^2b-3\,ab}{12\,a^2b^2+30\,ab^2}$

11. $\dfrac{4\,ay-4\,cd}{5\,a^2y-5\,acd}$

Do as indicated:

12. $\frac{2a}{3} + \frac{5b}{6}$

13. $1 + \frac{a}{c}$

14. $\frac{a+7}{a-2} + \frac{2a-4}{a+7}$

15. $\frac{x+y}{x^2-y^2} - \frac{2x-y}{x^2-y^2}$

16. $\frac{c^2}{c^2-d^2} \div \frac{c}{c-d}$

17. $\frac{x}{x+1} \div \frac{x^2}{x^2-1}$

18. $\frac{2x}{2x+x^2} \times \frac{x^2-4}{x^2}$

19. $\frac{a}{b} \div \frac{2a^2}{b^2}$

20. $\frac{b^2-2b-24}{5} \div \frac{b^2+8b+16}{10}$

Change to simple fractions:

21. $3\frac{2}{5}$

22. $a - \frac{1}{a}$

23. $x - 1 + \frac{x}{2}$

24. $x - 1 - \frac{1}{x}$

25. $c + d - \frac{3}{c}$

26. $a - 3 + \frac{1}{3}$

Change into integral expressions:

27. $\frac{3125}{5}$

28. $\frac{100}{20}$

29. $\frac{256}{2}$

30. $\frac{x^2+x}{x}$

31. $\frac{a^2-ab-2b^2}{a-2b}$

32. $\frac{x^2+xy}{x+y}$

Change into mixed expressions:

33. $\frac{10x^2-15x+3}{5}$

34. $\frac{3ab-3c+1}{3}$

35. $\frac{x^2+7x+1}{x+1}$

36. $\frac{y^2-3y+4}{y-2}$

B

Reduce to lowest terms:

37. $\frac{2c^2-cd-d^2}{3cd-5c^2+2d^2}$

38. $\frac{x^4-1}{5x^2-5}$

39. $\frac{2a^2-2b^2}{a^2+ab-2b^2}$

40. $\frac{3xy-3xy^2}{3x-6xy-9xy^2}$

41. Combine: $\frac{x-4}{x+4}+\frac{x+4}{x-4}-\frac{2x^2}{x^2-16}$

42. Simplify: $\dfrac{a+1-\frac{a}{6}}{a-1-\frac{a}{2}}$

43. Divide: $\frac{y^2-y}{x^4-x^2}\div\frac{y^3+3y^2-4y}{2x^2-x-1}$

GENERAL REVIEW

A

1. Simplify: $2ab+3cd+4ab-c^2-4cd$

If $r=2$, $s=3$, and $t=-5$, find the value of

2. $\frac{r^2+s^2+t}{2r}$

3. $\frac{3r+6r^2}{5}$

4. $\frac{t^3+r^3-3}{t}$

5. $\frac{2r^2+s^2}{s+t}$

6. $(rst)^3$

7. $st+tr$

Solve:

8. $x-3=2$

9. $x+6=0$

10. $7x-1.1=.3$

11. $40x-5=0$

12. $8c-3=5c+9$

13. $5(x+2)=37-4x$

14. $\frac{x}{20}+\frac{x}{3}-\frac{x}{6}=13$

15. $30(m-2)+\frac{m}{3}=\frac{5m+481}{16}$

16. $.5y+8=20.5$

17. $\frac{1}{4}(12x-16)-\frac{1}{2}(8x-4)=0$

18. $\frac{2(7x-1)}{7}-\frac{3(3x+5)}{14}=0$

Solve for x:

19. $ax-b=0$

20. $ax-b=c$

21. $a+bx=c$

22. $\pi rx=16$

23. $a(7-x)=5$

24. $5cx-3=7-5cx$

Solve by factoring:

25. $x^2 = x + 6$ **27.** $4x^2 - 12x + 9 = 0$

26. $x^2 = -4x$ **28.** $b^2 - 10b = 56$

29. Copy and complete the following table for $x - 3y = 6$:

x	−8	−6	−4	−2	0	2	4	6
y	?	?	?	?	?	?	?	?

30. Find the following products mentally:

a. $(a - b)(a + b)$ **d.** $(2x - 5)(3x - 4)$

b. $(a + 3)(a + 3)$ **e.** $(x - 3y)^2$

c. $2c^2(c^2 - c + 1)$ **f.** $2\pi r(r + h)$

31. Solve for x and y:
$$3x - 4y = 11$$
$$2x + 5y = -8$$

32. How many pounds of 80-cent tea must be mixed with 30 pounds of 90-cent tea to make a mixture of 85-cent tea?

33. An agent sells hose at 78 cents a pair, making a profit of 20% based on the cost. If there is no overhead, what is the per cent of profit based on the selling price?

34. If school paper costs x dollars per thousand sheets, how many dollars do y sheets cost?

35. Factor into prime factors:

a. $x^3 - x$ **c.** $4p^2 - 16p + 12$

b. $3x^2 - 3x - 60$ **d.** $x^4 - y^4$

36. Solve for x: $2bmx + 1 = x + 4b^2m^2$

37. Solve for x: $(x - a)(x - b) + a = x^2$

38. Solve for y: $y^2 - 3ay + 2a^2 = 0$

39. Solve for m: $2m^2 + 5cm = 12c^2$

40. Solve for m and n:
$$\frac{m}{4} - \frac{n}{6} = \frac{1}{2}$$
$$\frac{m}{3} - \frac{n}{2} = -1$$

41. The perimeter of a rectangle is 204 feet, and the length is 22 feet more than the width. What are the dimensions of the rectangle?

42. $i = prt$. What principal in $2\frac{1}{2}$ years at 5% will produce $52.50 simple interest?

43. A man invested $3000, part at 6% and the remainder at 5%. The annual income from the 6% investment was $48 greater than the income from the 5% investment. How much was invested at each rate?

B

44. Solve:
$$\frac{x+y}{2} - \frac{x-3y}{3} = 10$$
$$\frac{x-y}{5} + \frac{y-3x}{2} = 6$$

45. Solve: $x - \dfrac{x-2}{3} = 6$

46. Divide: $\dfrac{2b^2-8}{4b^2} \div \dfrac{3b+6}{2b}$

47. Combine: $\dfrac{1}{y-7} - \dfrac{y+2}{y^2+2y-35}$

48. Divide: $12y^3 + 2y - 30y^2 - 5$ by $2y - 5$

49. A customer bought a table for $22.05 at a 10%-discount sale. What was the original price of the table?

50. Solve $S = \dfrac{a+l}{2}$ for l.

51. If you should make 12 hits in 42 times at bat, what batting average would you have?

52. Solve $x(x-3) + 1 - x(x-5) = 0$.

CHAPTER TEST [A]

Reduce to lowest terms:

1. $\frac{2a^3}{4a}$

2. $\frac{x}{xy}$

3. $\frac{6(a-b)}{18(a-b)}$

4. $\frac{2m}{m(m+1)}$

5. $\frac{7x-21}{8x-24}$

6. $\frac{a^2-2ab+b^2}{a^2-b^2}$

Multiply or divide as indicated:

7. $\frac{a}{8} \div \frac{a}{4b}$

8. $\frac{x}{y} \cdot \frac{y^2}{3x}$

9. $\frac{2a}{5b} \cdot \frac{b^2-b}{2ab-2a}$

10. $\frac{x^2-1}{y^3} \cdot \frac{y^2}{x^2-2x+1}$

11. $\frac{x^2-5x+6}{x^2+8x+12} \cdot \frac{x^3+6x^2}{x^2+3x}$

12. $\frac{m^2-6m+5}{m^2-1} \div \frac{m^2-5m}{m^2+4m+3}$

Combine:

13. $\frac{2}{a}+\frac{3}{a}$

14. $\frac{a}{c}+\frac{b}{c}$

15. $\frac{x}{y}+1$

16. $\frac{2a}{3}-\frac{a}{6}$

17. $\frac{x+y}{3}+\frac{x-y}{4}$

18. $\frac{3x^2-6x+1}{8x^2}-\frac{x-3}{4x}$

19. $\frac{3}{a+2}+\frac{5}{a-2}$

20. $\frac{b}{b-3}-\frac{b}{b+2}$

CHAPTER TEST [B]

Reduce to lowest terms:

1. $\frac{3x^2y^3}{21y^3z^2}$

2. $\frac{6abc}{4ab+4bc}$

3. $\frac{1-x^2}{4x^2+x-3}$

Change to mixed numbers or integral expressions:

4. $\frac{x^3+2x^2+1}{x}$

5. $\frac{a^3+1}{a+1}$

6. $\frac{5a^2-7a+1}{a-2}$

Change to simple fractions:

7. $c + 1 - \frac{1}{c}$

8. $x - y + \frac{y^2}{x + y}$

9. $\dfrac{1 - \frac{y}{x}}{1 + \frac{y}{x}}$

Multiply or divide as indicated:

10. $3a \cdot \frac{a + 2}{6a^2 + 12a}$

11. $\frac{ax + ay}{x - 3y} \div \frac{ax^2 - ay^2}{x^2 - 4xy + 3y^2}$

Combine:

12. $\frac{x}{y^2} - \frac{y}{x^2}$

13. $\frac{x + 1}{1 - x} + \frac{x - 1}{1 + x}$

14. $\frac{3}{x + 4} - \frac{x - 3}{2x^2 + 7x - 4}$

15. $\frac{4}{a - 2} - \frac{3a + 2}{a^2 - 4}$

INSURANCE

WHEN WE THINK of insurance, we generally think of protection against sudden or unexpected losses, such as those caused by fire, storm, or accident, or against unforeseen expenses such as might arise in the event of sickness or death. This protection is furnished by an insurance company in return for payment of a relatively small amount of money, called a premium. Each premium is much smaller than the amount of protection which the person who pays it receives. However, a fire-insurance company, for example, knows that only a very few of the people who pay premiums to it will suffer loss from fire. The company will collect a great many small premiums during the year, but will be called upon to pay only a small number of claims. The number of dollars paid by a company in a year to persons who have lost property by fire should approximately balance the number of dollars paid to it in premiums, with due allowance for overhead and profit.

Insurance companies make extensive use of mathematics. They must determine the amount of premium which must be charged for insurance protection. These premiums are based upon the past experience of a great many companies, or upon government statistics of the number of fires, accidents, or deaths in relation to the total amount of property, or the total number of automobiles or people in the area which the company serves. In addition to the protection they afford, most life-insurance policies have cash and loan values, as well as options by which the policy may be discontinued and its cash value used to purchase a smaller paid up policy or to extend the original policy for a certain period of time without further payment. Many policy owners also receive yearly dividends. Tables of these premiums, values, and dividends are compiled by expert mathematicians called actuaries, of whom each insurance company has one or more. However, not all of the mathematical work in an insurance company is done by actuaries. A great deal of it is done by clerks who, working from the actuaries' tables, determine rates, values, and dividends for the thousands (in some large companies, millions) of individual policies. These clerks must be accurate with figures, and have a good working knowledge of high-school mathematics.

Courtesy John Hancock Mutual Life Insurance Company

Both Men and Women Are Employed in This Actuarial Department of a Large Insurance Company

CHAPTER XIV

Fractional Equations

How to Solve Fractional Equations [A]

Not all equations containing fractions are fractional equations. In a fractional equation the letter representing the unknown number appears in a denominator. The equation $\frac{x}{2} - \frac{x}{4} = 1$ is an equation containing fractions, but is not a fractional equation. The equation $\frac{2x - 3}{x} + 1 = 2$ is a fractional equation.

A fractional equation is solved as any other equation containing fractions. You should keep in mind the following steps for solving an equation containing one unknown letter:

*1. If the equation contains fractions, clear it of fractions by multiplying both members by the L.C.D.

2. Remove parentheses.

3. If it is a first-degree equation, transpose so that all terms containing the unknown letter are in the left member and all other terms are in the right member.	3. If it is a second-degree equation, transpose so that all terms are in the left member.
4. Simplify each member.	4. Factor the left member.
5. Divide each member of the equation by the coefficient of the unknown letter.	5. Set each factor containing the unknown letter equal to zero and solve the resulting equations.

---

* In some cases it is advisable to remove parentheses before clearing of fractions. This is often the case in decimal equations.

The following is an equation containing fractions, but it is not a fractional equation.

Example 1. Solve $\frac{x}{3} + \frac{2x - 3}{5} = 6$

Solution. The L.C.D. is 15.

$$15\left(\frac{x}{3}\right) + 15\left(\frac{2x-3}{5}\right) = 15(6)$$

$$5x + 6x - 9 = 90$$

$$5x + 6x = 90 + 9$$

$$11x = 99$$

$$x = 9$$

Check. Does $\frac{9}{3} + \frac{18 - 3}{5} = 6?$

Does $3 + 3 = 6?$ Yes.

EXERCISES [A]. REVIEW

Solve, and check as directed by your teacher:

1. $\frac{2x}{3} = 6$
2. $\frac{7x}{8} = 16$
3. $\frac{3}{5}x = 10$
4. $\frac{2x - 5}{3} = 15$
5. $\frac{3x - 3}{4} = 3$
6. $\frac{1}{3}x = \frac{2}{5}$
7. $\frac{3x + 1}{5} = 7$
8. $\frac{4x}{9} = \frac{2}{7}$
9. $\frac{6x}{8} = \frac{4}{7}$
10. $\frac{x}{2} - \frac{x}{3} = 5$
11. $\frac{2x}{3} = 5 - \frac{x}{4}$
12. $\frac{2y - 1}{2} = \frac{4y - 1}{7}$
13. $\frac{x - 7}{2} + \frac{3x - 5}{4} = -7$
14. $\frac{x - 7}{2} - \frac{x - 15}{3} + \frac{1}{6} = 0$
15. $\frac{x - 3}{3} + 3 = \frac{x + 4}{5}$
16. $\frac{3x - 2}{10} + \frac{1}{5} = \frac{2x - 7}{5}$
17. $\frac{m + 6}{9} - 7\frac{2}{9} = \frac{5m - 1}{3}$
18. $\frac{x + 5}{4} - \frac{2x - 1}{5} = \frac{23}{20}$
19. $\frac{y - 1}{6} + \frac{2}{3} = 11\frac{1}{2}$

20. $\dfrac{c-6}{2}-\dfrac{2(6-c)}{3}=\dfrac{-7}{6}$

21. $\dfrac{8b-1}{3}=3-\dfrac{10b+7}{6}$

In the following decimal equations remember that $.2=\frac{2}{10}$, $.25=\frac{25}{100}$, and $.004=\frac{4}{1000}$.

22. $.25x=12$

23. $.04m=8$

24. $.5y+.2y=1.4$

25. $.03x-.05=.1$

26. $.05m-.14=.02$

27. $4+.02p=.25p-.6$

28. $.1n-.04=.004-.3n$

29. $3(2x-3)+8.1=0$

30. $.05(700-3y)=34.25$

31. $.2(6x+7)-4.5=.1(2x-1)$

32. $8(1.5c-3)-.4(c+3)=-4$

33. $.07(x-6)+.3(2x+9)=4.96$

The following is a fractional equation.

Example 2. Solve $\dfrac{21}{2y}-4=\dfrac{9}{y}$.

Solution. The L.C.D. is $2y$.

$$2y\left(\frac{21}{2y}\right)-2y(4)=2y\left(\frac{9}{y}\right)$$

$$21-8y=18$$

$$-8y=18-21$$

$$-8y=-3$$

$$y=\tfrac{3}{8}$$

EXERCISES [A]. FRACTIONAL EQUATIONS

Solve and check:

1. $\dfrac{2}{x}+\dfrac{3}{x}=5$

2. $\dfrac{7}{x}-\dfrac{5}{x}=1$

3. $1=\dfrac{2}{y}$

4. $\dfrac{5}{c}=3$

5. $\dfrac{3}{h}=-1$

6. $4-\dfrac{1}{y}=0$

7. $\dfrac{1}{y}+6=-2$

8. $\dfrac{1}{x}-9=0$

9. $\dfrac{1}{3x}-7=4$

10. $\frac{3}{p} + \frac{1}{5} = 5$

11. $\frac{2}{b} + \frac{4}{b} - 3 = 0$

12. $\frac{2}{3x} + \frac{1}{2} = \frac{4}{x} + \frac{4}{3}$

13. $\frac{4}{x} - \frac{2}{x} = \frac{2}{3} - \frac{3}{x}$

14. $\frac{3}{2x} - 1 = -\frac{7}{2x} - \frac{3}{8}$

15. $\frac{50}{m} - 2 = \frac{10}{m}$

Example 3. Solve $\frac{x+3}{x-1} - \frac{3}{x} = 1$.

Solution. The L.C.D. is $x(x-1)$.

$$x\cancel{(x-1)}\frac{x+3}{\cancel{x-1}} - \cancel{x}(x-1)\frac{3}{\cancel{x}} = x(x-1)1$$
$$x^2 + 3x - 3x + 3 = x^2 - x$$
$$x^2 - x^2 + 3x - 3x + x = -3$$
$$x = -3$$

Check. Does $\frac{-3+3}{-3-1} - \frac{3}{-3} = 1$?

Does $0 + 1 = 1$? Yes.

Example 4. Solve $\frac{3x}{2x+1} - \frac{7}{2x^2+x} = \frac{3}{2}$.

Solution. All denominators should be factored.

Then $$\frac{3x}{2x+1} - \frac{7}{x(2x+1)} = \frac{3}{2}$$

The L.C.D. must contain all the factors found in the denominators in order to be exactly divisible by each denominator. Here the monomial factors of the denominators are 2 and x. The binomial factor is $2x + 1$. Hence the L.C.D. is $2x(2x + 1)$.

$$2x\cancel{(2x+1)}\frac{3x}{\cancel{2x+1}} - 2\cancel{x}\cancel{(2x+1)}\frac{7}{\cancel{x}\cancel{(2x+1)}} = \cancel{2}x(2x+1)\frac{3}{\cancel{2}}$$
$$6x^2 - 14 = 6x^2 + 3x$$
$$6x^2 - 6x^2 - 3x = 14$$
$$-3x = 14$$
$$x = -\frac{14}{3}, \text{ or } -4\tfrac{2}{3}$$

EXERCISES

A

Solve and check:

1. $\frac{6}{y+1}=3$
2. $\frac{12x}{x+3}=48$
3. $x-1=\frac{30}{x}$
4. $y-\frac{3}{2}=\frac{7}{y}$
5. $\frac{x-7}{x+1}+\frac{3x-1}{3}=0$
6. $\frac{x+1}{x-3}=3$
7. $\frac{3x-2}{5(x-2)}=\frac{2}{5}$
8. $\frac{y-5}{y-3}-1=\frac{2}{y}$
9. $\frac{c+2}{c-1}=\frac{c-3}{c+2}$
10. $\frac{3}{x}=\frac{2}{x-4}+3$
11. $\frac{14}{x-2}+\frac{12}{x+2}=9$
12. $\frac{4}{x-3}+\frac{6}{x+3}=\frac{2}{3}$
13. $\frac{3}{x+1}-\frac{x}{x-1}=\frac{3-x^2}{x^2-1}$
14. $\frac{y+1}{y+3}-\frac{y-2}{y-3}=\frac{2}{y-3}$
15. $\frac{4}{y}+\frac{2}{y^2+3y}=\frac{5}{y+3}$
16. $\frac{x}{x-4}-\frac{5x}{3x-12}+5=0$
17. $\frac{2}{x-1}+\frac{2}{x+1}=\frac{8}{x^2-1}$
18. $\frac{6}{y+1}+\frac{4}{y-1}=\frac{3}{y^2-1}$
19. $\frac{x+6}{x+4}+\frac{2}{x+1}=\frac{3x}{x+1}$
20. $\frac{4x-1}{x+3}=\frac{2}{x+3}+\frac{x^2+11}{3x+9}$
21. $\frac{2x+1}{x-1}-\frac{3x}{x+2}=\frac{5x-2}{x^2+x-2}$

15
12

B

Equations 22–26 reduce to linear equations and equations 27–31 reduce to quadratic equations. Solve:

22. $\frac{2x}{x+3}-\frac{3x-4}{2x+6}=\frac{5}{x+3}$
*23. $\frac{3x}{x^2}-\frac{5}{x+6}-\frac{1}{x^2+6x}=0$

---

* All fractions in equations should be reduced to lowest terms before clearing equation of fractions.

24. $\frac{4x}{x^2+5x} - \frac{3}{x^2+5x} = \frac{3}{x}$

25. $\frac{y+2}{y-3} + \frac{y+3}{y+2} = \frac{2y^2+7}{y^2-y-6}$

26. $\frac{6x^2-2x+5}{3x^2-2} = 2$

27. $\frac{4}{x-3} - \frac{4}{x} = \frac{1}{15}$

28. $\frac{5}{x} - \frac{5}{x+2} = \frac{1}{12}$

29. $\frac{7}{x+5} + \frac{4}{x} - \frac{3}{2} = 0$

30. $\frac{60}{x+3} - \frac{60}{x} + 1 = 0$

31. $\frac{3x-5}{x^2-3x+2} + \frac{2x}{x-1} = \frac{x+5}{x-2}$

C

32. $\frac{3}{x} + \frac{5}{2x} = \frac{x+27}{x+2}$

33. $\frac{3}{3x-1} + \frac{6}{4x-3} = \frac{5}{2x+1}$

34. $\frac{x}{x+1} - \frac{3x}{1-x} = \frac{55}{12}$

35. $\frac{2x-5}{x^2+3x+2} + \frac{x-6}{x^2-4} = \frac{3x+1}{x^2-x-2}$

Solving Formulas [A]

A formula is a literal equation which tells how the value of one unknown letter may be found by performing certain operations upon one or more letters of known values.

The formula $V = lwh$ states that the volume of a rectangular solid is found by multiplying its length, width, and height together. Let us solve this formula for l.

$$V = lwh$$

$$-lwh = -V$$

D_{-1} $$lwh = V$$

D_{wh} $$l = \frac{V}{wh}$$

This last equation is a formula for finding the length of a rectangular solid when its volume, width, and height are known. The two formulas $V = lwh$ and $l = \frac{V}{wh}$ express the same relationship of the length, width, height, and volume of the rectangular solid, although given in different form.

Example. Solve $v = \frac{b - u}{k}$ for b.

Solution.

$$v = \frac{b - u}{k}$$

M_k

$$k(v) = k\left(\frac{b - u}{k}\right)$$

or

$$kv = b - u$$

$$-b = -kv - u$$

D_{-1}

$$b = kv + u$$

This formula tells how to find b when k, v, and u are known. What does $v = \frac{b - u}{k}$ tell?

EXERCISES. SOLVING FORMULAS

A

1. Solve $V = lwh$ for h and express the resulting formula as a rule.

2. Solve $\pi = \frac{c}{2r}$ for r and make a rule for finding the radius of a circle when the circumference is given.

3. Solve $v = \frac{b - u}{k}$ for u.

4. Solve $d = \frac{n + 2}{p}$ for p; for n.

5. Solve $F = \frac{W}{R + 1}$ for R; for W.

6. Solve $M = \dfrac{t}{g - f}$ for f; for t.

7. Solve $T = \dfrac{1}{a} + t$ for t; for a.

B

8. Solve $g = \dfrac{T + Mf}{M}$ for M.

9. Solve $A = 2\,\pi r(r + h)$ for h.

10. Solve $W = \dfrac{E^2 t}{R}$ for R.

11. Solve $S = \dfrac{c - b}{b - a}$ for b.

12. Solve $K = \dfrac{Wv^2}{2\,g}$ for v^2.

13. Solve $W = T_1V - T_2V$ for V. (T_1 and T_2 are different variables. T_1 is read "T sub one" and T_2 is read "T sub 2.")

14. Solve $n = \dfrac{s}{w} - 1$ for s.

15. Solve $F = .327\,vd^2$ for v.

C

16. Solve $c = \dfrac{1}{c_1} + \dfrac{1}{c_2} + \dfrac{1}{c_3}$ for c_1.

17. Solve $Ft = \dfrac{Wat}{g}$ for a. Solve it for F. Can you solve it for t?

18. Solve $Ft = \dfrac{Wv^2}{gr}$ for r.

19. Solve $W = P\dfrac{2\,R}{R - r}$ for R.

20. Solve $A = 2\,\pi r^2 + 2\,\pi rh$ for π.

21. Solve $h = \dfrac{2\,A}{b + b'}$ for A and express the resulting formula as a rule.

EXERCISES. LITERAL EQUATIONS

A

Solve the following equations for x or y:

1. $\frac{a}{x} + b = 0$

2. $\frac{1}{x} + \frac{2}{x} = \frac{1}{a}$

3. $\frac{ax}{c} + \frac{bx}{c} = 1$

4. $\frac{x}{m} - \frac{x}{n} = 1$

5. $x - 1 = a + \frac{3x - 12}{a}$

6. $\frac{m}{2}(x - m) = \frac{m}{3}$

7. $\frac{2y + a}{2y - a} = 4$

8. $\frac{a - b}{cx} = d$

9. $\frac{a^2 - b^2}{x} = \frac{1}{2}(a - b)$

10. $\frac{3}{y} = \frac{2y + 2}{cy} + \frac{c - y}{y}$

B

Solve for x:

11. $\frac{x - 1}{x + 1} = \frac{m - n}{m + n}$

12. $\frac{3x}{c - d} - 2 = \frac{5x}{c}$

13. $\frac{1}{5(x - c)} = \frac{1}{4(c - d)}$

14. $\frac{2a}{x} + \frac{1}{2x} = \frac{4a + 1}{4a}$

15. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{x}$

16. $\frac{c}{b} - \frac{c^2}{bx} = \frac{2b - 3c}{x} + 1$

Solve for x and y:

17. $ax = by$
$abx + aby = (a + b)^2$

18. $mx - ny = 0$
$mx + ny = 2mn$

19. $5x + 3y = 8a + b$
$3x + 5y = 8a - 8$

20. $mx + ny = 2m^2$
$nx + my = 2n^2$

Simultaneous Equations [A]

In solving pairs of simultaneous equations, clear the equations of fractions and write them in standard form before trying to eliminate any unknown.

EXERCISES

A

Solve:

1. $$\frac{x}{3}+\frac{y}{2}=3$$
$$4x+3y=24$$

2. $$\frac{3x}{8}+\frac{2y}{5}=\frac{7}{2}$$
$$\frac{x}{8}-\frac{y}{5}+\frac{1}{2}=0$$

3. $$\frac{m}{2}+\frac{n}{7}=9$$
$$\frac{m}{7}+5=\frac{n}{2}$$

4. $$x+y=7000$$
$$.04x+.05y=310$$

5. $$\frac{x}{y}=-4$$
$$2x+3y=5$$

6. $$\frac{1}{x}-\frac{1}{y}=\frac{2}{xy}$$
$$\frac{x}{y}=\frac{1}{3}$$

B

Solve:

7. $$\frac{a+1}{b-1}=\frac{4}{3}$$
$$\frac{2a-3}{b+1}=1$$

8. $$\frac{x+1}{y+5}=\frac{6}{5}$$
$$\frac{x+1}{y+1}=\frac{5}{4}$$

9. $$\frac{4a-7}{5}-\frac{5b+3}{7}+7=0$$
$$a+3=4-\frac{5b-4}{7}$$

10. $$\frac{x-1}{y}=\frac{4}{5}$$
$$\frac{1}{x+y}+\frac{1}{x-y}=\frac{x+y+8}{x^2-y^2}$$

C

Solve:

11. $$\frac{2x}{3}+\frac{y}{3}=2a$$
$$\frac{3x}{4}+\frac{2y}{5}=\frac{61a}{20}$$

12. $$\frac{3x}{b}-\frac{y}{2b}=5$$
$$\frac{7x}{2b}-\frac{3y}{b}=-\frac{3}{4}$$

13. $$\frac{1}{x}+\frac{1}{y}=\frac{2m}{xy}$$
$$\frac{2}{x}-\frac{3}{y}=\frac{3n}{xy}$$

14. $$\frac{b}{a+y}-\frac{a}{b-x}=0$$
$$\frac{a}{b-y}-\frac{b}{a+x}=0$$

EXERCISES [A]. REVIEW

In adding fractions and in solving equations containing fractions, we first find the L.C.D. Some pupils become confused when adding fractions and solving equations containing fractions. Work and study carefully the following exercises.

1. Combine: $\frac{2a}{3} + \frac{5a}{6}$

2. Solve: $\frac{2a}{3} + \frac{5a}{6} = 1$

3. How are fractions combined?

4. How are equations cleared of fractions?

5. Combine: $\frac{6x+7}{5} - \frac{9}{2} - \frac{2x-1}{10}$

6. Solve: $\frac{6x+7}{5} - \frac{9}{2} - \frac{2x-1}{10} = 0$

7. Combine: $\frac{x-4}{9} - \frac{x+2}{6} - \frac{x}{3}$

8. Solve: $\frac{x-4}{9} - \frac{x+2}{6} = \frac{x}{3}$

9. What is a ratio? a proportion?

10. How many terms are there in a ratio? in a proportion?

11. If $\frac{a}{b} = \frac{c}{d}$, show that $ad = bc$. Complete: In a proportion the product of the means is --?--.

12. Use Exercise 11 to solve $\frac{2x-1}{3x+5} = \frac{2x+1}{3x-4}$.

Problems Whose Equations Contain Fractions [A]

Many problems in the following sets are solved by using fractional equations. Some of the equations are of the first degree and some are of the second degree. The problems whose equations are of the first degree and have two unknowns may be solved with one letter or with two letters, but all problems whose equations are of the second degree should be solved by one letter. Do you know why?

Remember that problems whose equations are of the second degree have two answers for each unknown quantity.

PROBLEMS

A

1. A number increased by its square is 30. What is the number?

2. Twice the square of a certain number, decreased by five times the number, gives 12 as a result. Find the number.

3. The sum of a number and its reciprocal is $3\frac{1}{3}$. What is the number? (The reciprocal of 7 is $\frac{1}{7}$, and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.)

4. A number exceeds its reciprocal by $3\frac{3}{4}$. What is the number?

5. A number increased by three times its reciprocal is $9\frac{1}{3}$. Find the number.

6. The sum of two numbers is 8. If 3 is added to four times the smaller and the result is divided by the larger, the quotient is 3. What are the numbers?

7. A man lost $\frac{3}{4}$ of his money and then lost \$500 more. He then had $\frac{3}{16}$ of what he had at first. How much had he at first?

8. The difference of two numbers is 5. If three times the smaller is divided by the larger, the quotient is 1 and the remainder is 9. What are the numbers?

Let $x =$ the smaller number

Then $x + 5 =$ the larger number

$$\frac{3x}{x+5} = 1 + \frac{9}{x+5}$$

Complete the solution.

9. The numerator of a certain fraction exceeds the denominator by 3, and the value of the fraction is $\frac{4}{3}$. Find the fraction. (First find the numerator and denominator and then write the fraction.)

10. The denominator of a certain fraction is 2 greater than the numerator, and the value of the fraction is $\frac{4}{5}$. Find the fraction.

11. The numerator of a certain fraction is 4 less than its denominator. If each term of the fraction is increased by 1, the value of the fraction is $\frac{2}{3}$. Find the fraction.

12. The denominator of a certain fraction exceeds the numerator by 9. If the numerator is increased by 4, the value of the resulting fraction is $\frac{2}{3}$. What is the fraction?

13. Two numbers have a difference of 6. What are the numbers if the smaller is $\frac{3}{4}$ of the larger?

14. Mr. Jones has $2.90, consisting of nickels and dimes. If he has 36 coins in all, how many of each kind has he?

15. A boy has 10 coins, consisting of nickels and dimes. If the value of the nickels in cents is divided by the number of dimes, the quotient is $7\frac{1}{2}$. How many coins of each kind has he?

16. The width of a rectangle is $\frac{3}{4}$ of its length, and its perimeter is 126 feet. Find its dimensions.

17. The base of a rectangle is 4 feet longer than the altitude. Find the dimensions of the rectangle if its area is 45 square feet.

18. What are the dimensions of a rectangular flower bed containing 40 square feet if it takes 26 feet of fence to surround it?

19. Dick is half as old as Bill. Four years ago twice Dick's age added to Bill's age was 32 years. How old is each?

20. One half of Henry's age added to one third of Robert's age equals 11 years. In 6 years one third of Henry's age added to one seventh of Robert's age will equal 9 years. What are their present ages?

21. Frank can mow a yard in 2 hours less time than Herman. Together they can mow the lawn in $1\frac{7}{8}$ hours. How many hours does it take each working alone to mow the lawn?

22. Two men working together can paint a house in 6 days. If one man working alone can paint the house in 10 days, how long will it take the other man working alone to paint it?

23. What number increased by 20% of itself equals 74?

24. What number decreased by 25% of itself equals 69?

25. A merchant sold a dress costing \$2.25 for \$4.95. What was his per cent of gain based upon the cost? What was his per cent of gain based upon the selling price?

26. In running a race, one sprinter, who runs $8\frac{1}{3}$ yards a second, requires two more seconds than another sprinter, who runs 10 yards a second. What is the distance of the race?

27. A train runs between two cities 240 miles apart. If the train were to increase its speed 12 miles an hour, its running time would be lessened by one hour. What is its usual rate?

B

28. The sum of two fractions is $\frac{7}{10}$ and the smaller is $\frac{4}{5}$ of the larger. What are the fractions?

29. If each side of a square is decreased by 5 inches, the area of the square thus formed is 165 square inches less than that of the original square. Find the side and area of the original square.

30. The denominator of a certain fraction exceeds the numerator by 1. The reciprocal of the fraction exceeds the fraction by $\frac{7}{12}$. Find the original fraction.

31. The sum of the numerator and the denominator of a fraction is 8. If the numerator is doubled and the denominator is increased by 1, the value of the resulting fraction is 1. Find the fraction.

32. Two pipes together can fill a swimming pool in 4 hours 48 minutes. How long will it take each pipe alone to fill the pool if the difference of their times is 4 hours?

33. A bus driver because of snow was forced to drive at 10 miles an hour less than his usual speed, and he arrived 2 hours late at his destination 315 miles away. Find his usual rate.

34. A man made a trip of 170 miles, going the first half of the distance at 35 miles an hour and the second half of the distance at 50 miles an hour. How long did it take him to make the trip?

35. If the man in Example 34 had made the trip by going half the time at 35 miles an hour and the rest of the trip at 50 miles an hour, how long would it have taken him to make the trip?

36. In a number of two digits, the tens' digit is four times the units' digit. If the difference of the digits is 6, what is the number?

37. The sum of the two digits of a certain number is 16. If the digits are interchanged, the number is decreased by 18. What is the number?

38. The sum of the two digits of a certain number is 6. If the number is divided by the number formed by interchanging the digits, the quotient is $\frac{5}{17}$. Find the original number.

C

39. Robert walked 6 miles into the country and ran back 2 miles an hour faster than he went out, making the round trip in $2\frac{1}{2}$ hours. What were his rates going and coming?

40. The tens' digit in a number of two digits exceeds the units' digit by 5. Find the number if it exceeds 10 times the tens' digit by 3.

41. An excavating machine can dig a basement in five days, and a team of horses can do the same work in forty days. An excavation is started by machine, but the machine breaks down after 3 days. How many days are needed to finish the work by team?

42. A boy sold his bicycle for $24. If the number of per cent gained, based on the cost, was equal to the number of dollars in the cost, what was the cost?

43. A baseball team has won 70 games and lost 60 games. How many of its remaining 20 games must it win in order to have a winning percentage of 52%?

44. A tank can be filled by one pipe in 8 hours and emptied by another in 12 hours. If both pipes are turned on when the tank is half full, how many hours will it take to fill the tank?

45. An iron bar of uniform cross section weighed 480 pounds. When the bar was made 4 feet longer by rolling it in a bar mill, it weighed 6 pounds less per linear foot. What was the original length of the bar?

Wind and River Problems [B]

Most of you know how difficult it is to swim or to row a boat up a swiftly moving stream, and how easy it is to swim or to row a boat downstream. When you are swimming downstream, your distance from the starting point is equal to the distance you actually swim plus the distance that you are carried by the current. When you are swimming upstream, your distance from the starting point is equal to the distance you actually swim less the distance you are carried back by the current.

ORAL EXERCISES [B]

1. If a motorboat can go 10 miles an hour on a calm lake, how fast can it go downstream on a river flowing 2 miles an hour? How fast can it go upstream on the river?

2. How fast can an airplane with a speed of 150 miles an hour go with the wind, when the wind is blowing 30 miles an hour? How fast can it go against the wind?

3. A motorboat can go x miles an hour in still water. How fast can it go downstream on a river flowing 3 miles an hour? How fast can it go up the river?

Example. A river's current is 4 miles an hour. What speed must a motorboat have in order to go 24 miles down the river and back in $2\frac{1}{2}$ hours?

Solution. Let $x =$ the rate of the boat in miles an hour in still water

Then $x + 4 =$ the rate in miles an hour downstream

and $x - 4 =$ the rate in miles an hour upstream

$$\frac{24}{x+4} + \frac{24}{x-4} = \frac{5}{2}$$

Complete the solution.

EXERCISES [B]

1. A man can row a boat 3 miles an hour in still water. How far down a river flowing 2 miles an hour can he go and return in his boat in 12 hours?

2. An aviator flies 240 miles with the wind in 2 hours. He returns flying against the wind in 3 hours. Find the rate of the airplane and of the wind.

3. An airplane whose rate in still air is 120 miles an hour can go 600 miles with the wind in the same time that it can go 360 miles against the wind. What is the velocity of the wind?

4. An airplane can go 100 miles with the wind in 30 minutes, but the return trip against the wind requires 40 minutes. Find the rate of the wind and of the airplane in still air.

5. A motorboat can go 24 miles upstream in $1\frac{1}{2}$ hours and return in 1 hour. Find the speed of the boat in still water.

6. An airplane flew from one city to another in forty minutes with the wind, and returned in fifty minutes against the same wind. If the rate of the wind was 12 miles an hour, how far apart were the cities?

7. With the wind an airplane can go 200 miles in 1 hour and 20 minutes. Against the wind it can go 135 miles in $1\frac{1}{2}$ hours. Find the speed of the plane and the velocity of the wind.

WORD LIST

At this time you should be able to spell and use each of the following words or groups of words correctly:

abscissa (p. 287)
binomial (p. 13)
co-ordinates (p. 287)
digit (p. 356)
fractional equation (p. 398)
literal equation (p. 352)
means (p. 228)
product (p. 9)
ratio (p. 226)
reciprocal (p. 409)

CHAPTER REVIEW [A]

Solve:

1. $\frac{y}{5}=\frac{2y-9}{3}-11$

2. $\frac{5y}{6}+y=\frac{y-6}{3}$

3. $x+\frac{1}{x}=2$

4. $\frac{3}{x}+\frac{x}{3}=2\frac{1}{2}$

5. $\frac{7-4n}{6}-\frac{23-8n}{12}=\frac{2-n}{n}$

6. $\frac{5x-7}{3}+\frac{x-11}{x-5}=4$

7. $\frac{y-1}{y+1}-\frac{y-2}{y-1}+\frac{1}{12}=0$

8. $\frac{x+4}{x+3}-\frac{x-1}{x}=\frac{7}{2x+6}$

9. $\frac{1}{y}-\frac{2}{y-1}=\frac{6}{2y^2+y}$

10. $.3y-.02y=7$

11. $3.2y+.4=.8y+10$

Solve for x:

12. $\frac{x-b}{x}=\frac{x}{x+a}$

13. $a-\frac{a}{2x}=\frac{b}{x}$

14. Solve $b=\frac{m}{n-p}$ for p.

15. Solve $U=\frac{MV}{M+N}$ for M.

16. A rectangle is 3 rods longer than it is wide. If the width is divided by the length, the quotient is $\frac{4}{5}$. Find the dimensions of the rectangle.

17. One number is $\frac{1}{9}$ as large as another. If 3 is added to each number, then one of the numbers will be $\frac{1}{5}$ as large as the other. Find the numbers.

18. John has 27 coins in nickels and quarters. If their value is $4.95, how many are there of each?

19. An airplane travels twice as fast as an automobile. The airplane requires one hour longer to go 400 miles than the automobile takes to go 160 miles. Find the rate of each.

20. In 8 years Mary Jane will be $\frac{7}{12}$ as old as Betty, but 2 years ago she was $\frac{2}{7}$ as old as Betty. How old is each girl now?

GENERAL REVIEW

A

Solve:

1. $5(x+4)+9=49$

2. $y(y-2)=24$

3. $x(2x-1)+6x=3$

4. $y(y-6)=2y-15$

5. $3(y+11)-7(y-2)=57$

6. $11c-(12-8c)=2(7c+9)$

7. Solve the set of equations:

$$\begin{aligned} 3a-4b&=9 \\ 3a+8b&=0 \end{aligned}$$

8. Find the volume of a pyramid whose base is a square 3 feet on a side and whose height is 5 feet. $\left(\text{Use } V=\frac{s^2h}{3}.\right)$

9. Find the approximate horsepower of a six-cylinder engine on a tractor if the diameter of each piston is 4 inches. The formula is $P=\frac{nd^2}{2.5}$, in which n is the number of cylinders and d is the diameter of each piston.

10. Using the formula $c=2\pi r$, copy and complete the following table. Use 3.14 for the value of π.

r	0	3.5	7	10.5	14	17.5
c	?	?	?	?	?	?

Graph the formula $c=2\pi r$.

11. Factor:

a. $\pi R-\pi$

b. $4x^2-8$

c. $4x^2+8x$

d. $6s^2+12s$

e. e^2-e-20

f. e^3-e^2-e

g. $4x^2-324$

h. $3x^2+13xy-10y^2$

12. Solve $A=hb$ for h; for b.

13. Solve $P=2l+2w$ for l.

14. Reduce $\frac{8a^2bc}{12ac^2}$ to lowest terms.

15. Reduce $\frac{x^2+x-6}{2x^2-x-21}$ to lowest terms.

Do as indicated:

16. $\frac{x^2 - 6x}{4} \cdot \frac{x}{x - 6}$

17. $\frac{x^2 + 4x}{2x + 1} \div \frac{x^2 - 16}{4x^2 - 1}$

18. $\frac{a}{b} + 1$

19. $\frac{y}{y^2 - 3y - 4} + \frac{1}{y - 4}$

20. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

21. $\frac{2y}{y^2 + y} - \frac{4y + 3}{y^2 - 1}$

22. What is the haul of each of two trucks if 4 trips of the smaller and 3 trips of the larger make a total haul of 23 tons, whereas 5 trips of the smaller and one trip of the larger make a total haul of 15 tons?

23. Three pineapples and four heads of cabbage sell for 39 cents, while four pineapples and three heads of cabbage sell for 45 cents. What is the selling price of each?

24. Solve graphically: $y - 3x = 9$
$x + 2y = 4$

If $a = 3$, $b = -1$, and $c = 0$, find the value of

25. $\frac{a^3b - bc}{9b}$

26. $(a - b)^3 + 4ab$

27. $abc - 4a^2$

28. $\frac{15 + a^2b^3}{-6}$

29. $a^2b^2 - 25$

30. $4a^2b^3 + (ab)^3$

Copy and complete the following sentences:

31. A quadratic equation is an equation of the --?-- degree.

32. A trinomial has --?-- terms.

33. In the division of fractions, the divisor is --?--.

34. Before fractions can be combined they must have the same --?--.

35. The numerator of the fraction $\frac{a}{b}$ is --?-- and the denominator is --?--.

B

36. Solve $S = \frac{1}{2}gt^2$ for g.

37. Solve $\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}$ for d_1.

38. Solve $s = \frac{n}{2}(a + l)$ for a.

39. Factor:

a. $x^4 - 81$

b. $a^3 - a$

c. $5x^2 - 10x + 25$

d. $6x^2 + 6x - 72$

e. $5a^2 - 5b^2$

f. $2bx - 6b^2x + 8bx^2$

g. $a^2y^2 - 5a^2y + 6a^2$

h. $15x^3y - 38x^2y^2 + 24xy^3$

40. Simplify: $\frac{3x^3 - 6x^2 - 24x}{2x^4 + 3x^3 - 2x^2}$

41. Combine: $\frac{x + y}{x - y} - \frac{x - y}{x + y} - \frac{4xy}{x^2 - y^2}$

42. Multiply: $\frac{2ax^2 - 2ay^2}{x^2 - 3xy + 2y^2} \cdot \frac{4x - 8y}{ax + ay}$

43. Divide $\frac{2a^3 + 2a^2b - 24ab^2}{5a^2 - 80b^2}$ by $\frac{4ab - 12b^2}{a^2 - 4ab}$.

Solve:

44. $\frac{2c}{2c - 3} - \frac{3c}{2c + 3} = \frac{15 - 2c^2}{4c^2 - 9}$

45. $\frac{y - 1}{y} = \frac{y^2 + y - 1}{y^2 - y} - \frac{y}{y - 1}$

46. $\frac{8}{n^2 - 1} = \frac{n}{n - 1} + \frac{n}{n + 1}$

47. A field can be plowed by a team of horses in 10 days and by a tractor in 3 days. How long will it take the team and the tractor together to plow the field?

48. Jennie can wash the dishes in 30 minutes and Harriet can wash them in 25 minutes. After Jennie has been working for 10 minutes, Harriet begins to help her. How long will it take both girls to finish the dishes?

CHAPTER TEST [A]

Solve:

1. $\frac{3}{5}x = 10$ 3. $\frac{2x - 3}{5} = 15$ 5. $\frac{2}{3}x - \frac{3}{5} = 0$

2. $\frac{3}{y} = 2$ 4. $\frac{3}{4x - 1} = 1$ 6. $\frac{2}{y} + \frac{4}{y} = 4$

7. $\frac{x - 7}{x + 3} - \frac{3x - 10}{3x} = 0$ 8. $\frac{x}{x - 6} - \frac{2x}{3x - 18} + \frac{4}{3} = 0$

9. Solve for x: $\frac{x}{a} + \frac{x}{b} = a + b$

10. The square of a certain number, decreased by $\frac{1}{3}$ of the number, gives 34 as a result. Find two numbers satisfying this condition.

11. The sum of two numbers is 12. If 5 times the larger is added to $\frac{1}{4}$ the smaller, the result is 41. Find the numbers.

CHAPTER TEST [B]

Solve:

1. $\frac{4}{y} - \frac{3}{y - 4} = \frac{7}{2}$ 2. $\frac{x + 4}{x - 2} = \frac{48}{x^2 - 4}$

3. $\frac{y + 6}{y - 5} + \frac{y + 2}{y - 2} = \frac{y^2 + 20}{y^2 - 7y + 10}$

4. $\frac{x + y}{2} + 2 = 0$

$\frac{8}{x + y} - \frac{8}{x - y} = \frac{x + y + 20}{x^2 - y^2}$

5. Solve for b: $m = \frac{a - b}{b - c}$

6. Solve for y: $\frac{2b}{y} + \frac{1}{2y} = \frac{4b + 1}{4b}$

7. The denominator of a certain fraction is 7 more than the numerator. If the numerator is increased by 8 and the denominator is decreased by 10, the result is 6 times the original fraction. What is the original fraction?

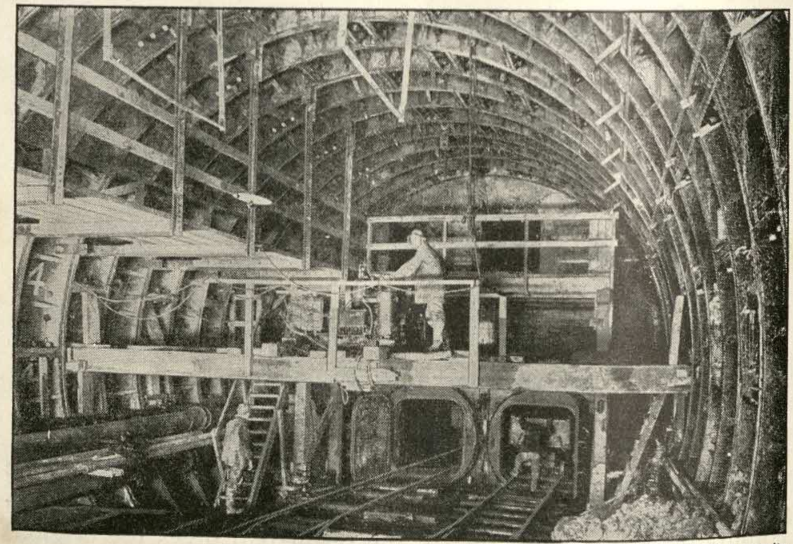
TUNNELS

TUNNELS, LIKE BRIDGES, are constructed to improve transportation. They are made through mountains in order to level roadways and to shorten distances. They are constructed under bodies of water to do the work of bridges and ferries.

The longest vehicular tunnel in the world is the Holland Tunnel, which passes under the Hudson River, connecting New York City with Jersey City.

This tunnel, consisting of two tubes, is more than one and a half miles long, and accommodates two lines of traffic each way. It is a busy thoroughfare, more than thirteen million cars passing through it each year.

The Lincoln Tunnel, lying between the Holland Tunnel and the George Washington Bridge, is the newest Hudson River crossing. Its construction was started at each of the



The Port of New York Authority

Holland Tunnel in the Process of Construction

The Port of New York Authority

South Tube of Lincoln Tunnel

two entrances. Here the laborers (sand hogs), working in large machines called shields, began the construction of the large cast iron shell for the outer wall of the tunnel. To counteract the weight of the earth and water above, the work was done under compressed air. The shields were pushed forward by large jacks. The work was strenuous, but each crew averaged about forty feet a day. The engineering on this work was so exact that when the first tube of the tunnel was "holed through," the shields met within a quarter of an inch.

The ventilating systems of the two tunnels are alike in most respects. The Holland Tunnel has a maximum ventilation of 3,760,000 cubic feet of fresh air per minute, and the Lincoln Tunnel has a maximum ventilation of 1,913,000 cubic feet per minute.

The planning and construction of tunnels such as these are testimonials to the practical value of mathematics. The mathematics involved in each project was immense, ranging from simple arithmetic to the mathematics taught in college.

CHAPTER XV

Powers, Roots, and Radicals

Squaring Numbers [A]

In this chapter you will learn more about powers and roots of numbers. When a number is used two or more times as a factor, the product is called a *power* of that factor.

Since $5^2 = 5 \times 5 = 25$, then 25 is the second power, or the square, of 5. Likewise x^8 is the square of x^4.

Example 1. $(4\,x^3)^2 = (4\,x^3)(4\,x^3) = 16\,x^6$

Example 2. $(-7\,a^5)^2 = (-7\,a^5)(-7\,a^5) = 49\,a^{10}$

Example 3. $(\frac{3}{4})^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Example 4. $\left(\frac{2\,a}{3\,b^2}\right)^2 = \frac{2\,a}{3\,b^2} \times \frac{2\,a}{3\,b^2} = \frac{4\,a^2}{9\,b^4}$

ORAL EXERCISES [A]

Study the four examples above and then supply the missing words:

1. The sign of the square of a positive number is --?--, and the sign of the square of a negative number is --?--.

2. The numerical coefficient of the square of a monomial is found by --?-- the numerical coefficient of the monomial.

3. The exponent of each letter in the square of a monomial is found by multiplying the exponent of that letter in the monomial by --?--.

4. To square a fraction, we place the --?-- of the numerator of the fraction over the square of the --?-- of the fraction.

Find the indicated squares:

5. 3^2
6. $(-5)^2$
7. $(-6)^2$
8. 9^2
9. 10^2
10. $(-12)^2$
11. $(-4a)^2$
12. $(7b)^2$
13. $(2a^2)^2$
14. $(xy)^2$
15. $(-abc^2)^2$
16. $(6x^2y^3)^2$
17. $(0.1)^2$
18. $(-1.3\,r^2s^3)^2$
19. $\left(\frac{2a^2c}{5m}\right)^2$
20. $\left(\frac{7a^5b^6}{8y^3}\right)^2$

The Zero Power of a Number[B]

So far we have been using exponents that are whole numbers. Let us now see what a zero exponent means. By the law of signs for division, $x^6 \div x^6 = x^{6-6} = x^0$. You know that $x^6 \div x^6 = 1$. From these results we define x^0 as 1, where x may have any value except zero. Thus $8^0 = 1$, $100^0 = 1$, and $852^0 = 1$.

The Square Roots of Numbers[A]

As you learned earlier, when a number has two equal factors, either factor is the *square root* of the original number. Since $9 = 3 \times 3$, the square root of 9 is 3. The square root of a number is the number which when it is squared will produce the given number.

We know that $9 = (-3)(-3)$. Then -3 is also a square root of 9. Every number has two square roots. The two roots of a number are equal numerically, but they are opposite in sign. The two square roots of $4x^6$ are $2x^3$ and $-2x^3$.

A *radical* is an indicated root of a number. The radical sign written with a number indicates that the positive square root of the number is to be taken. Thus $\sqrt{4} = +2$, $\sqrt{x^2} = +x$, and $\sqrt{9a^8} = +3a^4$. The *cube root* of a number is indicated by $\sqrt[3]{\ }$. Thus $\sqrt[3]{27} = +3$, $\sqrt[3]{8} = +2$. The small 3 above the radical sign is the *index* of the root. When no index is written, it is understood to be 2.

When the positive square root of a number is to be taken, the plus sign, or no sign at all, is written before the radical; when the negative square root of a number is to be taken, the minus sign is written before the radical. Thus $+\sqrt{25}$, or $\sqrt{25}$ means $+5$ and $-\sqrt{25}$ means -5. When both square roots of a number are to be taken, both the plus sign and the minus sign are written before the radical. Thus $\pm\sqrt{25}$ means $+5$ and -5. The positive square root of a number is called the *principal square root* of the number.

EXERCISES [A]

Find the indicated roots:

1. $\sqrt{9}$
2. $\sqrt{16}$
3. $\sqrt{25}$
4. $\sqrt{36}$
5. $\sqrt{81}$
6. $\sqrt{121}$
7. $\sqrt{144}$
8. $\sqrt{x^2}$
9. $\sqrt{y^6}$
10. $-\sqrt{9}$
11. $-\sqrt{25}$
12. $-\sqrt{64}$
13. $-\sqrt{y^6}$
14. $\pm\sqrt{36}$
15. $\pm\sqrt{49}$
16. $\pm\sqrt{121}$
17. $\pm\sqrt{400}$
18. $-\sqrt{900}$
19. $-\sqrt{a^4}$
20. $\sqrt{c^{10}}$
21. $\sqrt{4a^{10}}$
22. $-\sqrt{16x^4}$
23. $\sqrt{a^8b^{10}}$
24. $\sqrt{25c^6}$
25. $-\sqrt{81c^6}$
26. $\sqrt{a^6b^2c^{12}}$
27. $\sqrt{100x^6y^8}$
28. $\sqrt{36x^2y^4}$
29. $\sqrt{m^{10}y^{12}}$
30. $\sqrt{49y^8}$
31. $\sqrt{.36}$
32. $\sqrt{.49m^4}$
33. $\sqrt{169a^4}$
34. $-\sqrt{1.44a^{10}}$
35. $\sqrt{100m^4}$
36. $-\sqrt{81p^{12}}$

Using Tables to Find Squares and Square Roots of Numbers [A]

The easiest and shortest way to find the square or the square root of a number is to use a table of squares and square roots, such as the one on the next page.

EXERCISES [A]

1. For what does "No." in the table on page 425 stand? For what does "Sq." stand? For what does "Sq. Root" stand? This table also gives the cubes and cube roots of numbers.

2. Using this table, find the squares of

a. 18
b. 23
c. 36
d. 49
e. 92
f. 99
g. 78
h. 53

Table of Powers and Roots

NO.	SQ.	SQ. ROOT	CUBE	CUBE ROOT	NO.	SQ.	SQ. ROOT	CUBE	CUBE ROOT
1	1	1.000	1	1.000	51	2,601	7.141	132,651	3.708
2	4	1.414	8	1.260	52	2,704	7.211	140,608	3.733
3	9	1.732	27	1.442	53	2,809	7.280	148,877	3.756
4	16	2.000	64	1.587	54	2,916	7.348	157,464	3.780
5	25	2.236	125	1.710	55	3,025	7.416	166,375	3.803
6	36	2.449	216	1.817	56	3,136	7.483	175,616	3.826
7	49	2.646	343	1.913	57	3,249	7.550	185,193	3.848
8	64	2.828	512	2.000	58	3,364	7.616	195,112	3.871
9	81	3.000	729	2.080	59	3,481	7.681	205,379	3.893
10	100	3.162	1,000	2.154	60	3,600	7.746	216,000	3.915
11	121	3.317	1,331	2.224	61	3,721	7.810	226,981	3.936
12	144	3.464	1,728	2.289	62	3,844	7.874	238,328	3.958
13	169	3.606	2,197	2.351	63	3,969	7.937	250,047	3.979
14	196	3.742	2,744	2.410	64	4,096	8.000	262,144	4.000
15	225	3.873	3,375	2.466	65	4,225	8.062	274,625	4.021
16	256	4.000	4,096	2.520	66	4,356	8.124	287,496	4.041
17	289	4.123	4,913	2.571	67	4,489	8.185	300,763	4.062
18	324	4.243	5,832	2.621	68	4,624	8.246	314,432	4.082
19	361	4.359	6,859	2.668	69	4,761	8.307	328,509	4.102
20	400	4.472	8,000	2.714	70	4,900	8.367	343,000	4.121
21	441	4.583	9,261	2.759	71	5,041	8.426	357,911	4.141
22	484	4.690	10,648	2.802	72	5,184	8.485	373,248	4.160
23	529	4.796	12,167	2.844	73	5,329	8.544	389,017	4.179
24	576	4.899	13,824	2.884	74	5,476	8.602	405,224	4.198
25	625	5.000	15,625	2.924	75	5,625	8.660	421,875	4.217
26	676	5.099	17,576	2.962	76	5,776	8.718	438,976	4.236
27	729	5.196	19,683	3.000	77	5,929	8.775	456,533	4.254
28	784	5.292	21,952	3.037	78	6,084	8.832	474,552	4.273
29	841	5.385	24,389	3.072	79	6,241	8.888	493,039	4.291
30	900	5.477	27,000	3.107	80	6,400	8.944	512,000	4.309
31	961	5.568	29,791	3.141	81	6,561	9.000	531,441	4.327
32	1,024	5.657	32,768	3.175	82	6,724	9.055	551,368	4.344
33	1,089	5.745	35,937	3.208	83	6,889	9.110	571,787	4.362
34	1,156	5.831	39,304	3.240	84	7,056	9.165	592,704	4.380
35	1,225	5.916	42,875	3.271	85	7,225	9.220	614,125	4.397
36	1,296	6.000	46,656	3.302	86	7,396	9.274	636,056	4.414
37	1,369	6.083	50,653	3.332	87	7,569	9.327	658,503	4.431
38	1,444	6.164	54,872	3.362	88	7,744	9.381	681,472	4.448
39	1,521	6.245	59,319	3.391	89	7,921	9.434	704,969	4.465
40	1,600	6.325	64,000	3.420	90	8,100	9.487	729,000	4.481
41	1,681	6.403	68,921	3.448	91	8,281	9.539	753,571	4.498
42	1,764	6.481	74,088	3.476	92	8,464	9.592	778,688	4.514
43	1,849	6.557	79,507	3.503	93	8,649	9.644	804,357	4.531
44	1,936	6.633	85,184	3.530	94	8,836	9.695	830,584	4.547
45	2,025	6.708	91,125	3.557	95	9,025	9.747	857,375	4.563
46	2,116	6.782	97,336	3.583	96	9,216	9.798	884,736	4.579
47	2,209	6.856	103,823	3.609	97	9,409	9.849	912,673	4.595
48	2,304	6.928	110,592	3.634	98	9,604	9.899	941,192	4.610
49	2,401	7.000	117,649	3.659	99	9,801	9.950	970,299	4.626
50	2,500	7.071	125,000	3.684	100	10,000	10.000	1,000,000	4.642

3. From the table find the positive square roots of

a. 17 **c.** 2 **e.** 92 **g.** 78

b. 25 **d.** 3 **f.** 99 **h.** 53

4. Using the table find the squares of

a. 1.414 **c.** 6.928 **e.** 9.950 **g.** 1.732

b. 3.162 **d.** 5.477 **f.** 9.110 **h.** 2.449

How to Find the Square Root of a Number [A]

The method of finding the square root of arithmetic numbers is based on algebra. First we shall find the square root of an algebraic expression. We know that $(a + b)^2 = a^2 + 2\,ab + b^2$. Then the principal square root of $a^2 + 2\,ab + b^2$ is $a + b$. The process is as follows:

$$\begin{array}{r|l} & a + b \text{ the square root} \\ \hline a & a^2 + 2\,ab + b^2 \\ & a^2 \\ \hline 2\,a + b & +\,2\,ab + b^2 \\ & +\,2\,ab + b^2 \\ \hline & \quad 0 \qquad 0 \end{array}$$

The square root of a^2 is a, the first term of the root.

Subtracting a^2 leaves $2\,ab + b^2$. We know that b is the other term of the root. To get b we must divide $2\,ab$ by $2\,a$, which is twice the root already found.

Adding b to $2\,a$ we get $2\,a + b$ for the divisor.

Multiplying $2\,a + b$ by b, the last term of the root, we obtain $2\,ab + b^2$, which, when subtracted, leaves no remainder.

If in the identity $(a + b)^2 = a^2 + 2\,ab + b^2$, we let $a = 40$ and $b = 6$, we have, $(40 + 6)^2 = 1600 + 480 + 36$. Then we find the square root of $1600 + 480 + 36$ as follows:

$$\begin{array}{r|l} & 40 + 6 \text{ the square root} \\ \hline 40 & 1600 + 480 + 36 \\ & 1600 \\ \hline 80 + 6 & +\,480 + 36 \\ & +\,480 + 36 \\ \hline \end{array}$$

The square root of 1600 is 40, the first term of the root.

Subtracting 40^2, or 1600, leaves $480 + 36$.

Multiplying the part of the root already found by 2, we have

$$2 \times 40 = 80$$

$$480 \div 80 = 6$$

Adding 6 to 80 we have 80 + 6 as the divisor, and 6 is the next term in the root.

Multiplying (80 + 6) by 6, we obtain 480 + 36, which, when subtracted, leaves no remainder.

Then 40 + 6 or 46 is the positive square root of 1600 + 480 + 36, or 2116.

The method of finding the square roots of arithmetic numbers based on the above solutions is given in the following examples. Study carefully the different steps in examples 1 and 2. It would be well to use pencil and paper to follow the solutions.

Example 1. Find the positive square root of 2116.

Solution. Separate the number into groups of two figures each, beginning to group at the *decimal point*. A bar can be placed over each group.

$$\overline{21}\ \overline{16}$$

The first divisor is also the first figure of the quotient, or root. The divisor is 4, because 21 lies between 4 × 4 and 5 × 5.

$$\begin{array}{r|l} & 4 \\ \cline{2-2} 4 & 21\ \overline{16} \end{array}$$

Place the 16 under the 21 and subtract, getting 5 as the remainder. Bring down the 16. The new dividend is 516.

$$\begin{array}{r|l} & 4 \\ \cline{2-2} 4 & 21\ \overline{16} \\ & 16 \\ \hline & \ \ 5\ 16 \end{array}$$

We are now ready for another division. To get the new divisor, we multiply the 4, the first figure in the root, by 2 (we always multiply by 2), and get 8 as the first figure of the second divisor.

$$\begin{array}{r|l} & 4 \\ \cline{2-2} 4 & 21\ \overline{16} \\ & 16 \\ \hline 8 & 5\ 16 \\ & \\ \cline{2-2} \end{array}$$

There is one more figure in the divisor, and it is the next figure of the root. As in long division, this is the largest figure possible.

By trial we find this figure to be 6. Then 86 is the complete divisor and 6 is the next figure in the quotient (root). Multiply 86 by 6, obtaining 516. Place the 516 under the 516 and subtract. Since there is no remainder, the positive square root of 2116 is 46.

$$\begin{array}{r|l}
\multicolumn{2}{r}{4\quad 6} \\
4 & \overline{21}\ \overline{16} \\
 & 16 \\ \hline
86 & 5\ 16 \\
 & 5\ 16 \\ \hline
\end{array}$$

Check. $46^2 = 46 \times 46 = 2116.$

Example 2. Find the positive square root of 44,436.64.

Solution. Beginning at the decimal point, separate the number into groups of two figures each. The first divisor is 2, which is the first figure of the root, 2 × 2 = 4. Subtract 4 from 4. The remainder is zero. Bring down 44 as the next dividend.

Double the 2 in the divisor, obtaining 4 as the first figure of the second divisor. By trial the next figure of the divisor is 1, which is the second figure of the root. Multiply the divisor 41 by 1 and place the product under 44. Subtract, getting 3. Bring down the group 36. The third dividend is 336.

Multiply 21, the part of the root already found, by 2, obtaining 42, which is the first part of the third divisor. The last figure of the divisor and the third figure of the root is 0. Multiply 420 by 0, obtaining 0. Subtract, getting 336 as the first part of the fourth dividend.

Double 210, the part of the root already found, getting 420 as the first part of the fourth divisor. The last figure of the divisor and the fourth figure of the root is 8. Multiply 4208 by 8, getting 33,664. Subtract. There is no remainder.

Place the decimal point of the root above the decimal point of the original number. Then 210.8 is the required square root. Note that each group of the original number has a figure of the root above it.

$$\boxed{\begin{array}{r|l}
\multicolumn{2}{r}{\overline{2}\ \ \overline{1}\ \ \overline{0.}\ \overline{8}} \\
 & 04\ 44\ 36.64 \\
2 & 4 \\ \hline
41 & 0\ 44 \\
 & 41 \\ \hline
420 & 3\ 36 \\
 & 0\ 00 \\ \hline
4208 & 3\ 36\ 64 \\
 & 3\ 36\ 64 \\ \hline
\end{array}}$$

Proof. $210.8^2 = 44{,}436.64$

EXERCISES [A]

Find the positive square roots of

1. 729
2. 1156
3. 2809
4. 4624
5. 5329
6. 60.84
7. 88.36
8. 2.1316
9. 114.49
10. 21025
11. 1.3689
12. 1056.25
13. 7.0756
14. 445.21
15. 10816
16. 138384
17. 357.21
18. 0.183184
19. 813604
20. 316.84
21. 1210000
22. 86.8624
23. 485.3209
24. 0.030625

Computing Approximate Square Roots [A]

Most numbers are not perfect squares. The number 7 is not a perfect square, since there is no whole number or fraction whose square is 7. Since we cannot find the exact square roots of such numbers, we find their approximate square roots by computing their square roots to one or more decimal places. In most cases it is sufficient to find the roots to two or three decimal places.

Example. Find the approximate square root of 5 to the nearest hundredth.

Solution. Since the square root is to be found to the nearest hundredth, the root should be computed to three decimal places. Three groups of zeros must be added to give three decimal places in the root. The square root of 5 to three decimal places is 2.236. The square root of 5 to the nearest hundredth is 2.24, since 6 is more than 5.

$$
\begin{array}{r|l}
 & 2.\ \ 2\ \ \ 3\ \ \ 6 \\
2 & 5.\overline{00}\ \overline{00}\ \overline{00} \\
 & 4 \\
\hline
42 & 1\ 00 \\
 & 84 \\
\hline
443 & 16\ 00 \\
 & 13\ 29 \\
\hline
4466 & 2\ 71\ 00 \\
 & 2\ 67\ 96
\end{array}
$$

EXERCISES [A]

Find the approximate square root of each number to the nearest hundredth:

1. 2
2. 3
3. 6
4. 11
5. 20
6. 28
7. 33.33
8. 1.82
9. .032
10. 32.4
11. 42.17
12. 99

If $A = \sqrt{b^2 - 4\,ac}$, find A when

13. $a = 1$, $b = 8$, and $c = 9$

14. $a = 1$, $b = 22$, and $c = 2$

15. $a = 2$, $b = -7$, and $c = 3$

16. $a = 7$, $b = -49$, and $c = 0$

17. $a = -5$, $b = 14$, and $c = -8$

18. $a = 1$, $b = -1$, and $c = -1$

Square Roots of Fractions [A]

The square root of a fraction may be found by dividing the square root of the numerator by the square root of the denominator.

Example 1. $\sqrt{\frac{4}{25}} = \frac{2}{5}$ **Example 2.** $\sqrt{\frac{16\,a^4}{49\,b^6}} = \frac{4\,a^2}{7\,b^3}$

When the terms of a fraction are not perfect squares, the approximate square root may be found in three ways. These ways will be illustrated in finding the square root of the fraction $\frac{2}{3}$.

Example 3. Find the approximate square root of $\frac{2}{3}$.

Solution 1. Using the method of examples 1 and 2,

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{1.414}{1.732} = .816$$

Solution 2. Changing $\frac{2}{3}$ to a decimal, we have

$$\sqrt{\tfrac{2}{3}} = \sqrt{.666666} = .816$$

Solution 3. In this solution the denominator is made a perfect square by multiplying it by 3. Then the numerator must be multiplied by 3. The solution follows:

$$\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{2.449}{3} = .816$$

In most cases the third method is used, since it is usually the shortest.

EXERCISES [A]

Find the indicated roots:

1. $\sqrt{\frac{1}{4}}$

2. $\sqrt{\frac{9}{25}}$

3. $\sqrt{\frac{x^2}{4y^2}}$

4. $\sqrt{\frac{y^6}{16m^2}}$

5. $\sqrt{\frac{25c^2}{36d^2}}$

6. $\sqrt{\frac{9m^8}{49}}$

7. $\sqrt{\frac{m^{10}}{64x^6}}$

8. $\sqrt{\frac{16a^4}{121b^4}}$

Find the approximate square roots of the following fractions:

9. $\frac{3}{4}$

10. $\frac{3}{8}$

11. $\frac{2}{5}$

12. $\frac{1}{3}$

13. $\frac{4}{7}$

14. $\frac{1}{2}$

15. $\frac{2}{9}$

16. $\frac{5}{8}$

Find the values of:

17. $\sqrt{a} + \sqrt{b}$ when $a = 25$ and $b = 64$.

18. $\sqrt{a^2 + b^2}$ when $a = 5$ and $b = 12$.

19. $\sqrt{a^2 + b^2}$ if $a = 16$ and $b = 30$.

20. $2\sqrt{m} - 3\sqrt{n}$ when $m = 31{,}625$ and $n = 256$.

Degree of a Radical [A]

The *degree* or *order* of a radical is the same as the index of the root. The number expression under the radical sign is called the *radicand*. Thus 3 is the radicand in $\sqrt{3}$.

Rational and Irrational Numbers [A]

Numbers such as 7, -6, $\frac{1}{8}$, $\sqrt{25}$, $\sqrt{\frac{1}{4}}$, and $\sqrt{0.25}$ are *rational numbers.* A rational number can be expressed as an integer or a common fraction.

Some numbers like $\sqrt{2}$, $\sqrt{15}$, and π cannot be changed into whole numbers or simple fractions. One may find the square root of 2 to any desired number of decimal places, but only the approximate value of $\sqrt{2}$ can be found. The value of π has been computed to more than seven hundred decimal places, but no common fraction or decimal fraction can express its true value. Numbers like these are *irrational numbers.*

Two Important Principles of Radicals [A]

In simplifying, adding, multiplying, and dividing radicals, two important principles are used:

Principle 1. The square root of a product is equal to the product of the square roots of its factors; and conversely, the product of two square roots is equal to the square root of the product of the radicands. $\sqrt{ab} = \sqrt{a}\sqrt{b}$; $\sqrt{a}\sqrt{b} = \sqrt{ab}$.

Examples.

$$\sqrt{36} = \sqrt{4}\sqrt{9} \qquad \sqrt{9}\sqrt{36} = \sqrt{324}$$
$$\sqrt{100} = \sqrt{4}\sqrt{25} \qquad \sqrt{49}\sqrt{2} = \sqrt{98}$$
$$\sqrt{8} = \sqrt{4}\sqrt{2} \qquad \sqrt{5}\sqrt{2} = \sqrt{10}$$

Principle 2. The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator; and conversely, the quotient of two square roots is equal to the square root of the quotient of the radicands.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Examples.

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3} \qquad \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$
$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5} \qquad \frac{\sqrt{16}}{\sqrt{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$
$$\sqrt{\frac{7}{8}} = \frac{\sqrt{7}}{\sqrt{8}}$$

EXERCISES [A]

1. Is $\sqrt{4}$ a radical? Is $\sqrt{4}$ a rational number?

2. State which of the following numbers are rational: 3; -8; $\frac{1}{4}$; $\sqrt{20}$; $\sqrt{16}$; $\sqrt{40}$.

Multiply or divide, using Principles 1 and 2:

3. $\sqrt{4}\sqrt{25}$
4. $\sqrt{2}\sqrt{3}$
5. $\sqrt{9}\sqrt{3}$
6. $\sqrt{12} \div \sqrt{3}$
7. $\sqrt{15} \div \sqrt{3}$
8. $\sqrt{27} \div \sqrt{9}$
9. $\sqrt{64} \div \sqrt{4}$
10. $\sqrt{8} \div \sqrt{2}$
11. $\sqrt{10} \div \sqrt{5}$
12. $\sqrt{18} \div \sqrt{2}$
13. $\sqrt{3} \div \sqrt{1}$
14. $\sqrt{6} \times \sqrt{2}$

Complete:

15. $\sqrt{20} = \sqrt{?}\sqrt{5} = ?\sqrt{5}$
16. $\sqrt{8} = \sqrt{?}\sqrt{2} = ?\sqrt{2}$
17. $\sqrt{98} = \sqrt{?}\sqrt{2} = ?\sqrt{2}$
18. $\sqrt{24} = \sqrt{?}\sqrt{6} = ?\sqrt{6}$
19. $\sqrt{80} = \sqrt{?}\sqrt{5} = ?\sqrt{5}$
20. $\sqrt{18} = \sqrt{?}\sqrt{2} = ?\sqrt{2}$

Simplifying Radicals[A]

A radical can be simplified if

1. *The indicated root can be found exactly.*
2. *The radicand contains a factor of which the indicated root can be taken.*
3. *The radicand is a fraction.*

In simplifying radicals you should remember that a radical is in its simplest form when the radicand is a whole number and is as small as possible.

Example 1. Simplify $\sqrt{25}$.

Solution. $\sqrt{25} = 5$

Example 2. Simplify $3\sqrt{32}$.

Solution. The radicand 32 contains the perfect square factors 4 and 16. We must use the factor 16. Can you tell why?

$$3\sqrt{32} = 3\sqrt{16}\sqrt{2} = 3 \times 4\sqrt{2} = 12\sqrt{2}$$

EXERCISES [A]

Simplify the following radicals:

1. $\sqrt{9x^2}$

2. $\sqrt{8}$

3. $\sqrt{12}$

4. $\sqrt{49}$

5. $\sqrt{98}$

6. $\sqrt{20}$

7. $\sqrt{24}$

8. $\sqrt{40}$

9. $\sqrt{\frac{1}{16}}$

10. $\sqrt{50}$

11. $\sqrt{60}$

12. $\sqrt{100}$

13. $\sqrt{80}$

14. $\sqrt{27}$

15. $\sqrt{18}$

16. $\sqrt{64}$

17. $\sqrt{128}$

18. $\sqrt{16}$

19. $\sqrt{48}$

20. $\sqrt{c^2d^4}$

21. $3\sqrt{64}$

22. $4\sqrt{27}$

23. $\sqrt{36x^{10}}$

24. $5\sqrt{28}$

25. $7\sqrt{25}$

26. $5\sqrt{72}$

27. $\sqrt{75}$

Example 3. Simplify $\sqrt{\frac{2}{3}}$.

Solution. This radical can be simplified because its radicand is a fraction. To make the denominator a perfect square we multiply it by 3. Then the numerator must be multiplied by 3.

Then
$$\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3} \quad \text{or} \quad \frac{1}{3}\sqrt{6}$$

Example 4. Simplify $\sqrt{\frac{4}{5}}$.

Solution. Multiplying both numerator and denominator of $\frac{4}{5}$ by 5, we have,

$$\sqrt{\frac{4}{5}} = \sqrt{\frac{20}{25}} = \frac{\sqrt{20}}{\sqrt{25}} = \frac{2\sqrt{5}}{5} \quad \text{or} \quad \frac{2}{5}\sqrt{5}$$

EXERCISES [A]

Transform the following radicals into their simplest forms:

1. $\sqrt{\frac{1}{2}}$

2. $\sqrt{\frac{1}{3}}$

3. $\sqrt{\frac{2}{5}}$

4. $\sqrt{\frac{a^2}{b^2}}$

5. $\sqrt{\frac{1}{6}}$

6. $4\sqrt{\frac{1}{7}}$

7. $3\sqrt{\frac{1}{3}}$

8. $\sqrt{\frac{1}{8}}$

9. $\sqrt{\frac{1}{12}}$

10. $\sqrt{\frac{1}{20}}$

11. $\sqrt{\frac{4}{7}}$

12. $\sqrt{\frac{9}{5}}$

13. $\sqrt{\frac{2}{9}}$

14. $\frac{1}{3}\sqrt{\frac{1}{2}}$

15. $\frac{1}{2}\sqrt{\frac{3}{8}}$

Example 5. Simplify $\sqrt{\frac{1}{a^5}}$.

Solution. If a^5 is multiplied by a, the product is a perfect square.

Then $$\sqrt{\frac{1}{a^5}} = \sqrt{\frac{a}{a^6}} = \frac{\sqrt{a}}{\sqrt{a^6}} = \frac{\sqrt{a}}{a^3} \quad \text{or} \quad \frac{1}{a^3}\sqrt{a}$$

EXERCISES[B]

Simplify the following radicals:

1. $\sqrt{x^3}$
2. $\sqrt{x^5}$
3. $\sqrt{4a}$
4. $\sqrt{7x^2}$
5. $\sqrt{32a^3}$
6. $\sqrt{8y^2}$
7. $\sqrt{m^3n^2}$
8. $\sqrt{a^3b}$
9. $\sqrt{ab^4}$
10. $\sqrt{8x^3}$
11. $\sqrt{\frac{1}{a}}$
12. $\sqrt{\frac{1}{a^2}}$
13. $\sqrt{\frac{2}{x^5}}$
14. $\sqrt{\frac{4}{ab}}$

Combining Radicals[A]

Similar radicals are radicals having the same index and the same radicand. Thus $2\sqrt{5}$, $6\sqrt{5}$, and $\frac{1}{2}\sqrt{5}$ are similar radicals. Similar radicals can be added or combined by adding their coefficients.

Example 1. Combine: $6\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

Solution. $$6\sqrt{2} + 3\sqrt{2} - 7\sqrt{2} = 2\sqrt{2}$$

Example 2. Combine similar radicals:
$$2\sqrt{3} + 4\sqrt{5} + 6\sqrt{3} - 6\sqrt{5}$$

Solution. $2\sqrt{3}$ and $6\sqrt{3}$ are similar radicals and $4\sqrt{5}$ and $-6\sqrt{5}$ are similar radicals.

Then $$2\sqrt{3} + 4\sqrt{5} + 6\sqrt{3} - 6\sqrt{5} = 8\sqrt{3} - 2\sqrt{5}$$

EXERCISES[A]

Simplify by combining like terms:

1. $4\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}$

2. $\sqrt{2} + \sqrt{2} + \sqrt{5}$

3. $\sqrt{5} - \sqrt{5}$

4. $3\sqrt{6} - 4\sqrt{6} - \sqrt{6}$

5. $3\sqrt{3} + 9\sqrt{3} - 2\sqrt{3}$

6. $\sqrt{7} + \sqrt{7} + \sqrt{7}$

7. $5\sqrt{5} - 4\sqrt{5} - \sqrt{5}$

8. $\sqrt{2} + 6\sqrt{2} - 6$

9. $10 - \sqrt{3} - 2\sqrt{3}$

10. $4\sqrt{5} - \sqrt{6} - \sqrt{5}$

11. $10 - 2\sqrt{2} + 2\sqrt{2}$

12. $7 + 5 - \sqrt{5}$

13. $6 + \sqrt{6} - 10\sqrt{6}$

14. $\sqrt{a} - 7\sqrt{a} + 3$

15. $\sqrt{a} + \sqrt{ab} + 2\sqrt{a}$

16. $\sqrt{x} + \sqrt{y} + 9\sqrt{x}$

Example 3. Simplify the radicals and combine:

$$3\sqrt{8} - \sqrt{50} + 6\sqrt{32}$$

Solution.

$$\begin{aligned} &3\sqrt{8} - \sqrt{50} + 6\sqrt{32} \\ &= 3\sqrt{4}\sqrt{2} - \sqrt{25}\sqrt{2} + 6\sqrt{16}\sqrt{2} \\ &= 6\sqrt{2} - 5\sqrt{2} + 24\sqrt{2} \\ &= 25\sqrt{2} \end{aligned}$$

Example 4. Simplify the radicals and combine:

$$\sqrt{18} + \sqrt{20} - \sqrt{\tfrac{1}{2}} + \sqrt{125}$$

Solution.

$$\begin{aligned} &\sqrt{18} + \sqrt{20} - \sqrt{\tfrac{1}{2}} + \sqrt{125} \\ &= \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{5} - \sqrt{\tfrac{2}{4}} + \sqrt{25}\sqrt{5} \\ &= 3\sqrt{2} + 2\sqrt{5} - \tfrac{1}{2}\sqrt{2} + 5\sqrt{5} \\ &= \tfrac{5}{2}\sqrt{2} + 7\sqrt{5} \end{aligned}$$

EXERCISES[B]

1. $\sqrt{48} - \sqrt{75} + \sqrt{12}$

2. $\sqrt{8} - \sqrt{48} + \sqrt{50}$

3. $\sqrt{5} + \sqrt{20} + \sqrt{7}$

4. $3\sqrt{6} - \sqrt{24} + \sqrt{54}$

5. $4\sqrt{7} - \sqrt{28} - \sqrt{63}$

6. $\sqrt{75} + 4\sqrt{3} + \sqrt{18}$

7. $\sqrt{25} + \sqrt{125} - \sqrt{20}$

8. $\sqrt{\tfrac{1}{2}} + \sqrt{8} - \sqrt{98}$

9. $2\sqrt{18}+3\sqrt{12}-\sqrt{\frac{1}{2}}-6\sqrt{\frac{1}{3}}$

10. $4\sqrt{y}-\sqrt{x}+9\sqrt{y}-\sqrt{9x}$

11. $2\sqrt{12}-\sqrt{75}+\sqrt{3}-\sqrt{\frac{1}{3}}$

12. $\sqrt{25a}+\sqrt{4a}-\sqrt{9a}+\sqrt{16a}$

13. $6\sqrt{a}+\sqrt{25a}-\sqrt{b}+\sqrt{a}$

14. $\sqrt{2}+3\sqrt{27}+2\sqrt{50}-\frac{1}{3}\sqrt{3}$

Multiplying Monomials Containing Radicals [A]

Radicals having the same index can be multiplied. The product of the square roots of two or more numbers is equal to the square root of their product.

Example 1. $\sqrt{6}\times\sqrt{5}=\sqrt{30}$

Example 2. $3\sqrt{2}\times 4\sqrt{6}=12\sqrt{12}=12\sqrt{4}\sqrt{3}=24\sqrt{3}$

EXERCISES [A]

Multiply:

1. $\sqrt{7}\sqrt{5}$

2. $\sqrt{3}\sqrt{2}$

3. $\sqrt{7}\sqrt{9}$

4. $\sqrt{6}\sqrt{2}$

5. $\sqrt{2}\sqrt{18}$

6. $3\sqrt{6}\cdot 4\sqrt{5}$

7. $\sqrt{3}\cdot\sqrt{6}$

8. $2\sqrt{2}\cdot\sqrt{8}$

9. $3\sqrt{2}\cdot 5\sqrt{8}$

10. $\sqrt{3}\cdot\sqrt{15}$

11. $\sqrt{5}\sqrt{12}$

12. $5\sqrt{2}\cdot\sqrt{2}$

13. $\sqrt{\frac{2}{3}}\times\sqrt{\frac{3}{2}}$

14. $\sqrt{\frac{5}{9}}\cdot\sqrt{\frac{2}{3}}$

15. $\sqrt{5}\cdot\sqrt{5}$

16. $\sqrt{\frac{1}{2}}\sqrt{2}$

17. $\sqrt{\frac{1}{3}}\sqrt{6}$

18. $\sqrt{\frac{1}{2}}\sqrt{8}$

19. $2\sqrt{5}\cdot 3\sqrt{5}$

20. $(\sqrt{2})^2$

21. $\sqrt{5}\sqrt{125}$

22. $3\sqrt{6}\cdot 2\sqrt{3}$

23. $5\sqrt{2}\cdot 6\sqrt{2}$

24. $3\sqrt{6}\cdot 5\sqrt{3}$

25. $4\sqrt{3}\cdot\sqrt{3}$

26. $\sqrt{11}\sqrt{11}$

27. $\sqrt{12}\cdot\sqrt{3}$

28. $7\sqrt{8}\cdot\frac{1}{2}\sqrt{2}$

29. $3\sqrt{5}(-4\sqrt{5})$

30. $(2\sqrt{3})^2$

Rationalizing Denominators [B]

You have already learned that a radical is not in its simplest form if its radicand is a fraction. We also say that a fraction is not in its simplest form if it has a radical in its denominator. The process of transforming a fraction with an irrational denominator to one with a rational denominator is called *rationalizing the denominator.*

Example 1. Rationalize the denominator of $\frac{2}{\sqrt{3}}$.

Solution. $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$, or $\frac{2}{3}\sqrt{3}$

Example 2. Rationalize the denominator of $\frac{4}{\sqrt{5}}$.

Solution. $\frac{4}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{25}} = \frac{4\sqrt{5}}{5}$, or $\frac{4}{5}\sqrt{5}$

EXERCISES [B]

Rationalize the denominators of the following fractions:

1. $\frac{1}{\sqrt{2}}$

2. $\frac{1}{\sqrt{3}}$

3. $\frac{1}{\sqrt{5}}$

4. $\frac{1}{\sqrt{7}}$

5. $\sqrt{\frac{1}{a}}$

6. $\sqrt{\frac{1}{b}}$

7. $\frac{4}{\sqrt{6}}$

8. $\frac{2}{3\sqrt{2}}$

9. $\frac{a}{\sqrt{c}}$

10. $\frac{2\sqrt{3}}{\sqrt{5}}$

11. $\frac{5}{\sqrt{8}}$

12. $\frac{4\sqrt{5}}{\sqrt{6}}$

REVIEW EXERCISES [A]

1. Complete: The square of either a positive or a negative number is --?--.

2. Find the following indicated powers:

a. $(-2a)^2$

b. $(3ac)^2$

c. $(mn)^2$

d. $(-4x^2)^2$

e. $(-5x)^2$

f. $(x^2y)^2$

3. Find the two square roots of 184.96.

4. Square the following:

a. $\sqrt{3}$ **b.** $\sqrt{-4}$ **c.** $\sqrt{\frac{1}{3}}$ **d.** $\sqrt{5}$ **e.** $\sqrt{100}$ **f.** $\sqrt{141}$

5. The formula for finding the area of a triangle when the lengths of its three sides are known is $A = \sqrt{s(s-a)(s-b)(s-c)}$. In this formula a, b, and c are the lengths of the sides, and $s = \frac{1}{2}(a+b+c)$. Find A when $a = 10$, $b = 24$, and $c = 26$.

Equations Containing Radicals [C]

A *radical equation* is one in which the unknown letter is contained in the radicand. Thus $\sqrt{x} = 3$ and $\sqrt{2x-1} = 5$ are radical equations. The equation $2x - \sqrt{5} = 0$ is not a radical equation, since x is not in the radicand. The solution of a radical equation makes use of another axiom:

> The same powers of equals are equal.

Example 1. Solve $2 + \sqrt{x} = 4$.

Solution. $2 + \sqrt{x} = 4$

$\sqrt{x} = 2$

* P_2 $x = 4$

Check. Does $2 + \sqrt{4} = 4$?

Does $2 + 2 = 4$? Yes.

NOTE. Since the radical always means the positive square root, equations such as $\sqrt{x} = -5$ and $\sqrt{x} + 1 = 0$ are not solvable.

> **Directions for Solving an Equation Containing One Radical:**
>
> 1. Transpose so that one member of the equation contains only the radical term.
> 2. Square both members of the equation.

NOTE. These directions are for equations whose radicals have the index 2.

* P_2 means that both members of the equation are squared.

EXERCISES [C]

Solve and check:

1. $\sqrt{x} = 3$
2. $\sqrt{x} = 1$
3. $\sqrt{3x + 1} = 2$
4. $\sqrt{x - 1} = 2$
5. $\sqrt{x + 4} = 3$
6. $\sqrt{2y - 1} = 7$
7. $2\sqrt{c + 2} = 6$
8. $5 = \sqrt{4b + 1}$
9. $3\sqrt{x + 1} = 8$
10. $\sqrt{x} = 5.1$
11. $2\sqrt{x} - 1 = 0$
12. $3 + \sqrt{x} = 6$
13. $2 + 4\sqrt{y} = 3$
14. $\sqrt{2x - 10} - 6 = 0$
15. $3 - \sqrt{2c - 7} = 0$
16. $\sqrt{2c - 7} - 5 = 0$
17. $\sqrt{k - 5} - 3 = 0$
18. $\sqrt{x + 5} - 5 = 8$
19. $2\sqrt{x} - 14 = 0$
20. $\sqrt{y} - 48 = 2$
21. Solve $h = \frac{s}{4}\sqrt{3}$ for s.
22. Solve $r = \sqrt{\frac{y}{11}}$ for y.

CHAPTER REVIEW

A

1. When you square a positive number, what is the sign of the result?

2. When you square a negative number, what is the sign of the result?

3. How many square roots does any positive number have?

4. Simplify:

a. $(-9)^2$

b. $(\sqrt{9})^2$

c. $(\sqrt{x})^2$

d. $(\sqrt{a^3})^2$

e. $(\sqrt{7})^2$

f. $\sqrt{(1087)^2}$

5. In the expression $4\sqrt[3]{5}$ what is the 3 called? the 5?

6. $c = \sqrt{a^2 + b^2}$. Find c if $a = 36$ and $b = 15$.

7. $a = \sqrt{c^2 - b^2}$. Find a when $c = 17$ and $b = 8$.

8. $b = \sqrt{c^2 - a^2}$. Find b if $c = 35$ and $a = 21$.

9. Find the value of $10 + 7\sqrt{81}$.

10. Find the value of $\dfrac{\sqrt{x} + \sqrt{y}}{y - x}$ if $x = 225$ and $y = 361$.

Find the positive square roots of:

11. 41616

12. 852.64

13. .045796

14. .2704

15. 4.5796

16. 11109.16

17. Find the positive square root of 17 to the nearest thousandth.

18. Find the positive square root of $\frac{5}{9}$ to the nearest hundredth.

Transform the following radicals into their simplest forms:

19. $\sqrt{20}$

20. $\sqrt{16}$

21. $\sqrt{24}$

22. $\sqrt{125}$

23. $\sqrt{\frac{1}{2}}$

24. $\sqrt{\frac{1}{3}}$

25. $\sqrt{108}$

26. $3\sqrt{18}$

27. $5\sqrt{45}$

Simplify:

28. $\sqrt{3}\sqrt{5}$

29. $\sqrt{5}\sqrt{3\frac{1}{5}}$

30. $\sqrt{12}\sqrt{3}$

31. $\sqrt{2}\sqrt{6}$

32. $\sqrt{\frac{1}{2}}\sqrt{2}$

33. $\sqrt{3}\sqrt{16}$

34. $\sqrt{3}\sqrt{3}$

35. $\sqrt{2}\sqrt{12.5}$

36. $\sqrt{5}\sqrt{35}$

B

Simplify the radicals and combine similar radicals:

37. $\sqrt{12} - \sqrt{80} - \sqrt{108} - \sqrt{5}$

38. $\sqrt{28} - \sqrt{6} + \sqrt{63} - \sqrt{24}$

39. $\sqrt{18} - \sqrt{5} + \sqrt{20} - \sqrt{50}$

40. $\sqrt{75} - 2\sqrt{5} + \sqrt{108}$

41. $6\sqrt{\frac{1}{3}} + 5\sqrt{\frac{1}{5}} + \sqrt{3}$

42. $2\sqrt{\frac{1}{2}} + 3\sqrt{\frac{1}{5}} + \sqrt{50}$

43. Simplify:

a. $\sqrt{\frac{2}{5}}$

b. $3\sqrt{\frac{1}{3}}$

c. $6\sqrt{\frac{2}{3}}$

d. $8\sqrt{\frac{1}{8}}$

44. Find the value of $\dfrac{2 + \sqrt{34}}{5}$ to the nearest hundredth.

GENERAL REVIEW

A

1. Find the number of gallons in a cylindrical vessel whose height is 10 inches and whose base has a diameter of 10 inches. ($V = \pi r^2 h$, and 1 gal. = 231 cu. in.)

2. State the formula $r = \frac{d}{t}$ as a rule.

3. When the temperature Fahrenheit is 18°, what is the temperature centigrade?

4. If $a = -4$, $b = 2$, and $c = 0$, find the value of

a. $\dfrac{3a+2}{bc-6}$ **b.** $10 - 3ab$ **c.** $\dfrac{(ab)^3 + 300}{100 + b - a}$

5. $V = \frac{4}{3}\pi r^3$. Find V when $r = 2.1$ and $\pi = 3.14$.

6. $A = p + prt$. Find p when $A = \$836.20$, $r = 4\%$, and $t = 3$.

7. One of the roots of the equation $x^4 - 3x^3 + 6x^2 - 8x = 30$ is among the following numbers: 1, 2, 3, 4, − 1, − 2. Find the root by substitution.

8. Multiply:

a. $(-3ab)(4c)(b^2)$

b. $(-m^2n)(mn^3)$

c. $(a - b + 1)(a - b - 1)$

d. $(x^2 - 3x + 4)(2x - 1)$

9. Divide:

a. $(-24a^3) \div (-4a)$

b. $(x^3 - x^2 + x) \div (-x)$

c. $(x^2 - 6x + 9) \div (x + 1)$

d. $(17y^2 - 20y + 12y^3 - 20) \div (3y + 5)$

10. Combine:

a. $\dfrac{1}{3x} - \dfrac{1}{2x}$

b. $\dfrac{m}{n} - c$

c. $\dfrac{a-b}{2} + \dfrac{a-3b}{3}$

d. $\dfrac{x-2y}{x+y} - \dfrac{x+y}{x-y}$

11. Solve:

a. $5x - 2 + 2(x + 2) = 6$

b. $6.7 - 3(x + .2) = 7x + 1.1$

c. $2x - (x - 2) = -6$

d. $\frac{2 + 5x}{3} = \frac{x^2 - x + 4}{x}$

12. Solve:

a. $a + 2b = 8$
$3a - b = 3$

b. $\frac{3x}{8} + \frac{2y}{5} = \frac{7}{2}$
$\frac{y}{5} - \frac{x}{8} = \frac{1}{2}$

13. Divide: $\frac{x^2 - y^2}{x^2 - 3x - 4} \div \frac{x - y}{x^2 + x}$

14. Find the positive square root of 19.5364.

15. Find to the nearest thousandth the positive square root of 19.

B

16. Solve $\frac{x}{5} + \frac{y}{4} = 0$
$\frac{x}{3} - \frac{y}{6} = 7$

17. Simplify $\sqrt{18} - \sqrt{72} - \sqrt{98}$.

18. Simplify $\sqrt{50} - \sqrt{\frac{1}{2}} + \sqrt{54} + \sqrt{96}$.

19. Find the roots:

a. $\sqrt{64a^2}$

b. $\sqrt{\frac{1}{9}x^6}$

c. $\sqrt{81m^4}$

d. $\sqrt{\frac{a^2}{b^4}}$

e. $\sqrt{.49c^6}$

f. $\sqrt{.01x^4}$

20. Separate 290 into two parts such that when the greater is divided by the smaller, the quotient is 6 and the remainder is 10.

21. A painter can paint a house in 10 days and his helper can paint it in 16 days. How many days are needed to paint the house when they work together?

22. How many pounds of 90-cent tea should be mixed with 70-cent tea to make 60 pounds of tea worth 75 cents a pound?

23. Solve by substitution: $3x - 2y = 0$

$5x + 9y = 37$

24. Solve $a^2 = b^2 + c^2 + 2cn$ for n.

25. State the degree of each equation and solve it:

a. $3x = 10 + x$ **c.** $4x^2 - 25 = 0$

b. $x^2 - x = 0$ **d.** $y^3 - 4y = 0$

26. Change $\dfrac{x^3 - x^2 + 2x - 1}{x - 1}$ into a mixed number.

27. Multiply: $\dfrac{x^2 - 2x - 24}{x^2 + x} \cdot \dfrac{x^2 + 2x + 1}{x^2 - 6x}$

28. A boy and a girl balance on a teeterboard. The boy is 6 feet from the fulcrum and the girl is 8 feet from it. What are their weights if the boy weighs 24 pounds more than the girl?

29. Rationalize the denominators:

a. $\dfrac{1}{\sqrt{6}}$ **b.** $\dfrac{3}{\sqrt{3}}$ **c.** $\dfrac{2\sqrt{2}}{\sqrt{5}}$ **d.** $\dfrac{3\sqrt{10}}{\sqrt{5}}$

30. Find the value of

a. 10^0 **c.** $\dfrac{3}{6^0}$ **e.** $\sqrt{4^3}$ **g.** $\sqrt{a^4y^{10}}$

b. 8^0 **d.** $\sqrt{8^2}$ **f.** $\sqrt{64y^4}$ **h.** $\sqrt{.0016}$

CHAPTER TEST [A]

1. Find the indicated squares: **a.** $(4a)^2$ **b.** $(-5a^3)^2$

2. Find the indicated square roots: **a.** $\sqrt{81}$ **b.** $-\sqrt{36a^2}$

3. Find the two square roots of 6084.

4. Find the square root of 132 to the nearest tenth.

5. Simplify: **a.** $\sqrt{12}$ **b.** $\sqrt{45}$

6. Simplify: **a.** $\sqrt{64}$ **b.** $\sqrt{75}$

7. Simplify: **a.** $\sqrt{\frac{2}{5}}$ **b.** $\sqrt{\frac{1}{3}}$

8. Simplify: **a.** $\sqrt{\frac{4}{9}}$ **b.** $\sqrt{\frac{1}{20}}$

9. Simplify: **a.** $\sqrt{3}+5\sqrt{3}-2\sqrt{3}$

b. $\sqrt{5}+\sqrt{5}-3$

10. Simplify: **a.** $\sqrt{2}\sqrt{18}$ **b.** $\sqrt{3}\sqrt{6}$

CHAPTER TEST [B]

1. Find the two square roots of 7089 to the nearest tenth.

2. Find the positive square root of 42436.

3. Find the positive square root of 172.4 to the nearest tenth.

4. Simplify: **a.** $5\sqrt{28}$ **b.** $\sqrt{24\,a^3}$

5. Simplify: **a.** $\sqrt{\frac{1}{12}}$ **b.** $\sqrt{\frac{4}{5}}$

6. Simplify: **a.** $\sqrt{\dfrac{1}{a^3}}$ **b.** $\sqrt{\dfrac{9}{ab}}$

7. Simplify: **a.** $\sqrt{\frac{1}{2}}-\sqrt{8}+\sqrt{128}$

b. $2\sqrt{12}+\sqrt{3}-\sqrt{75}$

8. Multiply: **a.** $3\sqrt{2}\cdot\sqrt{8}$ **b.** $\sqrt{5}\cdot\sqrt{12}$

9. Rationalize the denominator: **a.** $\dfrac{3}{\sqrt{6}}$ **b.** $\dfrac{5}{3\sqrt{2}}$

10. Find the value of $\dfrac{3\sqrt{x}-\sqrt{y}}{2y-x}$ if $x = 100$ and $y = 64$.

MATHEMATICS IN BRIDGEBUILDING

A GOOD BRIDGE should have utility, strength, and beauty. To be beautiful a bridge should have simplicity, harmony of color, graceful lines, and good proportions.

Years ago small streams were crossed by fording or by using wooden bridges, and larger bodies of water were crossed by ferries. Today very few bridges are made of wood. The smaller bridges are made of steel or reinforced concrete, and the largest bridges are made wholly of steel.

Movable bridges are used to span rivers and bodies of water where stationary bridges interfere with boat traffic. There are various types of movable bridges. Some are lifted vertically, some open like double garage doors, and some are hinged at both ends and rotate upwards.

The arch bridge and the suspension bridge are the two main types of the large steel bridges.

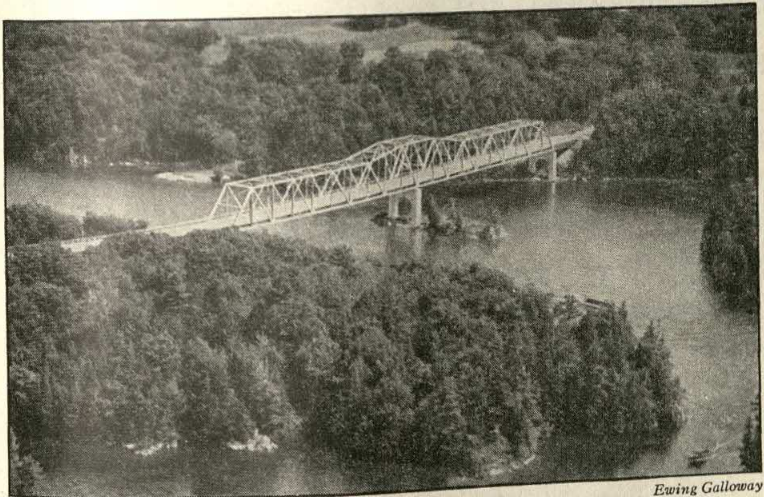
The Rainbow Bridge at Niagara Falls is a good example of the steel arch bridge. It was completed in 1941 at a cost of about $3,000,000. It is the longest bridge of its type in the world. It has a span of 960 feet and the rise of its arch is 150 feet. The deck of the bridge is of reinforced concrete and is 60 feet wide, providing for two lanes of vehicular traffic and one 10-foot sidewalk.

This bridge was built on the cantilever plan from each abutment, the two crews of workmen erecting the bridge from the abutments to meet at the middle.

The Golden Gate Bridge at San Francisco is an excellent example of the suspension bridge. It was completed in 1937. It has the longest clear span in the world, being 4200 feet long. Each of the two side spans is 1125 feet long, making the total length of the bridge 6450 feet. The total length of the bridge plus its approaches is about 7 miles. The steel towers which support the cables rise 746 feet above the low-water level. The cables contain 80,000 miles of steel wire and have a sag of 470 feet. The bridge is 90 feet wide and has six vehicular traffic lanes and two walks for pedestrians.

In a suspension bridge the weight is borne by heavy cables, which consist of a large number of strands of wire bound together. The cables of a suspension bridge form curves called parabolas. (If the cables supported only their own weight, the curves would be catenaries.)

The mathematics needed in the designing and construction of a large suspension bridge can be acquired in a college of engineering. The bridge engineer knows trigonometry, analytic geometry, calculus, mechanics, and differential equations.



Ewing Galloway

The Thousand Islands Bridge · This international bridge, connecting Canada and the United States, is $8\frac{1}{2}$ miles long. In designing the bridge, the engineers utilized the islands to reduce the span lengths required

CHAPTER XVI

Quadratic Equations

Kinds of Quadratic Equations [A]

A quadratic equation is one of the second degree. A quadratic equation in one unknown letter contains the second power of the letter but no higher power.

If a quadratic equation contains the first power of the unknown letter, as $3x^2 + x - 2 = 0$, it is called a *complete* quadratic equation. If it does not contain the first power of the unknown letter, as $3x^2 - 27 = 0$, it is called an *incomplete*, or *pure*, quadratic equation.

In Chapter XII you solved both complete and incomplete quadratic equations by the factoring method. In this chapter other methods of solving them will be studied.

How to Solve an Incomplete Quadratic Equation [A]

In the solution of incomplete quadratic equations use is made of another general principle or axiom:

> **The same roots of equals are equal.**

Example 1. Solve $x^2 - 9 = 0$.

Solution. First solve $x^2 - 9 = 0$ for x^2

$$x^2 = 9$$

*R_2 $$x = \pm 3$$

Check. Does $(+3)^2 - 9 = 0$? Yes.

Does $(-3)^2 - 9 = 0$? Yes.

Although the square roots of x^2 are $\pm x$, we write only the positive x, since we are solving simply for $+x$.

* R_2 means that the square roots of both members are taken.

Example 2. Solve $3x(x-4) - x(x+3) = 3(6-5x)$.

Solution.

$$\begin{aligned} 3x(x-4) - x(x+3) &= 3(6-5x) \\ 3x^2 - 12x - x^2 - 3x &= 18 - 15x \\ 3x^2 - x^2 - 12x - 3x + 15x &= 18 \\ 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

R_2

Note that the equation was first solved for x^2.

To solve an incomplete quadratic equation,

1. Solve the equation for x^2 (or the square of the unknown letter).
2. Find the square roots of both members.

EXERCISES. EQUATIONS

A

Solve the following quadratic equations. If any root is irrational, express it both as a radical in its simplest form and as a decimal.

1. $x^2 = 16$
2. $y^2 = 36$
3. $m^2 = 81$
4. $h^2 = 100$
5. $2x^2 = 200$
6. $3y^2 = 27$
7. $4x^2 = 4$
8. $5c^2 = 125$
9. $x^2 - 64 = 0$
10. $y^2 - 1 = 0$
11. $2a^2 - 98 = 0$
12. $4p^2 - 144 = 0$
13. $225 - y^2 = 0$
14. $2n^2 - 1682 = 0$
15. $\frac{1}{x^2} - 1 = 0$
16. $\frac{x^2 - 4}{4} = 3$
17. $\frac{3x^2}{4} - 3 = 6$
18. $x(2x+3) = 3x + 50$
19. $(x-2)^2 = 16 - 4x$
20. $\frac{x^2 - 1}{2} = 1$
21. $(y-1)(y+1) = 3$
22. $4x^2 - 150 = x^2 - 3$
23. $(2x+5)^2 = 20x$
24. $\frac{x^2 - 3}{2} = \frac{2x^2}{9} + 1$
25. $\frac{2x+5}{2} - x = \frac{x^2}{3}$

B

Solve for x:

26. $ax^2 = 4a$

27. $ax^2 = b$

28. $cx^2 = 2c$

29. $2mx^2 = n$

30. $mx^2 = 1$

31. $\pi x^2 - A = 0$

32. $3x^2 = a - b$

33. $bx^2 = b^2 - 1$

34. $x(x + a) = ax + c$

35. $\frac{2x}{3p} = \frac{x}{6p} + \frac{5}{x}$

36. $8x - \frac{2}{3} = \frac{x(24 - x)}{3}$

37. $2ax^2 - ab = 3ax^2$

38. $\frac{x^2 - a}{b} = \frac{x^2 - b}{a}$

39. $x(4x + 3) - 2x = 3 - (1 - x)$

40. $(2x - 3)(x + 5) = 3(x - 4) + 4x + 1$

41. $(2x - 1)^2 - (x + 2)^2 = -4(2x - 3)$

EXERCISES [B]. SOLVING FORMULAS

Example. Solve the formula $s = \frac{1}{2}gt^2$ for t and find the value of t when $s = 100$ and $g = 32$.

Solution. $s = \frac{1}{2}gt^2$

M_2 $2s = gt^2$

$gt^2 = 2s$

D_g $t^2 = \frac{2s}{g}$

R_2 $t = \pm\sqrt{\frac{2s}{g}} = \pm\sqrt{\frac{2s}{g} \times \frac{g}{g}} = \pm\sqrt{\frac{2gs}{g^2}} = \pm\frac{1}{g}\sqrt{2gs}$

Substituting 32 for g and 100 for s,

$$t = \pm\frac{\sqrt{2 \times 32 \times 100}}{32} = \pm\frac{\sqrt{6400}}{32} = \pm 2.5$$

Since time is always positive, $t = 2.5$.

1. Solve $E = \frac{1}{2}mv^2$ for v and find v when $m = 4$ and $E = 20$.

2. Solve $A = \pi r^2$ for r and find the value of r when $A = 154$ and $\pi = 3\frac{1}{7}$.

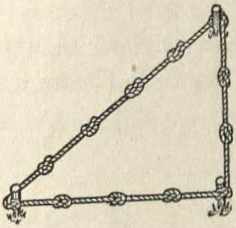
3. Solve $\frac{a}{x} = \frac{x}{b}$ for x and find x when $a = 64$ and $b = 225$.

4. Solve $F = \frac{v^2s}{8\ \pi d^2}$ for d and find d when $v = 16$, $s = 32$, $\pi = 3\frac{1}{7}$, and $F = 160$.

5. Solve $V = \frac{1}{3}\ \pi r^2h$ for r, and find r when $V = 352$, $h = 21$, and $\pi = 3\frac{1}{7}$.

The Pythagorean Theorem[A]

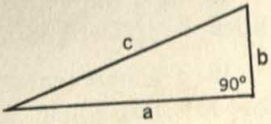
As you know, a right triangle is a triangle having one of its angles a right angle. Right triangles were known and studied thousands of years ago. When the early Egyptians wished to make a right angle they would often make a right triangle by dividing a rope into twelve equal parts and stretching the rope around three posts driven into the ground as shown in the diagram. The sides of this triangle are 3, 4, and 5 units in length. The right angle is opposite the side 5 units long. Does $5^2 = 4^2 + 3^2$?



Years later a very important theorem (a statement to be proved) dealing with all right triangles was discovered and proved. It is believed that this theorem was first proved by Pythagoras, a very able Greek scholar. For this reason it is called the *Pythagorean Theorem.* It is stated as follows:

> **In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.**

The *hypotenuse* of a right triangle is the side opposite the right angle. In the figure the side c is the hypotenuse. By the Pythagorean Theorem, $c^2 = a^2 + b^2$.



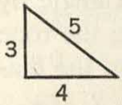
EXERCISES

A

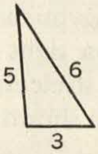
It has been proved that if the sum of the squares of two sides of a triangle is equal to the square of the third side, the triangle is a right triangle.

Tell which of the following triangles are right triangles:

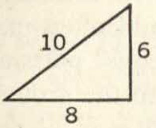
1.



2.



3.



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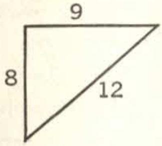

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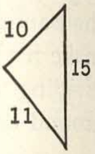

The following figures are right triangles or contain right triangles. Apply the Pythagorean Theorem and find the values of x.

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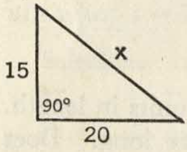

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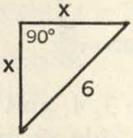

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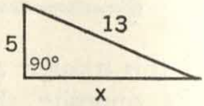

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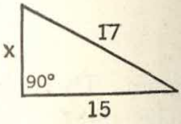

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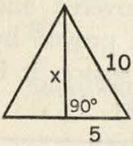

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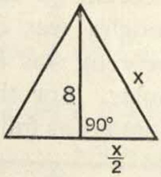

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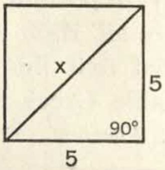

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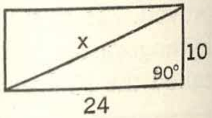
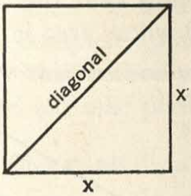


14. The hypotenuse of a right triangle is 25 feet and one side is 15 feet. How long is the other side?

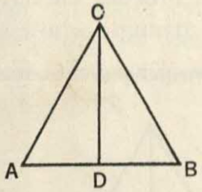
15. Find the hypotenuse of a right triangle whose other two sides are 16 feet and 30 feet.

16. The hypotenuse of a right triangle is 30 feet and the other two sides are equal. How long is each?

17. Find the side of a square whose diagonal is $12\sqrt{2}$ feet.



Ex. 17



Ex. 18

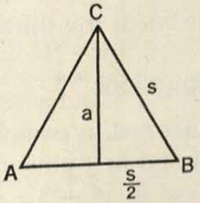
18. *ABC* is an equilateral triangle. Its altitude *CD* is perpendicular to *AB* and bisects (divides into two equal parts) *AB*. If each side of the triangle is 10 inches long, how long is the altitude *CD*?

19. The length of a rectangle is three times its width. If the diagonal of the rectangle is 15 feet, what are its length and width?

20. Find the altitude of an equilateral triangle each of whose sides is 8 inches.

B

21. *ABC* is an equilateral triangle whose side is s and whose altitude is a. Show that the altitude a of an equilateral triangle is given by the formula $a = \frac{s}{2}\sqrt{3}$.



Using the formula $a = \frac{s}{2}\sqrt{3}$, find the altitudes of the equilateral triangles having the following sides:

22. 8 inches **23.** 10 feet **24.** $5\sqrt{3}$ feet **25.** $25\sqrt{3}$ rods

26. What is the base of the equilateral triangle in Fig. 1? What is its altitude in terms of s? Show that the area of an equilateral triangle whose side is s and whose area is A is given by the formula $A = \frac{s^2}{4}\sqrt{3}$.

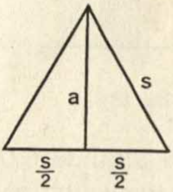


Fig. 1

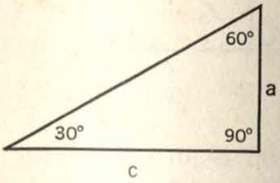


Fig. 2

Using the formula $A = \frac{s^2}{4}\sqrt{3}$, find the areas of the equilateral triangles which have the following sides:

27. 6 inches **28.** 16 feet **29.** $8\sqrt{3}$ inches **30.** $20\sqrt{3}$ feet

If the acute angles of a right triangle contain 30° and 60° respectively, the triangle is known as a 30–60° right triangle. In such a triangle the shortest side is half the hypotenuse. In Fig. 2, $a = \frac{1}{2}c$.

31. The hypotenuse of a 30–60° right triangle is 8 feet. How long are the other sides?

32. The shortest side of a 30–60° right triangle is 6 inches. Find the lengths of the other two sides.

33. Find the lengths of the hypotenuse and the shortest side of a 30–60° right triangle when the third side is $9\sqrt{3}$ inches.

Complete Quadratic Equations [A]*

As you have already learned, a complete quadratic equation contains both the second and first powers of the unknown. The equations $2x^2 + 5x - 3 = 0$, $x^2 - 3x = -2$, and $x^2 + x = 0$ are complete quadratics.

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* In most classes time will not be available to have all the pupils solve complete quadratic equations by completing the square and by formula. Either method may be omitted. Pages 458–461 show how to use the method of completing the square, and pages 463–464 give the formula method. The

There are four methods of solving complete quadratic equations. You know the factoring method. We shall now learn how to solve a complete quadratic by *completing the square.* Before you can use this method you must know how to square binomials quickly and to make perfect square trinomials.

Squaring Binomials [A]

The expression $(x+y)^2$ means $(x+y)(x+y)$. If the product of these equal binomials is found either by the long method shown at the right or by the method of finding special products given in Chapter XI, we find that $(x+y)^2 = x^2 + 2\,xy + y^2$. Notice how the three terms of this special product are found.

$$\boxed{\begin{array}{lll} x & +\,y & \\ \underline{x} & \underline{+\,y} & \\ x^2 + & xy & \\ & +\ \ xy & +\,y^2 \\ \hline x^2 & +\,2\,xy & +\,y^2 \end{array}}$$

1. The first term x^2 is the square of the first term x of the binomial.

2. The middle term $+\,2\,xy$ of the product is found by adding $+\,xy$ to $+\,xy$. Since these two cross products are alike, it is only necessary to find one of them and double it.

3. The last term $+\,y^2$ of the product is the square of y, the last term of the binomial.

Example 1. Square $(2\,x+3)$.

Solution. The first term of the product is the square of $2\,x$, which is $4\,x^2$. The middle term of the product is 2 times $2\,x$ times 3, which is $12\,x$. The last term of the product is the square of $+\,3$, which is 9.

Then $$(2\,x+3)^2 = 4\,x^2 + 12\,x + 9$$

Example 2. Find the value of $(3\,x - 5\,y)^2$.

Solution. $$(3\,x)^2 = 9\,x^2$$

$$2 \text{ times } (3\,x) \text{ times } (-\,5\,y) = -\,30\,xy$$

$$(-\,5\,y)^2 = 25\,y^2$$

Then $$(3\,x - 5\,y)^2 = 9\,x^2 - 30\,xy + 25\,y^2$$

solution by completing the square has the advantage of being understood by the pupils, and of giving practice in adding fractions and simplifying fractional radicands. The formula method requires less time to teach if the derivation of the formula is omitted.

Notice that if you square the *sum* of two numbers, the middle term of the product is *positive*, and that if you square the *difference* of two numbers, the middle term of the product is *negative.*

To square a binomial,

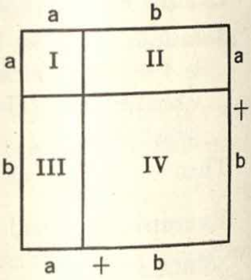
> **1. Square the first term.**
>
> **2. Add twice the algebraic product of the two terms.**
>
> **3. Add the square of the last term.**

EXERCISES [A]

1. $(x + 2)^2$
2. $(a + 3)^2$
3. $(c - d)^2$
4. $(4 - c)^2$
5. $(a + 7)^2$
6. $(m - 5)^2$
7. $(2x - 3)^2$
8. $(x - 6)^2$
9. $(4x - 1)^2$
10. $(x + 10)^2$
11. $(x - 9)^2$
12. $(4x - 7)^2$
13. $(x - \frac{1}{2})^2$
14. $(x + \frac{1}{3})^2$
15. $(x - .5)^2$
16. $(a - \frac{1}{5})^2$
17. $(x + .3)^2$
18. $(y - 1.2)^2$

19. Find the missing parts of these statements:

The area of square I = _ _?_ _ _.
The area of square IV = _ _?_ _ _.
The area of rectangle II = _ _?_ _ _.
The area of rectangle III = _ _?_ _ _.
The sum of the areas of the four figures = _ _?_ _ _.
The length of each side of the square = _ _?_ _ _.
The area of the square = $(? + ?)^2$.
Therefore $(? + ?)^2 = ?^2 + ?\,ab + ?^2$



Finding the Square Roots of Trinomials [A]

In the last list of exercises you squared binomials and obtained square trinomials as results. You will now reverse this process. That is, you will find the square roots of perfect square trinomials and obtain binomial results.

Finding the square root of a number is done by factoring the number into two equal factors, either of which is the square root of the number.

A trinomial is a perfect square if, when it is arranged in either descending or ascending powers of one letter, the first and third terms are positive perfect squares and the middle term, disregarding its sign, is equal to twice the product of the square roots of the first and third terms.

Example 1. Find the positive square root of $x^2 + 8x + 16$.

Solution. The first term is the square of x and the third term is the square of 4. Also, $8x$ is 2 times x times 4. Therefore $x^2 + 8x + 16$ is a perfect square. Since the sign of the middle term is positive, the positive square root of $x^2 + 8x + 16$ is $x + 4$.

Then $\sqrt{x^2 + 8x + 16}$ is $x + 4$.

Example 2. $\sqrt{x^2 - 10x + 25} = ?$

Solution. x^2 is the square of x, and 25 is the square of 5. Also $10x$ is twice x times 5. Therefore $x^2 - 10x + 25$ is a perfect square. The sign of $10x$ is negative. Then $\sqrt{x^2 - 10x + 25} = x - 5$.

EXERCISES [A]

Tell which of the following trinomials are perfect squares, and give the positive square roots of the perfect squares.

1. $x^2 - 4x + 4$

2. $m^2 - 6m + 9$

3. $x^2 + x + 1$

4. $4x^2 - 4x + 1$

5. $9x^2 - 6x + 1$

6. $x^2 - 2x + 1$

7. $y^2 + 12y + 36$

8. $100y^2 + 10y + 1$

9. $x^2 - 14x + 49$

10. $y^2 + 16y + 64$

11. $a^2 + a + \frac{1}{4}$

12. $2a^2 + 6ab + 9b^2$

13. $x^6 - 18x^3 + 81$

14. $c^2 + \frac{2}{3}c + \frac{1}{9}$

15. $h^2 + 3h + 2\frac{1}{4}$

16. $x^2 - 5x + 6\frac{1}{4}$

Making a Perfect Square Trinomial [A]

In solving a complete quadratic equation by completing the square, it is necessary to make one member of the equation a perfect square trinomial.

Example 1. Make a perfect square by adding a term to $x^2 + 8x$.

Solution. The square root of x^2 is x. Then x is the first term of the square root of this trinomial. Then we can write $x^2 + 8x + ? = (x + ?)^2$. Now $8x$ is twice the product of x and the missing term of the binomial. Then the missing term of the binomial is one half of 8 or 4. Now we can write $x^2 + 8x + ? = (x + 4)^2$. Then the missing term of the trinomial is 4^2 or 16. The trinomial is $x^2 + 8x + 16$.

EXERCISES [A]

Find the missing terms in the following to make them perfect square trinomials:

1. $x^2 + 6x + ?$
2. $x^2 + 2x + ?$
3. $c^2 - 6c + ?$
4. $m^2 - 8m + ?$
5. $r^2 + 10r + ?$
6. $x^2 - 12x + ?$
7. $y^2 - 4y + ?$
8. $r^2 - 2r + ?$
9. $a^2 - 16a + ?$
10. $x^2 + 20x + ?$
11. $x^2 + 5x + ?$
12. $x^2 - 7x + ?$
13. $c^2 - 3c + ?$
14. $y^2 - 9y + ?$
15. $p^2 + 11p + ?$
16. $x^2 - 15x + ?$
17. $y^2 + 13y + ?$
18. $x^2 + x + ?$
19. $x^2 - \frac{1}{2}x + ?$
20. $y^2 + \frac{1}{3}y + ?$
21. $y^2 - \frac{2}{3}y + ?$
22. $m^2 - .2m + ?$
23. $r^2 + .4r + ?$
24. $c^2 - \frac{2}{7}c + ?$
25. $x^2 + \frac{5}{3}x + ?$
26. $h^2 + \frac{2}{5}h + ?$
27. $k^2 - \frac{1}{5}k + ?$
28. $m^2 - 4mn + ?$
29. $a^2 + 2ab + ?$
30. $x^4 - 10x^2 + ?$

Solving Quadratics by Completing the Square [A]

When solving a quadratic equation by completing the square, the left member of the equation is made a perfect square so that its square root can be taken. The following example illustrates the different steps in a solution.

Example 1. Solve $x^2 - 6x + 8 = 0$.

Solution. We first transpose so that the constant term $+8$ is in the right member. We then have $x^2 - 6x = -8$. We next add $+9$ to the left member to make it a perfect square. Since we are adding $+9$ to the left member of the equation, we must also add $+9$ to the right member. Then $x^2 - 6x + 9 = 1$. We next find the square roots of both members, getting $x - 3 = \pm 1$. Transposing the -3, we have $x = 3 \pm 1$. Then $x = 4$ or 2.

Check. Does $(4)^2 - 6(4) + 8 = 0$ and does $(2)^2 - 6(2) + 8 = 0$?
Does $16 - 24 + 8 = 0$ and does $4 - 12 + 8 = 0$? Yes.

Directions for Solving a Quadratic Equation by Completing the Square:

1. Write the equation in the form $x^2 + bx = c$.
2. Add to each member the square of half of b.
3. Find the square root of each member, placing the ± sign before the square root of the right member.
4. Solve the two resulting equations.
5. Check both roots by substituting them in the original equation.

Example 2. Solve $x^2 - 11 = 10x$.

Solution. $x^2 - 11 = 10x$
Transposing $x^2 - 10x = 11$
A_{25} $x^2 - 10x + 25 = 36$
R_2 $x - 5 = \pm 6$
Transposing $x = 5 \pm 6$
$x = 11$ or -1

EXERCISES [A]

Solve the following equations by completing the square:

1. $x^2 - 6x = 8$
2. $y^2 - 4y = 5$
3. $m^2 + 2m = 24$
4. $h^2 - 10h = -24$
5. $c^2 - 12c = 13$
6. $x^2 - 16x = 17$

7. $x^2 - 12x + 11 = 0$
8. $x^2 - 8x - 20 = 0$
9. $x^2 + 18x + 17 = 0$
10. $x^2 + 6x = 55$
11. $x^2 + 4x = 117$
12. $x^2 + 6x - 135 = 0$
13. $x^2 - 6x - 91 = 0$
14. $b^2 - 20b = 96$
15. $x^2 + 20x + 75 = 0$
16. $h^2 - 10h = 336$
17. $p^2 - 28p + 180 = 0$
18. $x^2 + 20x = 1500$
19. $s^2 + 6s = 315$
20. $x^2 - 14x - 351 = 0$

Example 3. Solve $x^2 - 3x - 18 = 0$.

Solution. $x^2 - 3x - 18 = 0$

$$x^2 - 3x = 18$$

$$\tfrac{1}{2} \text{ of } 3 = \tfrac{3}{2} \quad \text{and} \quad \left(\tfrac{3}{2}\right)^2 = \tfrac{9}{4}$$

$A_{\frac{9}{4}}$ $$x^2 - 3x + \tfrac{9}{4} = \tfrac{81}{4} \qquad \left(\tfrac{18}{1} + \tfrac{9}{4} = \tfrac{72}{4} + \tfrac{9}{4} = \tfrac{81}{4}\right)$$

R_2 $$x - \tfrac{3}{2} = \pm \tfrac{9}{2}$$

$$x = \tfrac{3}{2} \pm \tfrac{9}{2}$$

$$x = 6 \text{ or } -3$$

Notice that we write $\frac{1}{2}$ of 3 in the form $\frac{3}{2}$, and not in the form $1\frac{1}{2}$. Do you know why?

EXERCISES [A]

Solve:

1. $x^2 + 5x = -4$
2. $y^2 - 9y = -20$
3. $x^2 - 3x = 10$
4. $x^2 - 11x = -28$
5. $x^2 - 3x - 40 = 0$
6. $x^2 - 7x + 6 = 0$
7. $c^2 - c - 56 = 0$
8. $x^2 + x = 72$
9. $m^2 - m - 30 = 0$
10. $h^2 - h = 110$
11. $y^2 + 45y + 450 = 0$
12. $m^2 - 5m - 750 = 0$
13. $x^2 - 4x = 780$
14. $r^2 + 7r = 540$

Express the roots of the following equations in simplest radical form:

15. $x^2 - 6x = 7$
16. $y^2 - 4y = 8$
17. $x^2 + 10x = 25$
18. $y^2 + 2y = 7$

19. $x^2 = 8x + 2$

20. $y^2 = 10y + 5$

21. $x^2 - 9 = 6x$

22. $h^2 = 3 + 6h$

23. $x^2 = 10x + 50$

24. $m^2 + 14m + 1 = 0$

Example 4. Solve $3x^2 + 7x - 5 = 0$.

Solution. $3x^2 + 7x - 5 = 0$

Transposing, $3x^2 + 7x = 5$. This equation should be divided by 3 to get *one* x^2 as the first term.

D_3 $\qquad x^2 + \frac{7}{3}x = \frac{5}{3}$

$$\tfrac{1}{2} \text{ of } \tfrac{7}{3} = \tfrac{7}{6} \text{ and } \left(\tfrac{7}{6}\right)^2 = \tfrac{49}{36}$$

Also, $\qquad \frac{49}{36} + \frac{5}{3} = \frac{49}{36} + \frac{60}{36} = \frac{109}{36}$

$A_{\frac{49}{36}}$ $\qquad x^2 + \frac{7}{3}x + \frac{49}{36} = \frac{109}{36}$

R_2 $\qquad x + \frac{7}{6} = \pm \frac{\sqrt{109}}{6}$

$$x = -\frac{7}{6} \pm \frac{\sqrt{109}}{6}$$

$$x = \frac{-7 \pm \sqrt{109}}{6}$$

In decimal form, $\qquad x = \frac{-7 \pm 10.44}{6} = .57 \text{ or } -2.91$

CAUTION. Be sure to learn and use all the different steps in the solution of a quadratic equation. The omission of steps usually results in errors.

EXERCISES [A]

The roots of some of the following equations contain radicals. Express these roots in their simplest radical form and also in decimal form as shown in Example 4.

1. $2x^2 + 7x = 15$

2. $2x^2 - 3x + 1 = 0$

3. $6x^2 + x = 2$

4. $5x^2 + 8x + 1 = 0$

5. $2x^2 - 7x = 15$

6. $4x^2 + 1 = 5x$

7. $6x^2 + 13x = 8$

8. $x^2 - x - 1 = 0$

9. $5y^2 + 11y - 83 = 0$

10. $2y^2 = 3y + 1$

11. $6x^2 = 1 - 5x$

12. $8c^2 + 6c + 1 = 0$

13. $4x^2 + 2x = 1$

14. $4x^2 - 1 = 3x$

15. $2x^2 - 10x = 9$

16. $2x^2 - 6x + 2 = 0$

17. $3m^2 - 11 = m$

18. $3x^2 - 3x = 36$

19. $3k^2 + 8k + 5 = 0$

20. $3y^2 - 2 + 5y = 0$

Example 5. Solve $\dfrac{1}{x-2} + 1 = \dfrac{6-x}{x^2-4} + \dfrac{1}{x+2}$.

Solution.

$$\frac{1}{x-2} + 1 = \frac{6-x}{x^2-4} + \frac{1}{x+2}$$

$$\frac{1}{x-2} + 1 = \frac{6-x}{(x-2)(x+2)} + \frac{1}{x+2}$$

The L.C.D. is $(x-2)(x+2)$.

$$\cancel{(x-2)}(x+2)\frac{1}{\cancel{x-2}} + (x-2)(x+2)1 =$$

$$\cancel{(x-2)}\cancel{(x+2)}\frac{6-x}{\cancel{(x-2)}\cancel{(x+2)}} + (x-2)\cancel{(x+2)}\frac{1}{\cancel{x+2}}$$

$$(x+2) + (x-2)(x+2)1 = (6-x) + (x-2)1$$

$$x + 2 + x^2 - 4 = 6 - x + x - 2$$

$$x^2 + x = -2 + 4 + 6 - 2$$

$$x^2 + x = 6$$

$A_{\frac{1}{4}}$ $$x^2 + x + \tfrac{1}{4} = \tfrac{25}{4}$$

R_2 $$x + \tfrac{1}{2} = \pm\tfrac{5}{2}$$

$$x = -\tfrac{1}{2} \pm \tfrac{5}{2}$$

$$x = 2 \text{ or } -3$$

EXERCISES [B]

1. $(x+4)^2 = 9x^2$

2. $(x+1)(x-4) = 50$

3. $\dfrac{x}{x-3} + \dfrac{8}{x+2} = 3$

4. $4x^2 = \dfrac{2x}{3} + \dfrac{1}{2}$

5. $(2x-1)(3x+2) = 55$

6. $\dfrac{1}{y} + \dfrac{1}{2} = \dfrac{1}{y-2}$

7. $\dfrac{x-4}{x+2} + \dfrac{x+3}{x-2} = \dfrac{9x+2}{x^2-4}$

Solving Quadratic Equations by Formula [A or B]

Any quadratic equation can be written in the general form $ax^2 + bx + c = 0$. In this equation a, b, and c represent known numbers and x stands for the unknown number. We shall now solve $ax^2 + bx + c = 0$ for x and obtain a formula for solving any quadratic equation.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$\text{D}_a \qquad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\frac{1}{2} \text{ of } \frac{b}{a} = \frac{b}{2a} \text{ and } \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\text{Also,} \qquad \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\text{A}_{\frac{b^2}{4a^2}} \qquad x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\text{R}_2 \qquad x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $$ax^2 + bx + c = 0,$$

then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this formula $+a$ is the number of x^2's, $+b$ is the number of x's, and $+c$ is the constant. Thus in the equation $2x^2 + 3x - 4 = 0$, $a = 2$, $b = 3$, and $c = -4$. Also in the equation $x^2 - x + 5 = 0$, $a = 1$, $b = -1$, and $c = 5$.

To determine the values of a, b, and c in the equation $4x^2 = 7 - x$, it must be written in the general form $4x^2 + x - 7 = 0$.

Example 1. Solve: $3x^2 - 5x = 2$

Solution. The right member must be zero. Then $3x^2 - 5x - 2 = 0$. Comparing this equation with $ax^2 + bx + c = 0$, we see that $a = 3$, $b = -5$, and $c = -2$. Write the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting $+3$ for a, -5 for b, and -2 for c in this formula, we get

$$x = \frac{+5 \pm \sqrt{25 - 4(+3)(-2)}}{6}$$

Simplifying $x = \dfrac{5 \pm \sqrt{49}}{6}$

$$x = \frac{5 \pm 7}{6}$$

$$x = 2 \text{ or } -\tfrac{1}{3}$$

The check is left to the pupil.

Example 2. Solve: $9x^2 + 12x = 1$

Solution. $9x^2 + 12x = 1$

Transposing $9x^2 + 12x - 1 = 0$

Write the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substituting in the formula $a = +9$, $b = +12$, $c = -1$, we get

$$x = \frac{-12 \pm \sqrt{144 - 4(+9)(-1)}}{18}$$

Simplifying $x = \dfrac{-12 \pm \sqrt{180}}{18}$

$$x = \frac{-12 \pm 6\sqrt{5}}{18}$$

Reducing the fraction $x = \dfrac{-2 \pm \sqrt{5}}{3}$

In decimal form $x = \dfrac{-2 \pm 2.236}{3} = +.079 \text{ or } -1.412$

CAUTION. In substituting in the formula, remember that $-b$ means -1 times b.

If you expect to continue the study of mathematics, review the quadratic formula each day until you can remember it.

EXERCISES [A or B]

Solve by the quadratic formula and check as indicated by your teacher:

1. $x^2 - 3x - 1 = 0$

2. $x^2 + x = 12$

3. $2x^2 - x - 3 = 0$

4. $x^2 + 2x - 5 = 0$

5. $x^2 - 6x + 2 = 0$

6. $3x^2 + 5x = 2$

7. $5x^2 + 9x - 2 = 0$

8. $x^2 - 3x + 2 = 0$

9. $x^2 + x = 20$

10. $x^2 - x = 1$

11. $x^2 + x = 2$

12. $-x^2 + 6x + 7 = 0$

13. $y^2 - 8y + 10 = 0$

y in this equation corresponds to x in the formula.

14. $x^2 + 54 = 15x$

15. $3y^2 = y + 4$

16. $y + 1 = 6y^2$

17. $m^2 - 3m = 5$

18. $3m^2 + 2m = 5$

19. $x^2 - 6x = 11$

20. $4y^2 + 7 = 12y$

21. $18x^2 = 2 - 24x$

22. $3x^2 + 2x = 5$

23. $-x^2 + 1 = 0$

EXERCISES

A

Solve the following equations by factoring and then by completing the square or by formula:

1. $x^2 - x - 6 = 0$

2. $x^2 - 2x = 24$

3. $2x^2 + 3x - 2 = 0$

4. $3x^2 + 13x + 4 = 0$

5. $x^2 + 5x = 36$

6. $4x^2 - x - 5 = 0$

7. $9x^2 + 4 = 15x$

8. $4x^2 + 14x = 8$

9. $y^2 + 6y = 16$

10. $y^2 + 12y = 45$

Simplify the following equations and solve them by the method indicated by your teacher:

11. $x(2x+3)=35$

12. $(x-3)^2=x-1$

13. $(x+2)(x-4)=0$

14. $x+\frac{1}{x}=\frac{3}{x}+3$

15. $(x+2)(x+3)=42$

16. $x^2+(x-2)^2=10$

17. $\frac{3y}{2}+\frac{2}{y}-\frac{2y}{3}=0$

18. $(1-x)^2+(2-x)^2=1$

19. $(1+x)^2+(2x+1)^2=34$

20. $2x(x+6)-3x=5$

21. $\frac{8}{x+2}+\frac{x}{x-3}=3$

22. $x(x-2)+(2x-3)+2=0$

23. $\frac{3}{x-5}=5-\frac{2x}{x-3}$

24. $1+\frac{3}{x}=\frac{70}{x^2}$

B

Solve:

25. $\frac{2}{3x}+4=\frac{5x}{6x+7}$

26. $\frac{x-3}{x+1}-\frac{x+2}{2x-2}=-\frac{12}{x^2-1}$

27. $\frac{2}{x-1}-\frac{3}{8}=\frac{1}{x+3}$

28. $\frac{8}{x}-\frac{2-3x}{4x^2}=15\frac{1}{2}$

29. $\frac{3}{2y+1}+\frac{2}{3}=\frac{5}{4y^2-1}$

30. $\frac{2x}{x-1}+\frac{x}{x+1}=\frac{4}{x^2-1}$

31. $\frac{7x+4}{x^2+2x}=\frac{14}{x+4}$

32. $\frac{1}{x-1}+\frac{1}{x-2}=\frac{1}{x-3}$

33. $\frac{8}{x-1}-\frac{4}{x+3}=1\frac{1}{2}$

34. $\frac{x}{x+3}+\frac{5}{x}=\frac{3}{2}$

35. $\frac{2}{x-1}-\frac{1}{3x+1}=\frac{1}{4}$

36. $\frac{3}{x^2-x}-\frac{5}{2x-2}=\frac{1}{2}$

37. $\frac{x}{2x-2}=\frac{4}{x^2-1}+\frac{5}{x+1}$

38. $\frac{2y+3}{y^2+y}-\frac{y+5}{y^2-y}=\frac{21}{8y^2-8}$

39. $\frac{x+5}{x^2+7x+12}=\frac{x-1}{2x^2+5x-3}$

40. $\frac{x}{x-3}+\frac{x}{x+3}=\frac{3+x}{3x+9}$

How to Solve Quadratic Equations Graphically [C]

A quadratic equation whose roots are real numbers can be solved very easily by the graphic method. Study the following example.

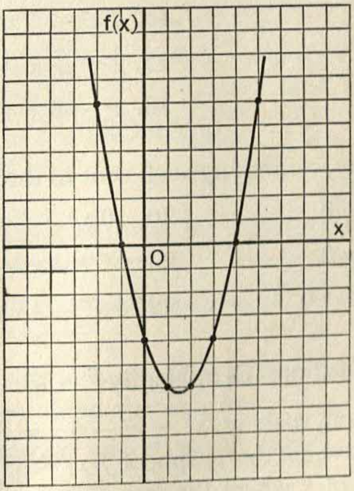
Example. Solve $x^2 - 3x = 4$ graphically.

Solution. The equation should be written as $x^2 - 3x - 4 = 0$, so that the right member is zero. The value of the left member $x^2 - 3x - 4$ depends upon the value of x. When different values are assigned to x, the expression $x^2 - 3x - 4$ takes on different values. Since the expression depends upon x for its value, it is called a *function of x*, abbreviated $f(x)$.

We next prepare a table for the equation $f(x) = x^2 - 3x - 4$.

If $x =$	5	4	3	2	1	0	-1	-2
Then $f(x)$ or $x^2 - 3x - 4 =$	6	0	-4	-6	-6	-4	0	6

The points whose co-ordinates are these pairs of values are plotted on graph paper as shown below. Then a smooth curve is drawn through the points in order. This curve is the graph of



$f(x) = x^2 - 3x - 4$. Solving the equation $x^2 - 3x - 4 = 0$ consists in finding the values of x which make $x^2 - 3x - 4$, or $f(x)$, equal to zero. The curve crosses the x-axis at $x = 4$ and at $x = -1$. At these two points the value of $x^2 - 3x - 4$, or $f(x)$, is zero. Then $x = 4$ and $x = -1$ are solutions of $x^2 - 3x - 4 = 0$.

The curve which is the graph of $f(x) = x^2 - 3x - 4$ is a *parabola.* The parabola has many important applications in engineering and science. The path of a projectile fired from a gun would be a parabola if there were no air resistance. Longitudinal sections of automobile headlights are modified parabolas. The strongest simple arch is parabolic in shape.

EXERCISES [C]

Solve the following equations graphically:

1. $x^2 - x - 6 = 0$

2. $x^2 - 2x = 8$

3. $x^2 - 4x - 5 = 0$

4. $x^2 - 5x = -6$

5. $x^2 - 2x = 0$

6. $x^2 + 3x = 0$

7. $x^2 - 9 = 0$

8. $x^2 - 4 = 0$

9. $2x^2 + 5x - 3 = 0$

10. $2x^2 - 9x + 9 = 0$

11. $x^2 - x = 12$

12. $x^2 - 6x + 9 = 0$

13. $3x^2 + 5x = 2$

14. $2x^2 - 5x = 18$

15. $2x^2 + x - 15 = 0$

16. $x^2 - 8x + 16 = 0$

Solve graphically, estimating each root to the nearest tenth:

17. $x^2 - 3x - 2 = 0$

18. $x^2 - 5x - 1 = 0$

19. $6x^2 - 7x + 2 = 0$

20. $10x^2 - 7x + 1 = 0$

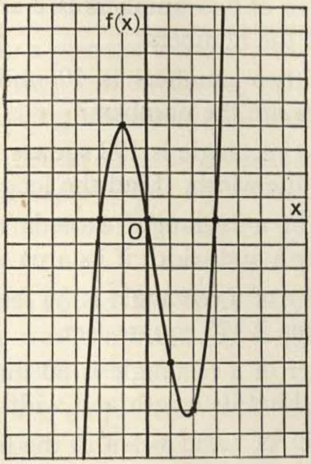
21. $x^2 + 4x - 1 = 0$

22. $6x^2 - 13x + 2 = 0$

Solving Cubic Equations Graphically [C]

A cubic equation such as $x^3 - x^2 - 6x = 0$ can be solved graphically by drawing the graph of $f(x) = x^3 - x^2 - 6x$ and finding the values of x where the curve crosses the x-axis. The graph of this equation crosses the x-axis at $x = -2$, $x = 0$, and $x = 3$. Then -2, 0, and 3 are the roots of the equation.

Notice that the curve has two turning points. A straight line cannot cut this curve in more than three points.



EXERCISES [C]

Graph the following equations and find their roots:

1. $x^3 - 4x^2 + 4x = 0$
2. $x^3 - x^2 - 4x + 4 = 0$
3. $x^3 + 3x^2 - 4x - 12 = 0$
4. $x^3 - 16x = 0$

PROBLEMS

A

The following problems lead to quadratic equations. Remember that each number sought will have two values.

1. The square of a certain number is 30 more than the number. What is the number?

2. Twice the square of a certain number equals 40 increased by twice the number. Find the number.

3. A certain number added to its square equals twelve. What is the number?

4. Twice the square of a certain number decreased by 6 times the number is 80. Find the number.

5. The sum of two numbers is 14 and their product is 48. What are the numbers?

6. The difference of two numbers is 2 and the sum of their squares is 52. Find the numbers.

7. The sum of two numbers is 10 and the sum of their squares is 58. What are the numbers?

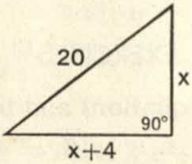
8. The area of a rectangle is 192 square feet and the length is 4 feet more than the width. Find the length and width.

9. The length of a rectangle exceeds twice its width by 6 feet. Find its length and width if its area is 176 square feet.

10. The perimeter of a rectangle is 56 feet. Find the length of each side if its area is 195 square feet.

11. The perimeter of a rectangle is 80 inches and its area is 396 square inches. Find its length and width.

12. Find the value of x and $x + 4$ in the figure below.



13. The hypotenuse of a right triangle is 85 inches long. Find the lengths of the two other sides if one exceeds the other by 35 inches.

14. The diagonal of a square is 50 feet. How long is each of its sides?

15. The diagonal of a square is 3 inches longer than one of its sides. Find the length of each of its sides.

16. Find the number of degrees in the angle which is 15° less than twice the square of its complement.

B

17. Find the legs of a right triangle if their difference is 7 and the hypotenuse is 13.

18. One leg of a right triangle is 10 inches. Find the length of the hypotenuse if it is 2 inches longer than the other leg.

19. Find the bases and altitude of a trapezoid whose area is 126 square inches if the altitude is one third of the lower base and the lower base is 6 inches longer than the upper base.

20. A man can do a piece of work in 2 days less than his son. If working together they can do the work in $4\frac{4}{9}$ days, how long will it take each working alone to do the work?

21. A fish pond can be filled in 4 hours by two pipes when both are used at the same time. How many hours are needed for each pipe to fill the pond if the smaller requires 3 more hours than the larger?

22. A man made a round trip of 36 miles in 54 minutes. If his speed returning was 9 miles an hour faster than his speed going, what was his rate each way?

23. A man makes a daily automobile trip of 80 miles at a certain rate. If he were to increase his rate 8 miles an hour he would make the trip in 30 minutes less time. What is his usual rate?

Historical Note [A]. The name of the person who first solved a quadratic equation is unknown. It is known, however, that Diophantus, who lived at Alexandria about 300 A.D., solved them by a method similar to our "completing the square" method.

During the next thousand years most of the work with quadratic equations was done by the Hindus. The most noted of the Hindu mathematicians were Brahmagupta (born about 598 A.D.), and Bhāskara (born about 1114 A.D.) The Hindu method of solving a quadratic equation follows:

$$\begin{array}{lr}
 & 2x^2 + 5x - 3 = 0 \\
 & 2x^2 + 5x = 3 \\
{}^{*}\mathrm{M}_8 & 16x^2 + 40x = 24 \\
\mathrm{A}_{25} & 16x^2 + 40x + 25 = 49 \\
\mathrm{R}_2 & 4x + 5 = 7 \\
 & 4x = 2 \\
 & x = \frac{1}{2}
\end{array}$$

---

* In the Hindu method both members of the equation are multiplied by 4 times the coefficient of x^2. Here $4 \times 2 = 8$.

The early Hindus used only the positive square root in the solution. It was not until the time of Gauss (1777–1855), the great German mathematician, that the two roots of quadratic equations were understood.

WORD LIST

Can you spell and use the following words?

circumference (p. 54)
equation (p. 21)
graphic solution (p. 467)
hypotenuse (p. 451)
linear equation (p. 289)
pure quadratic (p. 448)
Pythagoras (p. 451)
quadratic (p. 448)
right triangle (p. 451)
square root (p. 423)
trinomial (p. 13)

CHAPTER REVIEW [A]

Solve:

1. $x^2 = 16$

2. $3x^2 = 27$

3. $4m^2 = 128$

4. $4x^2 - 1 = 0$

5. $9x^2 - 4 = 0$

6. $x^2 - 5 = 15$

7. $x^2 = 25a^2$

8. $m^2 - 4m = -3$

9. $2x^2 + 5x + 2 = 0$

10. $3(x^2 - 7) = 28 - x^2$

11. $(2x - 3)^2 = 6(3 - 2x)$

12. $4x^2 + 12x + 8 = 0$

13. $\frac{2}{x} - 1 = \frac{5}{x^2} - 2$

14. $x - 3\frac{1}{2} = \frac{3}{x + 1}$

15. $2x + \frac{23}{x} = 8x - \frac{21}{x}$

16. $\frac{1}{x - 1} + \frac{1}{x + 1} = \frac{1 - x^2}{x^2 - 1}$

17. Solve $K = \frac{1}{2}mv^2$ for v and find v to the nearest tenth when $K = 80$ and $m = 5$.

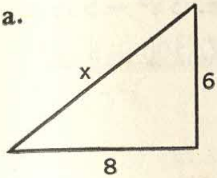
18. Solve $s = \frac{1}{2}gt^2$ for t and find t to the nearest tenth when $s = 120$ and $g = 32$.

19. Solve $A = \pi r^2$ for r and find r to the nearest tenth when $A = 55.23$ and $\pi = 3.14$.

20. State the Pythagorean Theorem.

21. Find the value x in the following right triangles:

a.



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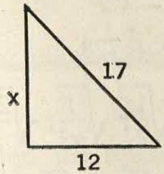

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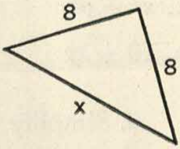

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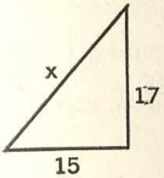

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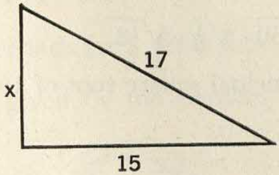
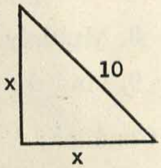


f.



22. The two legs of a right triangle are equal. Find their lengths if the hypotenuse is 16 inches.

23. Find the height of an equilateral triangle each of whose sides is 12 inches.

24. The diagonal of a square is $12\sqrt{3}$. How long is each of its sides?

25. A rectangular field is 80 rods long and 30 rods wide. How long is each of its diagonals?

GENERAL REVIEW

A

1. From the sum of $4a^3 - 6a + 7a^2 - 6$ and $3a^3 - a - 5a^2$ take $7a^3 - 4a + 3a^2 - a + 1$.

2. Find the products:

a. $(-3x)(4x^2)$

b. $(xy^2)(xy^3)$

c. $(-2ab)^3$

d. $\frac{2a}{3} \times \frac{9a}{10}$

e. $(x^2 - x + 1)(x - 3)$

f. $\frac{a^2 - a - 6}{a + 1} \cdot \frac{5a + 5}{a^2 - 3a}$

3. Multiply $x^2 - x - 12$ by $3x + 5$ and divide the product by $x + 3$.

4. Divide $18x^2y^3 - 12x^3y^2 - 6x^4y - 2xy$ by $2xy$.

5. If $x = -4$ and $y = 3$, find the value of:

a. $x^3 + y^3$ **c.** $3y^2 - 4x^2$ **e.** $3x^2 - 5xy$

b. $10 + xy$ **d.** $(xy)^3 + 728$ **f.** $100 + 5xy$

6. Simplify $\sqrt{x^6}$; $\sqrt{75}$; $\sqrt{\frac{1}{2}}$; $\sqrt{\frac{1}{\pi^2}}$; $\sqrt{\frac{25x^4}{9}}$.

7. Multiply $\sqrt{8} \cdot \sqrt{45}$; $\sqrt{20} \cdot \sqrt{72}$.

8. Multiply $\sqrt{50} \cdot \sqrt{\frac{1}{2}} \cdot \sqrt{18}$.

9. Find the principal square root of 191406.25.

Combine:

10. $\frac{5a}{6} - \frac{3a}{4}$ **12.** $\frac{m}{n} - 1$ **14.** $\frac{x-1}{3} - \frac{2x+1}{4}$

11. $\frac{a}{b} + 1$ **13.** $\frac{a}{b} - \frac{b}{a}$ **15.** $\frac{x+1}{8x} - \frac{x+5}{6x^2}$

16. Divide $\frac{a^2 + 4a}{2a+1}$ by $\frac{a^2 - 16}{4a^2 - 1}$.

Solve:

17. $2(x - 3) - 4(x + 2) = 0$ **19.** $c(c - 3) - (c - 5) = 4$

18. $x(x - 3) + 3x = 81$ **20.** $50.5r = 49 + 15.5r$

B

Solve:

21. $\frac{x-1}{x^2-2} = \frac{x+3}{x^2+2}$ **22.** $\frac{1}{x} + \frac{2x}{x-1} = \frac{8}{x^2 - x}$

23. $S = vt + \frac{1}{2}gt^2$. Find S when $v = 8$, $t = 6$, and $g = 32.2$.

24. $c = \frac{En}{R + rn}$. Find c when $n = 8$, $E = 30$, $R = 8$, and $r = 6$.

25. Solve graphically:

$$\begin{aligned} x + y &= 3 \\ 3x - y &= 11 \end{aligned}$$

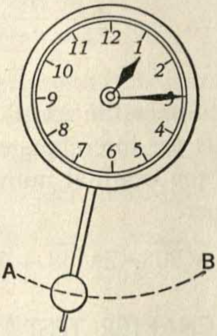
26. Reduce $\frac{2a^2 - 5a - 3}{8a^2 + 4a}$ to lowest terms.

27. Multiply $\frac{x^2 - xy}{4x^2 - 6xy}$ by $\frac{4x^2 - 9y^2}{x^2 - 2xy + y^2}$.

28. Tell which of the following are irrational: $\sqrt{4}$; $\sqrt{5}$; $\sqrt[3]{8}$; $\sqrt[3]{125}$; $\sqrt{\frac{1}{3}}$.

29. How do you find the square root of a product? of a fraction?

30. The time t in seconds for a pendulum to move from A to B or from B to A is given by the formula $t = \pi\sqrt{\frac{l}{g}}$. In this



formula l is the length of the pendulum in feet and g is the acceleration due to gravity. The clock ticks when the pendulum weight is at A and at B.

a. How many seconds are there between two successive ticks of a clock whose pendulum is 2 feet long if $g = 32.2$?

b. Find the length of the pendulum of a clock which ticks twice a second if the value of g is 32.

c. The value of g is less on a high mountain than at sea level. Will a clock run faster or more slowly on a mountain than at sea level?

d. Should the pendulum be shortened or lengthened to make the clock run faster?

Combine:

31. $\frac{3a+1}{6}+\frac{a+5}{3}-\frac{a-3}{5}$

32. $\frac{2a+1}{2a-1}+\frac{2a-1}{2a+1}+\frac{6}{4a^2-1}$

33. $\frac{a-3b}{a^2-ab-12b^2}-\frac{a+3b}{a^2-7ab+12b^2}$

Solve for x and y:

34. $\frac{x+2y}{x-2}=5$

$\frac{2x-3y}{2x+3}=\frac{1}{3}$

35. $\frac{x+2y}{x-y}=-\frac{1}{5}$

$\frac{x-y}{x+y}=5$

36. In a two-digit number the units' digit exceeds the tens' digit by 3. If the digits are interchanged, a number is formed which when added to the original number gives a sum of 77. Find the original number.

37. Simplify $\sqrt{12}-\sqrt{50}-2\sqrt{48}+2\sqrt{18}$

CHAPTER TEST [A]

Solve:

1. $x^2=64$

2. $(x-3)^2=2(17-3x)$

Solve by completing the square:

3. $x^2-7=6x$

4. $x^2+14x=15$

5. $4x^2+3=8x$

6. $3y^2-11=y$

7. $(x-1)^2+(x-2)^2-1=0$

8. The hypotenuse of a right triangle is 17 and one side is 8. Find the remaining side.

9. Find the length of the diagonal of a rectangle which is 40 feet long and 9 feet wide.

10. The sum of two numbers is 15 and the sum of their squares is 117. What are the numbers?

CHAPTER TEST [B]

Solve for x:

1. $bx^2 = 9b$. **2.** $ax^2 = 4$. **3.** $\pi x^2 - c = 0$.

4. Solve $E = \frac{1}{2}mv^2$ for v and find v when $m = 2$ and $E = 16$.

5. Find the altitude of an equilateral triangle each of whose sides is 10.

Solve by completing the square or by formula:

6. $x^2 - x + 1 = 0$ **7.** $3x + 1 = 4x^2$

8. $\dfrac{x-2}{x-4} + \dfrac{x+2}{x+4} = \dfrac{8(x+1)}{x^2-16}$

9. The area of a rectangle is 216 square feet and the length is 6 feet more than the width. What is the length of the rectangle?

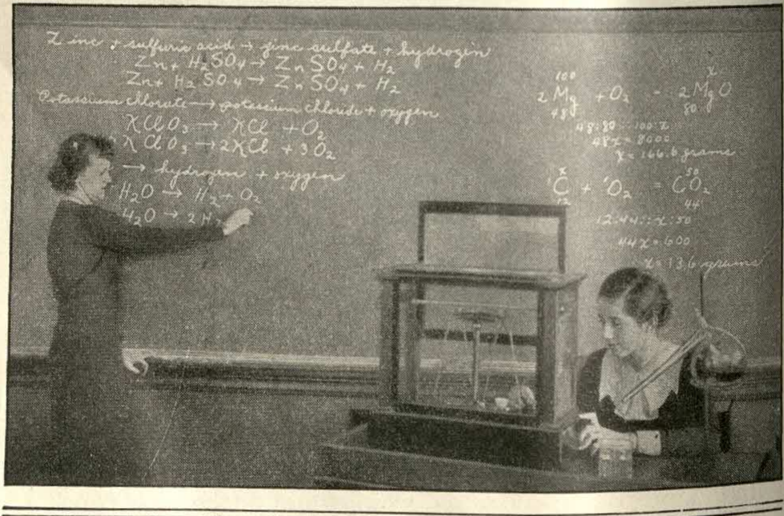
10. The hypotenuse of a right triangle is 30 feet. If one leg of the right triangle is 6 feet more than the other leg, find the length of the shorter leg.

MATHEMATICS AND CHEMISTRY

CHEMISTRY is the science which investigates the composition of substances and observes what changes are produced in them by the action of other substances and by outside forces.

Arithmetic and algebra are needed in the study of high-school chemistry. In the study of gases the student needs to be able to solve simple equations and to have a working knowledge of proportion and variation.

The story of a chemical reaction can be told by a chemical equation, such as $2\,Na + 2\,H_2O \rightarrow 2\,NaOH + H_2$. This equation states that 2 atoms of sodium added to 2 molecules of water produces 2 molecules of sodium hydroxide plus 2 atoms of hydrogen. From such an equation one can form a proportion showing how much of one kind of substance is needed to produce another kind.



Working Problems in Chemistry

CHAPTER XVII

Proportion and Variation

Proportion [A]

In this chapter you will continue the study of proportion, begin the study of variation, and learn how proportion and variation are related.

You know that the ratio of one number to another is found by dividing the first number by the second. You know that a ratio is a fraction and that a proportion is an equation whose two members are ratios. Thus $\frac{6}{15} = \frac{8}{20}$ and $\frac{a}{b} = \frac{c}{d}$ are proportions. The proportion $\frac{a}{b} = \frac{c}{d}$ can be written $a : b = c : d$. You will notice that the colon (:) means "divided by." The *terms* a, b, c, and d of the proportion are numbered from left to right — a being the first term, b the second, c the third, and d the fourth.

The middle terms, b and c, are the *means* and the outside terms, a and d, are the *extremes*. In the proportion $\frac{x}{3} = \frac{4}{7}$, 3 and 4 are the means and x and 7 are the extremes.

The Test of a Proportion [A]

If we clear the proportion $\frac{a}{b} = \frac{c}{d}$ of fractions by multiplying both members by bd, we get $bd\left(\frac{a}{b}\right) = bd\left(\frac{c}{d}\right)$, which simplifies to $ad = bc$. Reversing this process and dividing $ad = bc$ by bd, we get $\frac{ad}{bd} = \frac{bc}{bd}$, which becomes $\frac{a}{b} = \frac{c}{d}$. Then $ad = bc$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent equations. Since a and d are the extremes and b and c are the means, we know that

In a proportion the product of the means is equal to the product of the extremes.

Example 1. Clear the proportion $\frac{2x}{3} = \frac{4}{7}$ of fractions.

Solution. Since $\frac{2x}{3} = \frac{4}{7}$ is a proportion, we know that

$$(3)(4) = (2x)(7), \text{ or } 12 = 14x.$$

Example 2. Is $\frac{3}{4} = \frac{9}{13}$ a proportion?

Solution. If $\frac{3}{4} = \frac{9}{13}$ is a proportion, then $(4)(9) = (3)(13)$ and $36 = 39$. This is not true; then $\frac{3}{4} = \frac{9}{13}$ is not a proportion.

EXERCISES [A]

1. What is the ratio of 4 to 6? of 8 to 10? of x to y? of 2 to 5? of 5 to 2?

Which of the following are proportions?

2. $\frac{3}{7} = \frac{15}{36}$

3. $\frac{8}{7} = \frac{16}{14}$

4. $\frac{18}{27} = \frac{10}{15}$

5. $\frac{25}{36} = \frac{36}{52}$

6. $\frac{6a}{9b} = \frac{10a}{15b}$

7. $8 : 12 = 12 : 16$

Solve for x in each proportion:

8. $\frac{x}{3} = \frac{5}{6}$

9. $\frac{4}{x} = \frac{x}{16}$

10. $\frac{a}{b} = \frac{c}{x}$

11. $\frac{x}{5} = \frac{125}{4}$

12. $\frac{x-2}{3} = \frac{4}{x+2}$

13. $\frac{x-3}{x+3} = \frac{x+5}{x-7}$

14. State the principle of proportion that enables one to clear an equation such as $\frac{2x}{x+3} = \frac{x}{2}$ of fractions.

Solve for y:

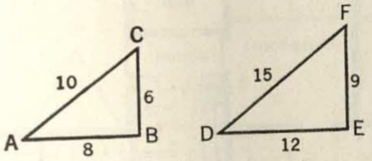
15. $\frac{y-6}{3} = \frac{1}{4}$

16. $\frac{3}{7} = \frac{1}{y}$

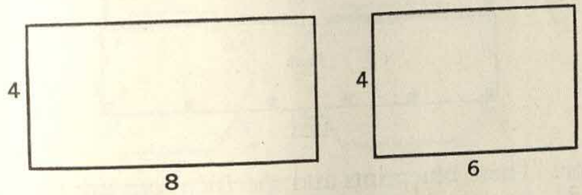
Similar Polygons [A]

Similar polygons are polygons which have the same shape. They are polygons whose corresponding angles are equal and whose corresponding sides are in proportion.

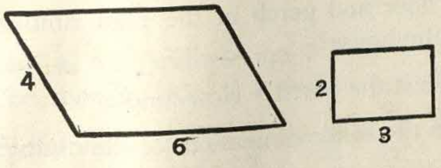
These two triangles (polygons with three sides) are similar ($\sim$). Notice that $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$. The sides are proportional, since $\frac{6}{9} = \frac{8}{12}$, $\frac{6}{9} = \frac{10}{15}$, and $\frac{8}{12} = \frac{10}{15}$.



These two rectangles are not similar. Their corresponding angles are equal but their corresponding sides are not proportional. Is $\frac{4}{4} = \frac{8}{6}$?



These two polygons are not similar. Since $\frac{4}{2} = \frac{6}{3}$, their corresponding sides are proportional; but the corresponding angles are not equal.

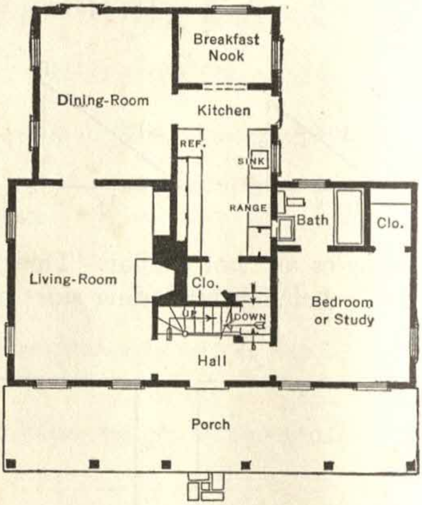


The study of similarity is not only interesting but useful. The architect, draftsman, and surveyor constantly use similarity in preparing their sketches and drawings. So do the dressmaker, photographer, and carpenter.

EXERCISES

A

This is a first-floor plan of a house. The architect makes the plans and specifications for a house. Then blueprints of the plans and copies of the specifications are given to the building contractor. These blueprints and specifications are the builder's guides. In this floor plan some figures have been erased, but 1 inch on the drawing represents 16 feet of the house. Questions 1–6 refer to this drawing.



You need a ruler for these exercises.

1. Are the floor and porch in the plan similar to the floor and porch in the house?

2. How wide is the porch? How long is it?

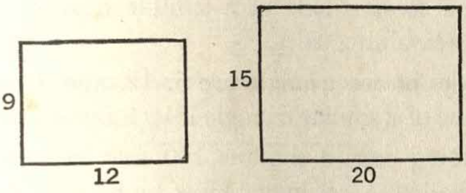
3. What are the inside dimensions of the dining room?

4. What is the length of the living room?

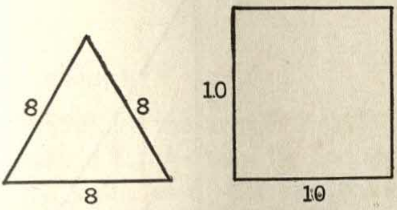
5. What are the inside dimensions of the breakfast nook?

6. What is the ratio of a line on the drawing to its corresponding line in the house?

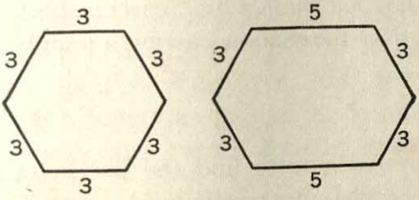
7. Are these rectangles similar? Why?



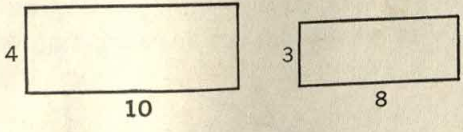
8. Are these figures similar? Why?



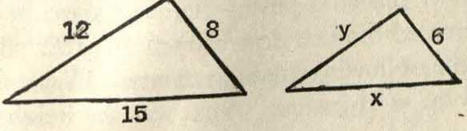
9. Give the reason why these polygons are not similar.



10. Are these rectangles similar? Why?



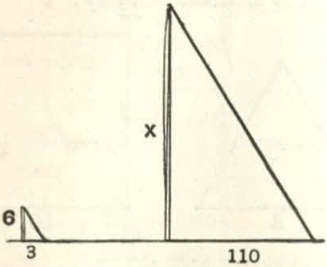
11. The sides of one triangle are 8 inches, 12 inches, and 15 inches. The shortest side of a similar triangle is 6 inches. Find the other two sides. $\left(\frac{x}{15} = \frac{6}{8} \text{ and } \frac{y}{12} = \frac{6}{8}\right)$



12. The sides of one triangle are 4 inches, 5 inches, and 6 inches. The shortest side of a similar triangle is 10 inches. Find the two remaining sides.

13. The sides of one triangle are 5, 12, and 13 inches. Find the shortest side of a similar triangle if its longest side is 91 inches.

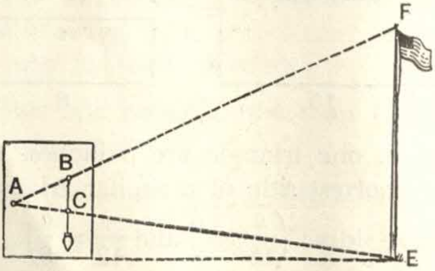
14. A building casts a shadow 110 feet long when a 6-foot post casts a shadow 3 feet long. How high is the building?



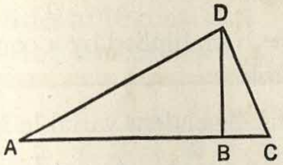
15. When a flagpole 80 feet high casts a shadow 55 feet long, what is the length of the shadow cast by a telephone pole 25 feet high?

B

16. Some boys wishing to find the height of a flagpole *EF* placed pins at *A* and *B* in a cardboard so that *A*, *B*, and *F* were in a straight line. From the pin at *B* they suspended a plumb bob. Then they placed a pin in the cardboard at *C* where the plumb line crossed the line drawn from *A* to *E*. By measuring they found the following measurements: $AE = 66$ feet, $AC =$ 6 inches, and $BC = 5\frac{1}{5}$ inches. What was the height of the pole?



17. $\triangle ABD$ is similar to $\triangle BCD$. Find BD if $AD = 10$, $AB = 8$, and $DC = 6$.



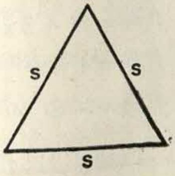
18. The hypotenuse of a right triangle is 20 feet and the base is 12 feet. Find the hypotenuse of a similar triangle whose base is 9 feet.

Variables and Constants [A]

One formula for finding the area of a circle is $A = \pi r^2$. You know that this formula can be used for finding the area of any circle, whatever the value of r. For each value of r there is a value of A. When r changes, so does A. Although r and A change in value, the value of π is always the same. The numbers A and r are called *variables* and the number π is called a *constant*. The area A is said to be a *function* of the radius r, since its value depends upon the value of r. Also, the expression πr^2, which equals A, is a function of r. In the formula $A = \pi r^2$, r is the *independent variable* and A is the *dependent variable*.

EXERCISES [A]

1. The formula for finding the perimeter of this triangle is $p = 3\,s$. Copy and complete the table below it:



If s =	1	2	3	4	5	6	7	8	9	10	30
Then p =	?	?	?	?	?	?	?	?	?	?	?

When s increases, how does p change? Why is s called the independent variable?

2. When one constant is multiplied by another, is the product a constant or a variable?

3. When a variable is multiplied by a constant, is the product a constant or a variable?

4. Tell which is the dependent variable in each formula:

a. $F = \frac{9}{5}C + 32$

b. $C = \frac{5}{9}(F - 32)$

c. $s = 16t^2$

d. $V = \frac{1}{6}\pi D^3$

5. The formula for finding the area of a rectangle is $A = bh$. In this formula b and h are independent variables.

a. What kind of variable is A?

b. What happens to A when h remains unchanged and b is doubled?

c. How does A change when both b and h are doubled?

d. How is A affected when h is halved and b is doubled?

e. How does A change when both b and h are trebled?

EXERCISES [B]

Example. $f(x) = x^2 - 3x$. Find $f(2)$.

Solution. The letter f stands for "function"; $f(x) = x^2 - 3x$ means that the function of x is $x^2 - 3x$. In $f(2)$ the (2) means that the value of x in the function is 2.

$$f(x) = x^2 - 3x$$
$$f(2) = 4 - 6$$
$$f(2) = -2$$

1. $f(x) = x^2 + 4x + 1$. Find $f(3)$; $f(-3)$.

2. $f(y) = 3y^2 - 4y + 1$. Find $f(4)$; $f(1)$; $f(0)$.

3. $f(x) = x^3 - 1$. Find $f(2)$; $f(-2)$.

4. $f(t) = 100t - 16t^2$. Find $f(4)$; $f(6)$.

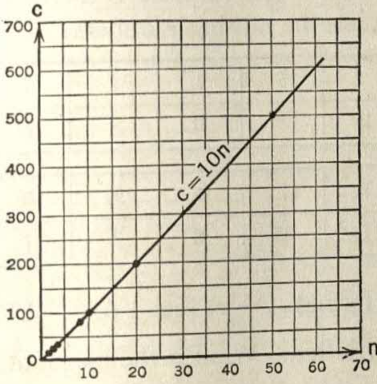
5. $f(x) = x^2 - 4x + 7$. Find $f(4)$.

Direct Variation[A]

The cost c of n articles at 10 cents each is given by the formula $c = 10\,n$. The following table is based on this formula:

When n =	0	1	2	3	8	10	20	50
c =	0	10	20	30	80	100	200	500

The graph of the formula is shown below. From the table and the graph you can see that c increases when n increases.



For each positive value of n there is a value of c. From the table you can see that any value of c divided by its corresponding value of n gives 10, and that any value of n divided by its corresponding value of c gives $\frac{1}{10}$. For this reason any number of proportions can be formed from these sets of values. Two examples are $\frac{3}{30} = \frac{50}{500}$ and $\frac{80}{8} = \frac{200}{20}$.

If we divide both members of $c = 10\,n$ by n, we get $\frac{c}{n} = 10$. This again shows that the ratio of c to n is always equal to 10. From this discussion you should be able to understand this:

> **If two variables change so that one divided by the other equals a constant, the numbers vary *directly*.**

When two numbers vary directly, either *varies directly* as the other and is *directly proportional* to the other.

EXERCISES [A]

1. In the table below, is the ratio of a to b constant for the whole table?

If b =	0	1	2	3	4	5	6	7
Then a =	0	5	10	15	20	25	30	35

Write the formula which shows this relation of a to b.

In exercises 2 and 3 one variable is directly proportional to the other. What are the missing numbers?

2.

When x =	2	5	?	7	?	20	25
Then y =	16	?	32	?	72	?	?

3.

When n =	30	?	36	90	48	?	0
Then p =	?	15	30	?	?	45	?

4. Change the formula $c = \pi d$ into a rule. What does $\frac{c}{d}$ equal? Is this a direct variation? What is the constant in $c = \pi d$?

5. In $c = 2\,\pi r$, which is the dependent variable? What does $\frac{c}{r}$ equal? How does c vary?

6. $i = 0.04\,p$. What is the constant in this formula? What does $\frac{i}{p}$ equal?

Inverse Variation [A]

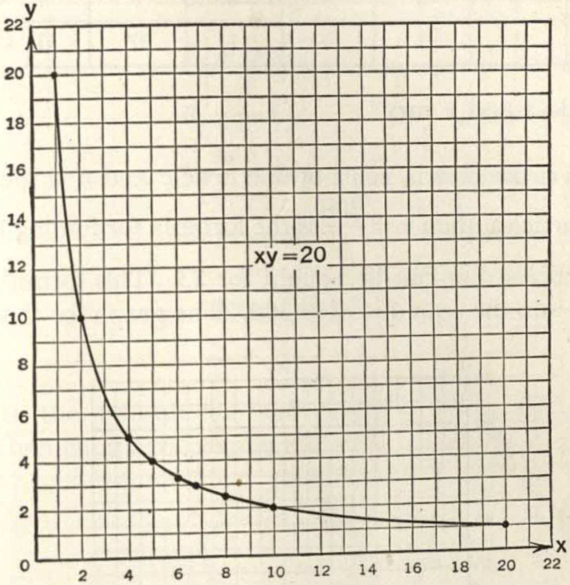
In direct variation the quotient of the two variables is constant and either variable increases when the other increases.

Now let us examine the following table:

When x =	1	2	4	5	8	20	10	6	3
Then y =	20	10	5	4	$2\frac{1}{2}$	1	2	$3\frac{1}{3}$	$6\frac{2}{3}$

You will observe that the product of any set of values of x and y is 20. The equation which shows this relation of x and

y is $xy = 20$. Here x *varies inversely* as y, and y *varies inversely* as x. The graph for this equation, using only the positive values of x and y, follows.



What do you know of the graph of an equation in which the variables vary directly? vary inversely? In the equation $xy = 20$, how does y change when x increases? Learn the following definition regarding inverse variation.

> **If two variables change so that their product is constant, they *vary inversely*.**

EXERCISES [A]

1. Is the product pv constant in the following table?

If p =	5	4	80	160	60	$\frac{2}{3}$	50	8
Then v =	16	20	1	$\frac{1}{2}$	$\frac{4}{3}$	120	$1\frac{3}{5}$	10

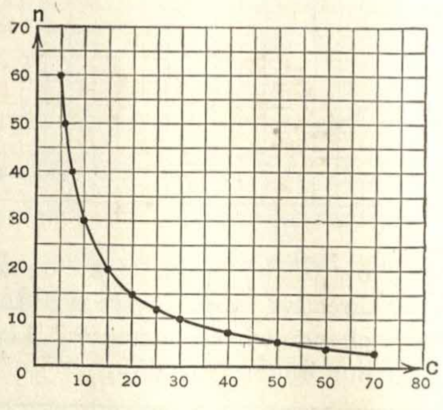
Write the formula for this table.

2. $xy = 48$. Copy and complete the following table for this formula:

x	1	3	?	6	?	?	24
y	?	?	4	?	12	1	?

How do x and y vary?

3. If c is the cost in cents of one article and n is the number of these articles, then $n = \frac{300}{c}$ is the formula for finding the number of articles that can be bought for $3. This formula can be changed into the equation $nc = 300$. The graph for this formula



is shown above. How does this graph compare with the graph of an equation expressing direct variation?

Direct and Inverse Variation[A]

The three equivalent equations which show that x and y vary directly are $\frac{x}{y} = k$; $x = ky$; and $y = \frac{x}{k}$. In each of these equations k is a constant. The three equivalent equations which show that x and y vary inversely are $xy = k$; $x = \frac{k}{y}$; and $y = \frac{k}{x}$.

ORAL EXERCISES [A]

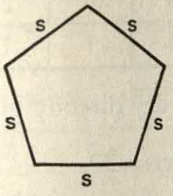
Tell how the variables vary in the following equations:

1. $x = 3y$
2. $xy = 10$
3. $x = \frac{7}{y}$
4. $y = \frac{6}{x}$
5. $h = \frac{1}{3}k$
6. $c = 2\pi r$
7. $m = \frac{1}{3n}$
8. $z = 5y$
9. $\frac{m}{n} = 30$
10. $pv = 40$
11. $h = 100s$
12. $S = \frac{1}{2}p$
13. $c = \pi d$
14. $nm = 45$
15. $s = 16t$

EXERCISES

A

1. Write the formula for the perimeter p of the polygon below. Then copy and complete the following table.



s	1	2	3	4	6	8	10	20
p	?	?	?	?	?	?	?	?

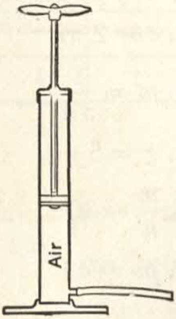
Do p and s vary directly or inversely? Is p a function of s? How does p change when s increases? What is the ratio of p to s? of s to p?

2. Copy and complete the following table, based upon the formula $p = s + 6$.

s	1	2	3	4	6	8	10	20
p	?	?	?	?	?	?	?	?

Do p and s vary directly? Do they vary inversely?

3. Here is a diagram of a bicycle pump filled with 20 cubic inches of air. The air pressure is 14.6 pounds per square inch.



The pressure and the volume of the air inside the pump obey the law expressed by the formula $pv = 292$ when no air is allowed to escape. Using this formula, copy and complete the following table:

v	1	2	4	8	10	16	50	100
p	?	?	?	?	?	?	?	?

Tell whether p and v vary directly or inversely.

4. Draw the graph of $xy = 32$.

5. Draw the graph of $\frac{x}{y} = 32$.

6. Copy and complete this table, based upon the equation $y = 4x^2$.

x	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
y	?	?	?	?	?	?	?	?	?	?	?	?	?

Does y vary directly as x? as x^2?

Draw the graph of the formula $y = 4x^2$. (It is not necessary to plot the points whose abscissas are -6, -5, -4, 4, 5, and 6.)

7. Let $a = 6$ in the formula $A = ab$. How does A vary? If b is doubled, how does A change?

8. In the formula $A = ab$, A varies jointly with a and b. If a is doubled and b is trebled, how is A changed?

9. $A = \pi r^2$. Does A vary directly or inversely as r? Does A vary directly as r^2?

B

10. $E = \frac{Wv^2}{32\,r}$, where W and v are variables. How does E change with respect to v? with respect to W? How does E change when W is unchanged and v is doubled?

11. The kinetic energy of a football player in motion can be expressed by the formula $E = \frac{Ws^2}{64}$. In this formula W is the weight in pounds of the player, s in his speed in feet per second, and E is the number of foot-pounds of energy.

Copy and complete the following table to show how hard each of four boys can hit the line of scrimmage:

PLAYERS	WEIGHT IN POUNDS	SPEED IN FEET PER SECOND	KINETIC ENERGY
Earl White	160	20	?
Bryant Foster	192	?	768
Chester Frank	?	18	729
Tony Pinelli	128	24	?

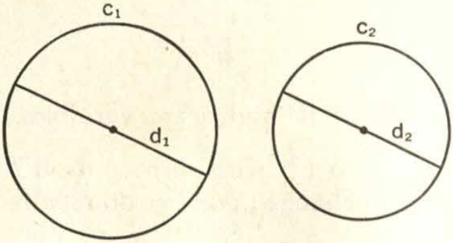
Which of these boys seems to be the best prospect for a fullback? for a guard?

12. $i = prt$ is the simple-interest formula. Which numbers are variables? If p and r are constant, how does i vary? If i and t are constant, how does p vary?

13. $V = .5236\,d^3$ is a formula for finding the volume of a sphere. Is d written as a function of V, or is V written as a function of d? Make a table for the formula letting d have the values 0, 1, 2, 3, 4, 5, 6, 7, and 8.

How to Express Direct Variations by Proportions [C]

The circumference of a circle is expressed by the formula $c = \pi d$. If we let d_1 and d_2 represent two different values of d, and let c_1 and c_2 represent the corresponding values of c, then $c_1 = \pi d_1$ and $c_2 = \pi d_2$. Now let us divide the members of the first equation by the members of the second equation. We get $\frac{c_1}{c_2} = \frac{\pi d_1}{\pi d_2}$, which simplifies to $\frac{c_1}{c_2} = \frac{d_1}{d_2}$.



This proportion is another way of expressing the fact that c varies directly as d. Can you remember how this proportion is written?

How to Express Inverse Variations by Proportions [C]

The formula for Boyle's law is $pv = c$ where c is a constant. In this formula p stands for the pressure of an enclosed gas and v stands for the volume of the gas. Let v_1 and v_2 represent two different volumes of the gas and let p_1 and p_2 represent the corresponding values of p. Then $p_1v_1 = c$ and $p_2v_2 = c$. By dividing the members of $p_1v_1 = c$ by the members of $p_2v_2 = c$, we get

$$\frac{p_1v_1}{p_2v_2} = 1$$

Clearing this equation of fractions, we have $p_1v_1 = p_2v_2$. From this last equation we obtain the proportion

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}$$

This proportion is another way of expressing the fact that p varies inversely as v. It is another way of expressing the inverse variation $pv = c$, or $p = \frac{c}{v}$.

ORAL EXERCISES [C]

1. How do a and b vary in the proportion $\frac{a_1}{a_2} = \frac{b_1}{b_2}$? in the proportion $\frac{a_1}{a_2} = \frac{b_2}{b_1}$?

2. How do x and y vary in the proportion $\frac{x_1}{x_2} = \frac{y_2}{y_1}$? in the proportion $\frac{y_2}{y_1} = \frac{x_1}{x_2}$?

3. Express the variation $c = 2\,\pi r$ by a proportion.

4. How does A vary in $\frac{A_1}{A_2} = \frac{{r_1}^2}{{r_2}^2}$?

5. How does A vary in the formula $A = s^2$? Express this variation by a proportion.

6. Using the constant k, change the proportion $\frac{c_1}{c_2} = \frac{n_1}{n_2}$ to a variation.

Example 1. x varies directly as y, and x is 60 when y is 52. Find x when y is 78.

Solution 1. Since x varies directly as y, $x = ky$, k being a constant.

Then $$60 = 52\,k$$

$$-\,52\,k = -\,60$$

$$k = \tfrac{15}{13}$$

Then $$x = \tfrac{15}{13}y$$

When $y = 78$ $$x = \tfrac{15}{13}(78) = 90$$

Solution 2. $$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

$$x_1 = 60,\ y_1 = 52, \text{ and } y_2 = 78$$

Then $$\frac{60}{x_2} = \frac{52}{78}$$

$$52\,x_2 = 60 \times 78$$

$$x_2 = 90$$

WRITTEN EXERCISES [C]

1. x varies directly as y and $x = 20$ when $y = 15$. Find x when $y = 27$.

2. x varies directly as y. If $x = 51$ when $y = 3$, find x when $y = 5$.

3. m varies inversely as n. If $m = 54$ when $n = 60$, find m when $n = 72$. $\left(\frac{m_1}{m_2} = \frac{n_2}{n_1}\right)$

4. p and v vary inversely. If $p = 55$ when $v = 28$, find v when $p = 22$.

5. x varies directly as y. If $x = 30$ when $y = 18$, find x when $y = 27$.

6. s varies directly as the square of r. If $s = 5026.56$ when $r = 20$, find s when $r = 25$.

7. A varies directly as the square of t. If $A = 2304$ when $t = 12$, find A when $t = 10$.

Direct Variations. 1. The total cost or selling price of like objects varies directly as the number of objects.

2. The amount of work done varies directly as the number of men (or machines) and as the time.

$$\left(\frac{W_1}{W_2} = \frac{n_1t_1}{n_2t_2}\right)$$

Inverse Variations. 1. The time required to go a given distance varies inversely as the speed.

2. The time required to do a given amount of work varies inversely as the number of persons doing the work.

3. If two pulleys are connected by a belt, their diameters vary inversely as the number of their revolutions.

4. The number of persons required to do a piece of work varies directly as the amount of work and inversely as the time for doing it.

$$\left(\frac{n_1}{n_2} = \frac{W_1t_2}{W_2t_1}\right)$$

8. x varies inversely as y. If $x = 24$ when $y = 8$, find x when $y = 6$.

9. x varies inversely as y. If $x = 66$ when $y = 2$, find x when $y = 12$.

10. m varies inversely as y. If $m = 6.5$ when $y = 5$, find m when $y = 17.5$.

11. Two numbers vary inversely. If the first is 10 when the second is 45, find the second when the first is 15.

12. A certain amount of air occupies 1500 cubic feet when its pressure is 30 pounds. How many cubic feet will it occupy when its pressure is increased to 70 pounds?

13. If 60 men can do a piece of work in 42 days, how long will it take 45 men to do the same work?

14. If 20,000 books sell for $18,800, what is the selling price of 15,500 of them?

15. If 10 six-inch pipes can fill a pool in 24 hours, how long will it take 8 of these pipes to fill the pool?

Example 2. If 12 men require 4 days to lay 40 rods of water pipe, how many men are needed to lay 160 rods of the same kind of pipe in 6 days?

Solution.
$$\frac{n_1}{n_2} = \frac{W_1 t_2}{W_2 t_1}$$

Then
$$\frac{12}{n_2} = \frac{40 \times 6}{160 \times 4} = \frac{3}{8}$$

$$3\,n_2 = 96$$

$$n_2 = 32, \text{ the number of men}$$

EXERCISES [C]

1. If 18 men in 10 hours can pick 1080 bushels of tomatoes, how long will it take 24 men to pick 1152 bushels of tomatoes?

2. If a crew of 75 men can construct 4 miles of road in 18 days, how much road can 45 men complete in 32 days?

3. The area of a triangle is expressed by the formula $A = \frac{1}{2}bh$. Express this fact by a proportion.

4. A motor with a speed of 1700 r.p.m. and a belt wheel $2\frac{7}{8}''$ in diameter is used to drive an air compressor in an electric refrigerator. How large should the drive wheel of the compressor be so that its speed is 650 r.p.m.?

5. The speed of a threshing machine cylinder should be 900 r.p.m. The belt wheel of the cylinder is 10 inches in diameter and is connected by a belt to the 36-inch flywheel of the engine. At what speed should the engine be run?

6. $I = \dfrac{E}{R}$ is a formula for finding I, the amount of current, when the number of volts, E, and the resistance, R, are known. (This formula is true for all direct currents.)

a. How does I vary with respect to E? to R?

b. Solve this formula for E; for R.

c. If an electric heater uses 22 amperes of electricity when the voltage is 110 volts, how much current will it use when the voltage drops to 100 volts? (In this problem R is constant.)

7. $d = .137\sqrt{h}$ is a formula for finding the distance, d, in miles one can see from a height h feet above the earth's surface.

a. How does d vary?

b. How far can one see on the ocean from a point 100 feet above the surface?

WORD LIST

Be sure you know the meanings of the following words and symbols:

constant (p. 485)
direct variation (p. 487)
extremes (p. 479)
inverse variation (p. 488)
means (p. 479)
Pythagorean Theorem (p. 451)
quadratic equation (p. 448)
radical (p. 423)
ratio (p. 479)

$\triangle$ $(\ )$ $\sim$ $\neq$
$=$ $[\ ]$ $\sqrt{\ }$ $\parallel$

CHAPTER REVIEW [A]

1. Solve for y the proportion $y : 6 = 7 : 3$.

2. Name the means and extremes of $x : y = 7 : 8$.

3. Complete: In a proportion the product . . .

4. How high is a telegraph pole which casts a shadow of 36 feet when a flagpole 120 feet high casts a shadow of 92 feet?

5. What is the ratio of 6 to 8? of $7x$ to $21x^2$?

6. Solve for m: $\frac{2m - 3}{6} = \frac{7}{m + 1}$

7. Form a proportion from $hk = mn$.

8. The sides of one triangle are 7 inches, 10 inches, and 13 inches. If the longest side of a similar triangle is 19.5 inches, find the length of the shortest side of it.

9. Suppose y varies directly as x.

a. Write this relation using c as the constant.

b. Express this relation by a proportion.

10. Suppose m varies inversely as n. Write this relation using k as the constant.

11. If y varies directly as x and $y = 14$ when $x = 3.5$, find the constant.

12. Copy and complete: $x^2 - 4x + 5$ is a --?-- of x.

13. $y = 2x^2 - x + 3$. Find y when $x = -3$.

14. How does x vary with respect to y in the following equations?

a. $7x = 8y$ **c.** $y = \frac{4}{x}$ **e.** $\frac{2}{x} = \frac{y}{10}$

b. $3xy = 7$ **d.** $y = 3x$ **f.** $cx = dy$

15. Copy and complete: In direct variation the --?-- increase together and --?-- together.

GENERAL REVIEW

A

1. Simplify $(-4x^3y)(-2xy)(-y^5)$.

2. Take $\frac{1}{2}x - \frac{1}{3}y + \frac{1}{4}$ from $\frac{1}{3}x - \frac{2}{3}y - \frac{1}{5}$.

3. Multiply $4a^3 - 3a^2b - 9ab^2 + 6b^3$ by $-4a^2b$.

4. Divide $21a^4 - 27a^3 - 12a^2 + 3a$ by $-3a$.

5. Simplify $4(x - a) - (5x + 3a) + (x - a)$.

6. Multiply $a^2 - 5ab - 2b^2$ by $2a - b$.

7. Divide $x^3 - 3x^2y + 3xy^2 + y^3$ by $x - y$.

8. Reduce $\frac{x^2 + 5x}{2x^2 - 50}$ to lowest terms.

9. Multiply $\frac{a^2 + ab - 2b^2}{3a^2 - 3ab}$ by $\frac{a^2 - ab}{a^2 + 4ab + 4b^2}$.

10. Combine: $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

11. Solve: $3x - 4y = 7$
$x + 2y = 3$

Solve:

12. $x + .01x = 1.515$

13. $4x^2 - 3x - 4 = 0$

14. $\frac{2x}{3} + \frac{6}{x} = 5$

15. $m^2 + 2m = 6$

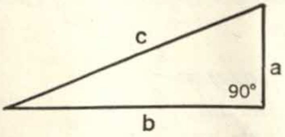
16. $x^2 - 9x = 3$

17. $3x(x + 2) - 5(x + 2) = 6$

18. $\frac{2x - 7}{x - 6} = \frac{6x + 1}{3x + 7}$

19. $c^2 - 4c = 16$

20. Tell which of the following statements concerning this triangle are true and which are incorrect:



a. $a^2 + b^2 = c^2$

b. $a + b = c$

c. $c^2 - a^2 = b^2$

d. $c = \sqrt{a^2 + b^2}$

e. $c - b = a$

f. $a^2 = \sqrt{c^2 - b^2}$

21. Factor:

a. $a^2 - 1$	**c.** $bx - b$	**e.** $5x^2 - 30x + 45$
b. $x^2 - 5x + 6$	**d.** $2x^2 + x - 15$	**f.** $3x^2 + 8x - 3$

22. Simplify $\sqrt{8}$; $\sqrt{50}$; $\sqrt{\frac{1}{2}}$.

23. Solve $6 : 10 = 16 : x$ for x.

24. Find $12\frac{1}{2}\%$ of 224.

25. Find what per cent 12 is of 72.

26. An electric refrigerator sold for $159 after a 40% discount was allowed. What was the original price?

27. Catherine thought of a number and squared it. Then she subtracted 7 times the number from twice its square and had 30 as the remainder. What was the number she had at first?

28. Two boys balance on a teeterboard when one is 6 feet and the other is 5 feet from the fulcrum. If each boy were 8 pounds lighter, they would balance when one is 9 feet and the other 11 feet from the fulcrum. Find the weight of each boy.

29. Find the hypotenuse of a right triangle if its legs are 18 feet and 20 feet long.

30. Find the side of a square whose diagonal is 30 feet.

31. Find the height of an equilateral triangle if one of its sides is 12 inches.

B

Solve:

32. $(3x + 5)^2 - 30x = 43$

34. $\dfrac{x}{x+3} - \dfrac{x}{x-3} = \dfrac{x^2+8}{x^2-9}$

33. $\dfrac{3}{x^2 - x} - \dfrac{4}{x^2 + x} = \dfrac{1}{2x}$

35. $\dfrac{c}{c+2} + \dfrac{1}{4c} = \dfrac{5}{c^2 + 2c}$

36. Which of the four values of x, $x = 1, 2, 3$, or 4, will give the fraction $\dfrac{3}{x+1}$ the greatest value?

37. How much smaller than $-3x$ is $9x$ when $x = -2$?

38. For what value of x does $\dfrac{3x}{4} = x + 6$?

Solve for x:

39. $bx - a = cx + d$

40. $\frac{x}{3} - \frac{a}{2} = \frac{b}{5}$

41. $a(x - a) = b(x + a - 3b)$

42. $\frac{x}{x - n} = \frac{x + m}{x}$

43. Two quarts of gasoline are mixed with five quarts of kerosene. How many quarts of kerosene must be added to the mixture to make it five-sixths kerosene?

C

Solve:

44. $\frac{10}{x} = \frac{x}{10 - x}$

45. $x^2 = 9(9 - x)$

46. $(x - 1)^2 + x(x + 1) = 3x(x + 1) + 4$

47. Combine: $\frac{a - b}{a + b} + \frac{2a + b}{b - a} - \frac{a^2 - 3b^2}{a^2 - b^2}$

48. Factor:

a. $\frac{1}{2}x^2 - \frac{1}{2}y^2$

b. $6p^2 - 5p - 1$

c. $5ay^2 - 25ay + 30a$

d. $x^4 - 256$

49. A man invested part of \$15,000 at 4% and the remainder at 5%. If the income from the 4% investment was \$375 more than the income from the 5% investment, how much was invested at each rate?

50. A train makes a run of 120 miles in a given time. If it were to make the run 3 miles an hour faster, its time for the run would be 10 minutes less. Find its usual rate.

51. Show that the altitude of an equilateral triangle with each side equal to s is $\frac{s}{2}\sqrt{3}$.

52. Solve $x^2 + 3x - 40 = 0$ by graphing.

53. Which is larger, and by how much, $5\sqrt{2}$ or $2\sqrt{5}$?

54. Simplify:

a. $\sqrt[3]{\frac{m^2}{m^3}}$

b. $2x\sqrt{x^5}$

c. $3\sqrt{\frac{1}{3}}$

d. $\sqrt{3}\sqrt{6}$

e. $x\sqrt{\frac{1}{xy}}$

f. $\sqrt{xy}\sqrt{x}$

CHAPTER TEST [A]

Solve for x in each proportion:

1. $\frac{x}{5} = \frac{3}{10}$ 2. $\frac{2}{x} = \frac{x}{8}$ 3. $\frac{a}{b} = \frac{c}{x}$

4. The sides of a triangle are 6, 8, and 12 inches respectively. The longest side of a similar triangle is 21 inches. Find the lengths of the other two sides.

5. In the formula $P = 4s$, when s increases how does P change?

6. In the formula $A = \pi r^2$, is π a variable or a constant?

7. In the formula $c = 2\pi r$, do c and r vary directly or inversely?

8. Tell how the two variables vary in each of the following equations:

a. $xy = 4$ **b.** $p = \frac{40}{v}$ **c.** $S = \frac{p}{2}$ **d.** $s = 16t$

9. In the formula $A = bh$, if both b and h are doubled, how is A changed?

10. In the equation $y = \frac{2}{x}$, if the value of x is multiplied by 4, how is y changed?

CHAPTER TEST [B]

Solve for x in each proportion:

1. $\frac{a}{x} = \frac{x}{b}$ 2. $\frac{x-4}{3} = \frac{x-2}{5}$

3. The length and width of one rectangle are 36 and 10 respectively. The length and width of a second rectangle are 54 and 15 respectively. Are the two rectangles similar?

4. The sides of one triangle are 5, 6, and 7 inches respectively. The shortest side of a similar triangle is 12. Find the two remaining sides.

5. The hypotenuse of a right triangle is 16 feet and its base is 10 feet. Find the hypotenuse of a similar right triangle if its base is 25 feet.

6. Which is the dependent variable in the formula $F = \frac{9}{5}C + 32$?

7. If $f(x) = x^2 - 2x + 3$, find $f(3)$.

8. In the equation $mn = 15$, how do m and n vary?

9. In the formula $s = 16t^2$, how does s vary with reference to t?

10. In the formula $E = \frac{Wv^2}{32r}$, how does E vary with respect to W? with respect to r?

CUMULATIVE TEST (CHAPTERS IX–XVII). Time 35 Minutes

On your paper write the word or number for each blank to make the following statements true:

1. A set of linear equations is --?-- when the graphs of the equations cross each other.

2. A set of linear equations is --?-- when the graphs of the equations are parallel.

3. If the edge e in the formula $V = e^3$ is multiplied by 5, the volume is multiplied by --?--.

4. A right angle contains --?-- degrees.

5. If A in the formula $A = 4\pi r^2$ is divided by 9, then r is --?-- by --?--.

6. Solve for x and y:

$$\begin{aligned} x + 5y &= 19 \\ 6x - 7y &= 3 \end{aligned}$$

7. Factor:

a. $6m - 6x$ **b.** $a^3 - a$ **c.** $4a^2 - 64$

8. Solve for x: $ax - bx = 2bx - a$

9. Solve by factoring: $y^2 - 56 + y = 0$

10. Find and express in its simplest form the ratio of 28 to 63.

11. Solve by completing the square: $3x^2 + 5x + 2 = 0$

12. Combine: $1 + \frac{c}{d}$

13. If 20 gallons of paint will cover 6600 square feet, how many gallons of paint will be needed to cover 2310 square feet? Solve by variation or proportion.

14. Simplify: **a.** $\sqrt{27}$ **b.** $\sqrt{\frac{2}{5}}$

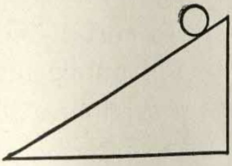
15. Divide $\frac{10n - 5}{3n^2}$ by $\frac{2n - 1}{15n}$.

16. How long will it take \$1250 to earn \$206.25 when it is invested at 3% simple interest?

17. Expand: $(x - 6y)^3$

18. A man has 25 coins in quarters and dimes. The value of the coins is \$4.00. Find the number of coins of each kind he has. Solve, using two letters.

19. The length of a rectangular flower bed is 50% greater than the width. Find the dimensions of the bed if its perimeter is 130 feet.



20. The distance d in feet that a ball will roll down an inclined plane in t seconds is given by the formula $d = kt^2$, k being a constant. By trial it was found that the ball rolled 27.9 feet in 3 seconds. How far will the ball go in 5 seconds?

CIVIL ENGINEERING

THERE ARE MANY KINDS of engineering. Among these are electrical engineering, mechanical engineering, civil engineering, hydraulic engineering, mining engineering, powerhouse engineering, and steam engineering.

Civil engineering is the science that relates to the building of highways and railroads, the construction of tunnels and canals, and the surveying of lands.

In civil engineering, distances are measured by the steel tape, and angles are measured by the transit. Indirect measurements are made by the use of trigonometry.

A city or county engineer measures lands, locates boundary lines, makes plans for sewers and levees, looks after the grades of streets, curbs, and sidewalks, and supervises the construction of public buildings.

Most civil engineers are graduates of engineering schools. Pupils who look forward to entering an engineering school should be proficient in mathematics in high school so that they may do well when they enter college.

Keystone View

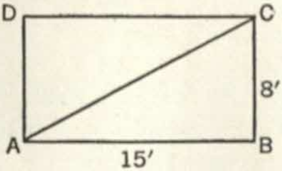
A Surveyor Working on the Detailed Mapping of a Small Area

CHAPTER XVIII

Numerical Trigonometry

The Two Kinds of Measurement [A]

There are two kinds of measurement, direct and indirect. A direct measurement is made by applying a ruler, yardstick, tape measure, protractor, or other measuring instrument to the object to be measured and noting how many times the dimension, or the angle, contains the unit of measure. An indirect measurement is made when it is *computed* from measurements already known. These two methods of measurement will be illustrated by two examples.



Suppose a man wishes to find the diagonal AC of room $ABCD$. He can measure AC directly by stretching a tape from A to C and reading the measurement on the tape. If he has no tape, he can use a yardstick or a foot rule and apply it end to end along line AC. The man may measure AC indirectly in many ways. For example, he can measure AB and BC directly and compute AC by the Pythagorean Theorem. Thus $AC = \sqrt{\overline{AB}^2 + \overline{BC}^2} = \sqrt{15^2 + 8^2} = 17$, the number of feet.

Suppose a farmer wishes to know how many bushels of wheat there are in a granary. He can measure the wheat directly by placing it in bushel baskets or by weighing it. He can measure the wheat indirectly by finding the length, width, and height of the granary by direct measurements and dividing the number of cubic inches in its volume by 2150.4, the number of cubic inches in a bushel.

Measurements Are Approximate[A]

Any measurement is only approximate. For example, if a boy is measuring the length of a room, he may use a yardstick on which the scale is not accurate, he may make an error in reading the numbers on the yardstick, he may not lay the stick exactly along the line of measurement, or he may make errors in placing the stick end to end.

The degree of accuracy in any measurement depends upon the delicacy of the instrument and upon the person doing the measuring. If a boy using a yardstick measures the width of a desk, he may say the width is 24 inches because the width is nearer 24 inches than 23 inches or 25 inches. The boy then is measuring the width to the *nearest inch.* In this measurement the figure 2 is exact and the figure 4 is approximate. If another boy using a ruler marked off in tenths of an inch measures the width, he may say that the width is 24.7 inches because the width is nearer 24.7 inches than 24.6 inches or 24.8 inches. This boy is measuring the width to the *nearest tenth* of an inch. The figures 2 and 4 are exact and the figure 7 is approximate. A skilled mechanic using a more delicate instrument may record the width as 24.73 inches. He would be measuring to the *nearest hundredth* of an inch.

Some measurements need to be more accurate than others. In rough timber work the carpenter needs only to measure to the nearest fourth of an inch, while the machinist frequently makes measurements to the nearest thousandth of an inch.

How Numbers Are Rounded Off[A]

No doubt you have heard a statement such as the following: "In round numbers Columbus, Ohio, is 300 miles from Chicago." The distance between Columbus and Chicago is approximately 314 miles. There may be some mistake in this measurement. Suppose we are not sure that the figures 1 and 4 in 314 are correct. Then we would say, "In round numbers the distance is 300 miles." In this case the number 314 has been rounded off to the nearest hundred. If we are sure of the 3 and

1 but not of the 4 in 314, we round off the number to the nearest ten and obtain 310.

When a measurement such as 7.23 is expressed correct to the nearest tenth as 7.2, the number 7.23 is said to be rounded off to the nearest tenth. When rounded off to the nearest tenth, the numbers 7.20, 7.21, 7.22, 7.23, and 7.24 become 7.2. When 7.26, 7.27, 7.28, and 7.29 are rounded off to the nearest tenth, they become 7.3. Since 7.25 is just as near 7.2 as 7.3, either 7.2 or 7.3 may be called the rounded off number. It is common practice in such a case to make the last retained figure *even.* According to this practice 7.25 rounded off to the nearest tenth becomes 7.2 and 7.15 rounded off to the nearest tenth becomes 7.2.

The number 134.589 rounded off to the nearest hundredth is 134.59; to the nearest tenth is 134.6; to the nearest unit is 135; to the nearest ten is 130; and to the nearest hundred is 100.

EXERCISES [A]

Round off the following numbers to the nearest tenth:

1. 7.32
2. 4.26
3. 17.31
4. 18.78
5. 2.31
6. 4.45
7. 4.35
8. .8453

Round off the following numbers to the nearest hundredth:

9. 9.215
10. 6.346
11. 7.545
12. 10.575
13. 2.630
14. 3.1652

Significant Figures [A]

Instead of saying that 4.3 is the result of rounding off 4.32 to the nearest tenth, we sometimes say that 4.3 is the result of rounding off 4.32 to two *significant figures.* Likewise, we can say that 1.73, 2.03, and .254 each contain three significant figures.

When a measurement is made, it should be recorded so as to show the accuracy of the measurement. Thus a length recorded as 52 feet means that the measurement has been made to the nearest foot; a length recorded as 52.3 feet means that the measurement has been made to the nearest tenth of a foot; and a length recorded as 52.36 feet means that the measurement has been made to the nearest hundredth of a foot. In each of these measurements all the figures except the last are exact and the error in the last figure is not greater than one half unit; that is, 52.3 feet means that the measurement is between 52.25 and 52.35 feet. The figures in each of these measurements are *significant* for they all have a real meaning, each one except the last being exactly correct, and the last having an error not greater than one half unit.

In general, all the figures of a number are significant. The only exception is in the case where zero is used merely as a place holder following a decimal point. The zeros in the numbers 20.5, 106.7, and 409.05 are significant, while the zeros in the numbers .025 and .0036 are not significant. The zero in the measurement 4.0 feet is significant, since it shows that the measurement has been made to the nearest tenth. Likewise, if the two zeros in the number 23.00 inches are used to show that the measurement is made to the nearest hundredth of an inch, they are significant.

The position of the decimal point in a number has no bearing on the number of significant figures in it. Thus each of the numbers 2.35, .0678, and 344 has three significant figures.

In a number such as 5200 we cannot be sure of the number of significant figures from the number itself. More information is necessary. If 5200 miles is measured to the nearest mile, it has 4 significant figures. If it is measured to the nearest 10 miles it has 3 significant figures, and if it is measured to the nearest 100 miles it has two significant figures.

ORAL EXERCISES [A]

Round off the following numbers to three significant figures:

1. 7843
2. 2.6185
3. .0946
4. 278.0
5. 192.5
6. 18.35
7. 86260
8. 12143

State the degree of accuracy to which the following recorded measurements have been made:

9. 7.3 ft.
10. 1.29 in.
11. 18.803 in.
12. 12.00 in.
13. 6 yd.
14. 70.0 ft.

Computations with Approximate Numbers [A]

When approximate numbers are added, subtracted, multiplied, or divided, the results also are approximate numbers. Let us see how these computations should be made.

Suppose that we wish to find the perimeter of a triangle whose sides are 8.16 feet, 10.3 feet, and 11.437 feet. If 8.16, 10.3, and 11.437 are the exact lengths of the sides, the perimeter of the triangle is exactly 29.897 feet.

If the numbers 8.16, 10.3, and 11.437 are found by measurement, they are only approximate. In such a case 8.16 stands for some number from 8.155 to 8.165 inclusive, 10.3 stands for a number between 10.25 and 10.35, and 11.437 stands for a number between 11.4365 and 11.4375.

$$\boxed{\begin{array}{l} 8.16\\ 10.3\\ 11.437\\ \hline 29.897 \end{array}}$$

From the two additions below, we know that the true perimeter is more than 29.8415 feet and less than 29.9525 feet.

$$\begin{array}{|l|l|} \hline 8.155 & 8.165\\ 10.25 & 10.35\\ 11.4365 & 11.4375\\ \hline 29.8415 & 29.9525\\ \hline \end{array}$$

The accepted way of finding the approximate perimeter consists in making the measurements to the same degree of accuracy and adding these measurements, obtaining a sum which has the same degree of accuracy as the measurements. In this example 10.3 is of the least degree of accuracy, being a measurement only to the nearest tenth. Accordingly, 8.16 and 11.437 are rounded off to the nearest tenth. By this method the perimeter is found to be 29.9 feet. Notice in this example that the same sum is obtained (1) when the numbers are added as given and the result rounded off to the nearest tenth; and (2) when the numbers are rounded off to the nearest tenth before addition.

$$\boxed{\begin{array}{l} 8.2\\ 10.3\\ 11.4\\ \hline 29.9 \end{array}}$$

When adding or subtracting approximate numbers you should follow the directions printed at the top of the next page:

To add or subtract approximate numbers, round off the numbers so that they have the same degree of accuracy.

The result will have the same degree of accuracy as the numbers.

Example. Add: $4.136 + 3.4 + 7.25$

Solution. 3.4 is of the least degree of accuracy. Then 4.136 and 7.25 must be rounded off to the nearest tenth to have the same degree of accuracy. Then $4.1 + 3.4 + 7.2 = 14.7$

Let us now consider a problem involving multiplication of approximate numbers. Suppose that we wish to find the area of a rectangle whose length is 7.3 inches and whose width is 4.6 inches. If 7.3 and 4.6 are the exact dimensions of the rectangle, its area is exactly 33.58 square inches.

7.3 4.6 4 38 29 2 33.58	7.25 4.55 3625 3 625 29 00 32.9875	7.35 4.65 3675 4 410 29 40 34.1775

If 7.3 and 4.6 are found by measurement, 7.3 is some number between 7.25 and 7.35, and 4.6 is some number from 4.55 to 4.65. Then the true area is more than 32.9875 and less than 34.1775. It is the practice of computers to round off the product 33.58 and say that the area is 34 square inches. This example illustrates the following rule for multiplying approximate numbers:

The product of two approximate numbers should not contain more significant figures than either the multiplier or the multiplicand.

Example. Multiply 7.312 by 4.3.

Solution. $7.312 \times 4.3 = 31.4416$

The number 7.312 has *four* significant figures and 4.3 has *two* significant figures. Then the product should have only two significant figures. Therefore $7.312 \times 4.3 = 31$.

The quotient of two approximate numbers should not contain more significant figures than either the divisor or dividend.

Example. Divide 1825.2 by 1.754.

Solution. $1825.2 \div 1.754 = 1040.5 +$

Since 1.754 contains only four significant figures, the quotient $1040.5 +$ must be rounded off to contain only four significant figures. Therefore, $1825.2 \div 1.754 = 1041$.

EXERCISES [A]

Exercises 1–6 deal with approximate numbers. Perform the indicated operations:

1. $1.6 + 3.15 + .068$

2. $172.3 - 42.55$

3. $17.2 + .12$

4. $21.36 \div 5.6$

5. 23.52×1.654

6. $140.0 \div 2$

Numerical Trigonometry [A]

The word *trigonometry* means "triangle measurement." However, the usefulness of trigonometry is not confined to the solution of triangles alone. Without trigonometry one could not go very far into the study of radio, light, heat, electricity, navigation, astronomy, or any field of engineering.

Facts about Angles [A]

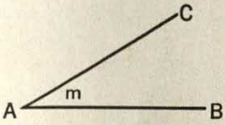
The following statements should be familiar to you:

1. An angle, abbreviated $\angle$, is the figure formed by two lines meeting at a point.
2. A right angle contains 90°.
3. An acute angle is less than a right angle.

4. An obtuse angle contains more than 90° and less than 180°.

5. Two lines are perpendicular ($\perp$) when they form a right angle.

6. An angle may be named in three ways: by a capital letter at its vertex, as $\angle A$; by a small letter within the angle, as $\angle m$; and by three capital letters, as $\angle BAC$. In reading an angle by three letters, the letter at the vertex is the middle letter.



Facts about Triangles [A]

The following facts are needed in the study of trigonometry:

1. The symbol for triangle is $\triangle$.

2. A right triangle has one right angle.

3. The sum of the angles of a triangle is 180°.

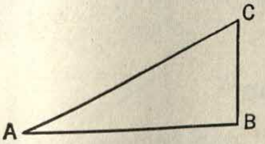
4. Any triangle has two acute angles.

5. Two triangles which have the same size and shape are congruent ($\cong$). Either will fit exactly upon the other.

6. Two triangles which have exactly the same shape are similar ($\sim$).

7. Two triangles are similar when their corresponding angles are equal and their corresponding sides are proportional.

8. The hypotenuse of a right triangle is the side opposite the right angle. In the figure the side AC opposite right angle B is the hypotenuse. The side BC is *opposite* $\angle A$ and the side AB is *opposite* $\angle C$. The side AB is *adjacent* to $\angle A$ and the side BC is *adjacent* to $\angle C$.

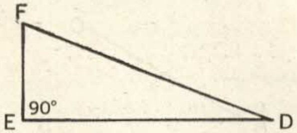


9. Two right triangles are similar if an acute angle of one is equal to an acute angle of the other.

ORAL EXERCISES [A]

1. What angle of $\triangle DEF$ is a right angle?

2. Which angles of $\triangle DEF$ are acute?



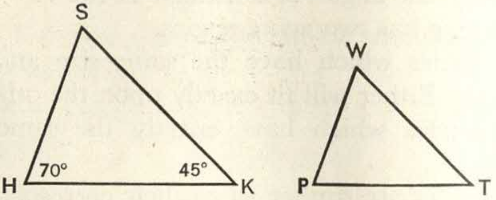
3. Using three letters, name $\angle E$; $\angle D$; $\angle F$.

4. What is the name of side DF?

5. Which two sides of $\triangle DEF$ are perpendicular?

6. How many degrees are there in the sum of $\angle D$ and $\angle F$?

$\triangle HKS$ and $\triangle PTW$ are similar, HS corresponding to PW and HK corresponding to PT.



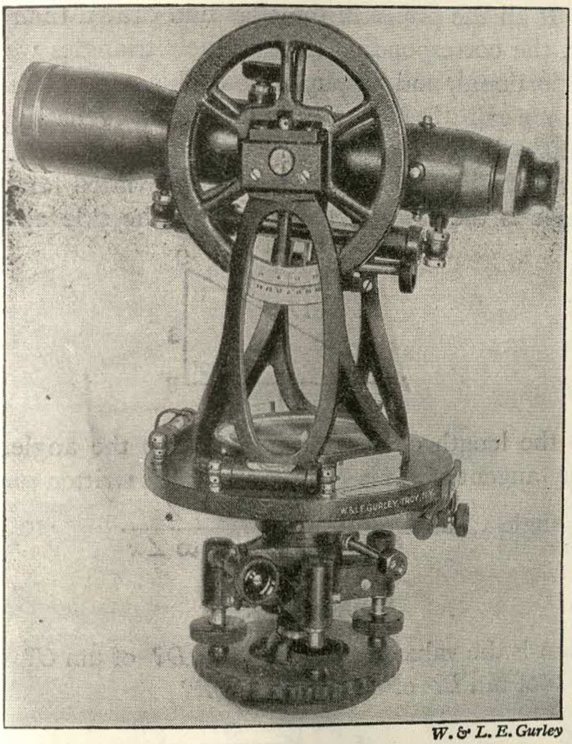
7. How many degrees are there in $\angle S$? in $\angle P$? in $\angle T$? in $\angle W$?

8. $\frac{HS}{PW} = \frac{SK}{WT}$. Why?

9. Does $\frac{HS}{SK} = \frac{PW}{WT}$?

Surveyors' Instruments [A]

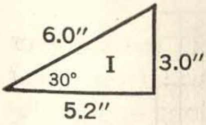
For making direct measurements out of doors the surveyor uses the *steel tape* for measuring linear distances and the *transit* for measuring angles. The steel tape is 100 feet long and graduated into tenths of a foot. The transit is made to measure both vertical and horizontal angles. The picture on the next page shows a transit. In use it would be mounted on a tripod.

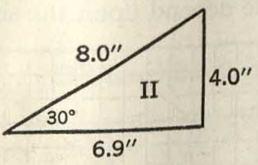


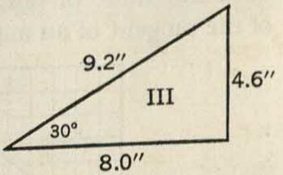
W. & L. E. Gurley

The Tangent of an Angle [A]

An algebra teacher asked each member of the class to draw a right triangle having an acute angle of 30°. Then the teacher asked the pupils to measure the sides of their triangles and find to the nearest tenth the ratio of the side opposite the 30° angle to the side adjacent to the 30° angle.

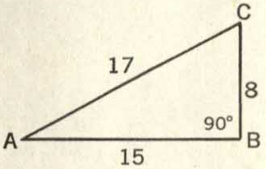






The pupil having triangle I divided 3 by 5.2, the pupil having triangle II divided 4 by 6.9, and the pupil having triangle III divided 4.6 by 8.0. All three of these pupils found the ratio to be 0.58. If all the pupils in the class had drawn their triangles correctly, the corresponding sides of their triangles would have been proportional, and all pupils would have found the ratio to be 0.58. We call this kind of ratio the *tangent* of the angle. Thus tangent $30^\circ = 0.58$.

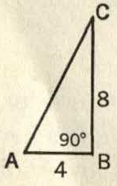
In any right triangle the tangent (abbreviated tan) of either acute angle is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle. In this figure the tangent of $\angle A$ is $\frac{8}{15}$ or .53. This is written $\tan A = \frac{8}{15}$.

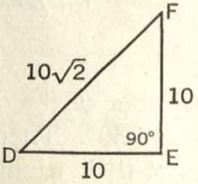


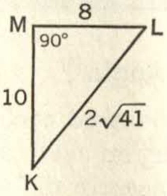
For any angle x, $\tan x = \dfrac{\text{side opposite } \angle x}{\text{side adjacent to } \angle x}$.

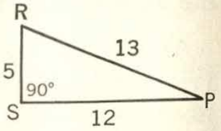
EXERCISES [A]

1. What is the value of tan A? of tan D? of tan C? of tan F? of tan K? of tan L? of tan P? of tan R?

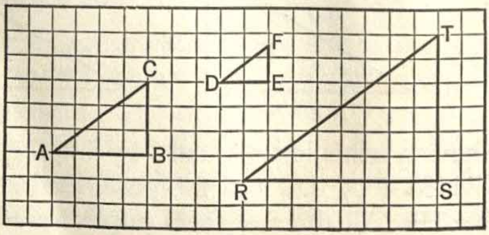






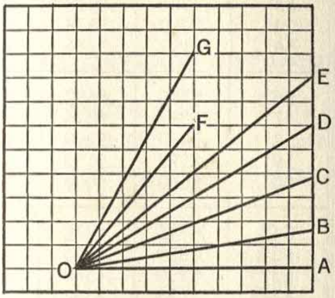


2. Below are three right triangles having $\angle A = \angle D = \angle R$. Find the value of tan A; of tan D; of tan R. Does the value of the tangent of an angle depend upon the size of the triangle?



3. In the figure below $\angle AOB$ contains 10°, $AO = 10$ units, and $AB = 1.8$ units. Then $\tan AOB = \frac{1.8}{10} = .18$. Using your protractor and ruler, complete the table in like manner.

Does the value of the tangent increase or decrease as the size of the angle increases? Give the reason why the tangent of an angle is a function of the angle.



∠	DEGREE	TAN
AOB	10°	.18
AOC	?	?
AOD	?	?
AOE	?	?
AOF	?	?
AOG	?	?

Table of Tangents[A]

In the last exercise you learned how to make a small table of tangents. These results by careful measurements may be made correct to two significant figures. By using methods of higher mathematics it is possible to make tables in which the values of the tangents are correct to four or more decimal places. On page 520 is a table of sines, cosines, and tangents given correct to four decimal places. This table gives the tangents of angles from 1° to 89°. To find tan 62° from this table, first look in the column headed "ANGLE" for the angle 62°. Then in the column headed "TAN," in the same horizontal line as 62°, find the number 1.8807. Then tan 62° = 1.8807 to the nearest ten-thousandth.

ORAL EXERCISES[A]

Using the table on page 520, find the following:

1. tan 32°
2. tan 45°
3. tan 16°
4. tan 20°
5. tan 10°
6. tan 34°
7. tan 19°
8. tan 1°
9. tan 89°
10. tan 80°
11. tan 71°
12. tan 65°
13. tan 60°
14. tan 30°
15. tan 15°

Table of Natural Functions

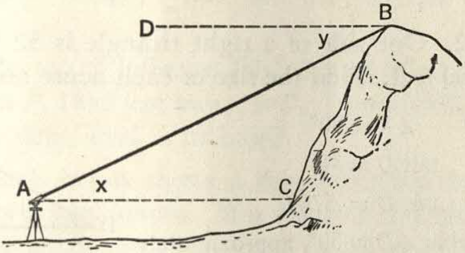
ANGLE	SIN	COS	TAN	ANGLE	SIN	COS	TAN
				45°	.7071	.7071	1.0000
1°	.0175	.9998	.0175	**46°**	.7193	.6947	1.0355
2°	.0349	.9994	.0349	**47°**	.7314	.6820	1.0724
3°	.0523	.9986	.0524	**48°**	.7431	.6691	1.1106
4°	.0698	.9976	.0699	**49°**	.7547	.6561	1.1504
5°	.0872	.9962	.0875	**50°**	.7660	.6428	1.1918
6°	.1045	.9945	.1051	**51°**	.7771	.6293	1.2349
7°	.1219	.9925	.1228	**52°**	.7880	.6157	1.2799
8°	.1392	.9903	.1405	**53°**	.7986	.6018	1.3270
9°	.1564	.9877	.1584	**54°**	.8090	.5878	1.3764
10°	.1736	.9848	.1763	**55°**	.8192	.5736	1.4281
11°	.1908	.9816	.1944	**56°**	.8290	.5592	1.4826
12°	.2079	.9781	.2126	**57°**	.8387	.5446	1.5399
13°	.2250	.9744	.2309	**58°**	.8480	.5299	1.6003
14°	.2419	.9703	.2493	**59°**	.8572	.5150	1.6643
15°	.2588	.9659	.2679	**60°**	.8660	.5000	1.7321
16°	.2756	.9613	.2867	**61°**	.8746	.4848	1.8040
17°	.2924	.9563	.3057	**62°**	.8829	.4695	1.8807
18°	.3090	.9511	.3249	**63°**	.8910	.4540	1.9626
19°	.3256	.9455	.3443	**64°**	.8988	.4384	2.0503
20°	.3420	.9397	.3640	**65°**	.9063	.4226	2.1445
21°	.3584	.9336	.3839	**66°**	.9135	.4067	2.2460
22°	.3746	.9272	.4040	**67°**	.9205	.3907	2.3559
23°	.3907	.9205	.4245	**68°**	.9272	.3746	2.4751
24°	.4067	.9135	.4452	**69°**	.9336	.3584	2.6051
25°	.4226	.9063	.4663	**70°**	.9397	.3420	2.7475
26°	.4384	.8988	.4877	**71°**	.9455	.3256	2.9042
27°	.4540	.8910	.5095	**72°**	.9511	.3090	3.0777
28°	.4695	.8829	.5317	**73°**	.9563	.2924	3.2709
29°	.4848	.8746	.5543	**74°**	.9613	.2756	3.4874
30°	.5000	.8660	.5774	**75°**	.9659	.2588	3.7321
31°	.5150	.8572	.6009	**76°**	.9703	.2419	4.0108
32°	.5299	.8480	.6249	**77°**	.9744	.2250	4.3315
33°	.5446	.8387	.6494	**78°**	.9781	.2079	4.7046
34°	.5592	.8290	.6745	**79°**	.9816	.1908	5.1446
35°	.5736	.8192	.7002	**80°**	.9848	.1736	5.6713
36°	.5878	.8090	.7265	**81°**	.9877	.1564	6.3138
37°	.6018	.7986	.7536	**82°**	.9903	.1392	7.1154
38°	.6157	.7880	.7813	**83°**	.9925	.1219	8.1443
39°	.6293	.7771	.8098	**84°**	.9945	.1045	9.5144
40°	.6428	.7660	.8391	**85°**	.9962	.0872	11.4301
41°	.6561	.7547	.8693	**86°**	.9976	.0698	14.3007
42°	.6691	.7431	.9004	**87°**	.9986	.0523	19.0811
43°	.6820	.7314	.9325	**88°**	.9994	.0349	28.6363
44°	.6947	.7193	.9657	**89°**	.9998	.0175	57.2900

Find the angles whose tangents are

16. .5317
17. .6745
18. 8.1443
19. 1.7321
20. 1.3270
21. .0175
22. .1405
23. 7.1154
24. .9325
25. 19.0811
26. .3249
27. .7002
28. .2126
29. .4245
30. 2.0503

Angles of Elevation and Depression[A]

In this sketch point A is in a valley and point B is on a hill. Lines AC and BD are horizontal lines in the same plane as AB. If you have a transit at A pointing along the horizontal line AC and wish to focus it on point B, you must turn the telescope through angle x. In the process the line of sight has been elevated from the horizontal line AC. We say "x is the *angle of elevation* of B from A." Now suppose the transit is placed at B and focused on A. In this case the line of sight is lowered from the horizontal line BD. We say "y is the angle of depression of A from B." Notice that the angle of elevation is above the horizontal, and that the angle of depression is below the horizontal. If you measure angles x and y you will find that the angle of elevation of B from A is equal to the angle of depression of A from B.



Using the Tangent to Solve Problems[A]

The following examples illustrate how the tangent is used in problems of indirect measurement:

Example 1. At a point 60 feet from the foot of a flagpole the angle of elevation of its top is 67°. How high is the pole?

Solution. $\angle B = 90°$

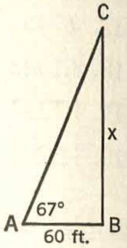
$$\frac{x}{60} = \tan 67°$$

$$x = 60 \tan 67°$$

From the table $\tan 67° = 2.3559$

Then $x = 60(2.3559)$

$$x = 141.354$$



Rounding off the result, $BC = 141.4$ feet. In this example no allowance is made for the height of the transit. What is the height of the flagpole in this example if the telescope is 4.5 feet above the ground?

Example 2. One side of a right triangle is 52 feet and the other side is 30 feet. Find the size of each acute angle.

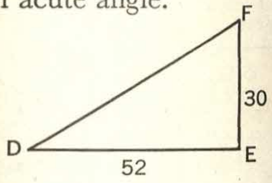
Solution. $\angle E = 90°$

$$\tan D = \tfrac{30}{52}$$

$$\tan D = .5769$$

From the table $\angle D = 30°$, approximately

$$\angle F = 90° - 30° = 60°\text{, approximately}$$



EXERCISES[A]

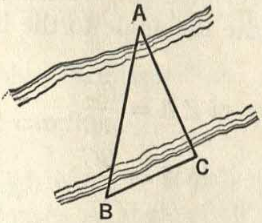
1. The angle of elevation of the sun is 42°. How high is a chimney which casts a shadow 240 feet long?

2. From an observation point the angle of elevation of an airplane was 82°. How high was the plane above the ground if the point directly beneath it was on the same level as, and 1500 feet from the point of observation?

3. Two Girl Scouts standing 300 yards from the foot of a cliff and on a level with it found the angle of elevation of its top to be 34°. How high was the cliff?

4. $ABCD$ is a rectangle with diagonal BD, $\angle ABD = 40°$, and $AB = 34$ inches. Find the area of the rectangle.

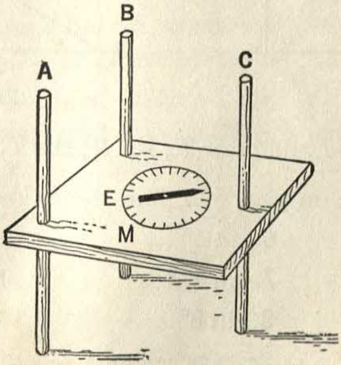
5. Two boys wish to find the distance from point A to point C across a river. They set a stake at C making $\angle BCA = 90°$. They find that $\angle B = 66°$ and $BC = 160$ feet. How long is AC?



6. When directly over one village the pilot of a plane observed that the angle of depression of another village on the same level as the first was 9°. If his altimeter showed him to be 8200 feet above the first village, what was the distance between the villages?

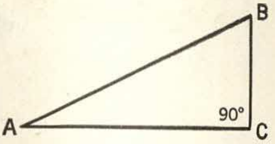
7. From the top of Jefferson High School the angle of depression of a point P, 1850 feet away, is 2°. How high is the building if P is on the same level as its base?

8. The sketch below shows a simple leveling instrument for measuring horizontal angles. M is a smooth, 1-inch board about a foot square. A, B, and C are small round sticks which fit into holes in M. E is a cardboard protractor circle glued to M. A pointer is fastened to M with a nail or screw. The board M can be leveled with a small spirit level by sliding the board on the sticks. Make an instrument of this kind.



The Sine of an Angle[A]

The sine of an angle is another important ratio. In any right triangle the *sine* (abbreviated sin) of either acute angle is the ratio of the side opposite the angle to the hypotenuse. Thus in triangle *ABC*,



$$\text{the sine of } \angle A = \frac{BC}{AB},$$

or,

$$\sin A = \frac{BC}{AB}$$

Also,

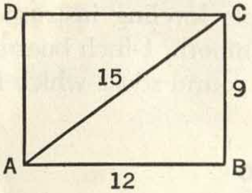
$$\sin B = \frac{AC}{AB}.$$

In general,

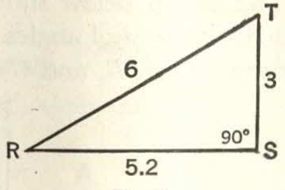
$$\sin x = \frac{\text{side opposite } x}{\text{hypotenuse}}.$$

ORAL EXERCISES[A]

1. *ABCD* is a rectangle. Find the tangent of $\angle BAC$; of $\angle BCA$. Find the sine of $\angle BAC$; of $\angle BCA$; of $\angle DAC$; of $\angle DCA$.



Ex. 1



Ex. 2

2. Using $\triangle RST$, find $\tan T$; $\sin R$; $\sin T$.

Using the table on page 520, find the following:

3. sin 24°
4. sin 63°
5. sin 81°
6. sin 5°
7. sin 85°
8. sin 8°
9. sin 15°
10. sin 36°
11. sin 18°

From the same table find the angles whose sines are

12. .8572
13. .7431
14. .0698
15. .2756
16. .9397
17. .9848
18. .9976
19. .0175
20. .7314

21. As an angle increases in size, how does the value of its sine change?

22. Can the sine of an angle be as small as zero?

23. Can the sine of an angle be as large as one?

Using the Sine to Solve Problems [A]

Study the example carefully.

Example. How high is a kite if it is at the end of a string 600 feet long which makes an angle of 48° with the ground?

Solution. $\frac{x}{600} = \sin 48°$

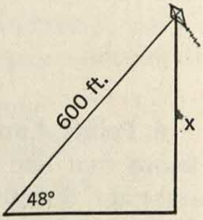
$x = 600 \sin 48°$

From the table $\sin 48° = .7431$

Then $x = 600(.7431)$

$x = 445.86$

To four significant figures, $x = 445.9$ feet

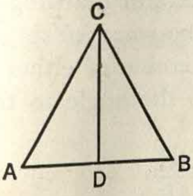


EXERCISES

A

1. A railroad track has an angle of elevation of 1°. What is the difference in altitude of two points on the track a mile apart?

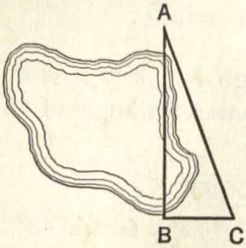
2. *CD* is the altitude of equilateral triangle *ABC*. How large is ∠*B*? ∠*BCD*? Using the sine of ∠*B*, find *CD* if *BC* = 10 inches. Using the sine of ∠*BCD*, find *BD*.



3. A 24-foot pole leaning against the side of a house makes an angle of 20° with the house. How far from the house is the foot of the pole?

4. The side of a square is 14 feet. Find the diagonal, using the sine function. Check your answer, using the Pythagorean Theorem.

5. A smokestack is 170 feet high. Find the length of a guy wire which is fastened to the stack 20 feet from the top and makes an angle of 40° with the ground.



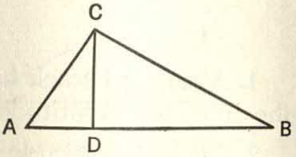
6. Points A and B are on the opposite sides of a lake. A boy knows that line AB runs north and south. Then he places a stake at C directly east of AB. He then measures AC and $\angle BCA$. If $AC = 1960$ feet and $\angle BCA = 42°$, what is the length of AB?

C

7. ABC is a triangle with $CD \perp AB$.

a. Show that $CD = AC \sin A$.

b. Show that the area of $\triangle ABC$ $= \frac{1}{2} AC \cdot AB \sin A$.



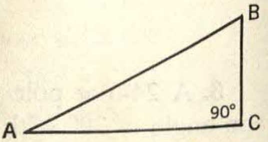
The Cosine of an Angle [A]

The third important ratio that depends upon the size of an angle is the cosine (pronounced kō′sīn and abbreviated cos). In any right triangle the cosine of either acute angle is the ratio of the side adjacent to the angle to the hypotenuse. Thus in $\triangle ABC$,

$$\cos A = \frac{AC}{AB}.$$

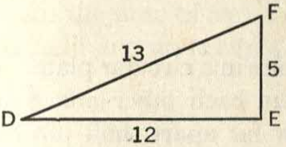
In general,

$$\cos x = \frac{\text{side adjacent to } x}{\text{hypotenuse}}.$$

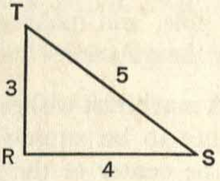


ORAL EXERCISES [A]

1. *DEF* is a right triangle with ∠$E = 90°$. Find tan D; tan F; sin D; sin F; cos D; cos F. How does sin D compare with cos F? How does cos D compare with sin F?



Ex. 1



Ex. 2

2. *RST* is a right triangle with ∠$R = 90°$. Find tan S; tan T; sin S; sin T; cos S; cos T. How does cos S compare with sin T?

Using the table on page 520, find the following:

3. cos 76°
4. cos 32°
5. cos 18°
6. cos 28°
7. cos 30°
8. cos 60°
9. cos 89°
10. cos 41°
11. cos 23°

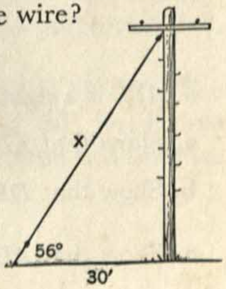
From this table find the angles whose cosines are

12. .6820
13. .4540
14. .9998
15. .7071
16. .9563
17. .7660

Using the Cosine of an Angle [A]

The following problem illustrates the use of the cosine.

Example. A guy wire attached to the top of a telephone pole reaches a stake in the ground 30 feet from the foot of the pole, and makes an angle of 56° with the line drawn from the stake to the foot of the pole. How long is the wire?



Solution. $\frac{30}{x} = \cos 56°$

Then $\frac{30}{x} = .5592$

M_x $30 = .5592\,x$

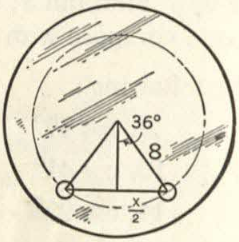
$x = 53.6$, the number of feet in the length of the wire.

EXERCISES

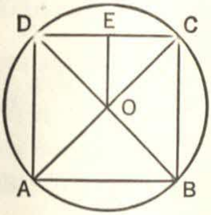
A

1. A guy wire of a pole meets the ground 42 feet from the foot of the pole, and makes an angle of 45° with the ground. How long is the guy wire?

2. A machinist wishes to drill 5 holes in a circular plate. These holes are to be equally distant from each other and 8 inches from the center of the plate. How far apart shall the centers of the holes be placed?



Ex. 2



Ex. 3

3. *ABCD* is a square inscribed in a circle with a 12-inch diameter.

a. How long is AO? DO?

b. $OE \perp DC$. How large is $\angle DOC$? $\angle DOE$?

c. Find the length of EO using the sine ratio.

d. Find the length of EO using the Pythagorean Theorem.

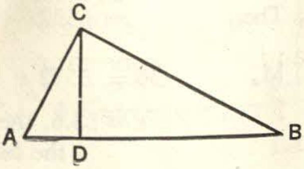
C

4. ABC is a right triangle with hypotenuse AB and $CD \perp AB$.

a. Show that $AD = AC \cos A$.

b. Show that $DB = BC \cos B$.

c. Show that $AB = \frac{AC}{\sin B}$.



5. The radius of the circle below is 16 inches. The line-segment AB is 7 inches, $AC = CB$, and $OC \perp AB$.

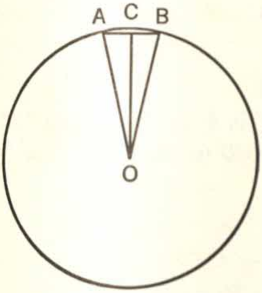
a. Find AC.

b. Find $\angle AOC$.

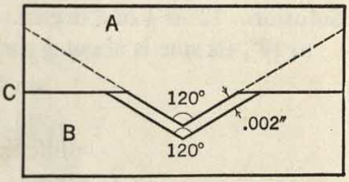
c. Find $\angle OAC$.

d. Find the area of $\triangle AOC$; of $\triangle AOB$.

e. Find the area of sector AOB. (A sector is a figure formed by two radii and an arc of a circle.)



Ex. 5



Ex. 6

6. The two metal plates A and B are placed together along line CD as shown in the figure. The angle parts fail to close by .002 of an inch. The plates are so made that they will fit exactly when a uniform width is ground off plate A along the straight part of CD. What is this width? (Draw a right triangle showing the width to be found.)

Interpolation [C]

When a right angle is divided into 90 equal parts, each part is called a degree. If an angle of one degree is divided into 60 equal parts, each part is called a minute (abbreviated ′). Then $1° = 60'$.

Suppose we wish to find the sine of an angle of $32° 30'$. The table on page 520 does not give the sine of $32° 30'$, but it does give the sines of 32 and 33 degrees and from these two sines we can find the approximate value of $32° 30'$.

From the table $\quad \sin 33° = .5446$
and $\quad \sin 32° = .5299$

Since 32° 30′ is halfway between 32° and 33°, its sine is about halfway between .5299 and .5446. To find the number halfway between .5299 and .5446, we first find the difference of these two numbers, which is .0147. Then we take half of this difference, which is $.0073\frac{1}{2}$. Then add $.0073\frac{1}{2}$ to the smaller number, .5299, and obtain .5372. Then $\sin 32^\circ 30' = .5372$.

This method of estimating a value that is not given in the table is called *interpolation*. Study the form and method of interpolation as given in the following examples:

Example 1. Find $\sin 18^\circ 12'$.

Solution. 12′ is $\frac{1}{5}$ of a degree. Since 18° 12′ is $\frac{1}{5}$ the way from 18° to 19°, its sine is about $\frac{1}{5}$ the way from .3090 to .3256.

$$\begin{aligned} \sin 19^\circ &= .3256 \\ \sin 18^\circ &= \underline{.3090} \\ \text{difference} &= .0166 \\ \tfrac{1}{5} \text{ of difference} &= .0033 \\ .3090 + .0033 &= .3123 \\ \text{Then} \qquad \sin 18^\circ 12' &= .3123 \end{aligned}$$

Example 2. Find $\tan 72^\circ 45'$.

Solution. 45′ is $\frac{3}{4}$ of a degree. Then tan 72° 45′ is about $\frac{3}{4}$ the way from 3.0777 to 3.2709.

$$\begin{aligned} \tan 73^\circ &= 3.2709 \\ \tan 72^\circ &= \underline{3.0777} \\ \text{difference} &= .1932 \\ \tfrac{3}{4} \text{ of difference} &= .1449 \\ 3.0777 + .1449 &= 3.2226 \\ \text{Then} \qquad \tan 72^\circ 45' &= 3.2226 \end{aligned}$$

EXERCISES [C]

Find the value of

1. $\sin 8^\circ 30'$

2. $\tan 24^\circ 15'$

3. $\sin 76^\circ 30'$

4. $\tan 20^\circ 30'$

5. $\sin 42^\circ 6'$

6. $\tan 24^\circ 24'$

7. $\sin 65^\circ 40'$

8. $\tan 47^\circ 36'$

9. $\sin 18^\circ 12'$

10. tan 77° 6′

11. sin 15° 10′

12. tan 53° 20′

13. sin 85° 50′

14. tan 4° 40′

15. sin 5° 48′

Example 3. Find cos 18° 15′.

Solution. 15′ is $\frac{1}{4}$ of one degree. Since the cosine becomes smaller as the angle increases, the difference .0014 must be subtracted from cos 18°.

$$\begin{aligned} \cos 18^\circ &= .9511 \\ \cos 19^\circ &= \underline{.9455} \\ \text{difference} &= .0056 \\ \tfrac{1}{4} \text{ of difference} &= .0014 \\ .9511 - .0014 &= .9497 \end{aligned}$$

Then $\cos 18^\circ\, 15' = .9497$

EXERCISES [C]

Find the value of

1. cos 25° 20′

2. cos 34° 6′

3. cos 12° 15′

4. cos 80° 5′

5. cos 4° 25′

6. cos 65° 20′

7. cos 12° 45′

8. cos 47° 40′

9. sin 28° 16′

10. tan 28° 16′

11. cos 19° 12′

12. cos 42° 45′

13. cos 84° 36′

14. cos 28° 16′

15. tan 81° 30′

Example 4. Find $\angle A$ if its sine = .3867.

Solution. In the table no angle is given whose sine is .3867. We can see that the angle is between 22° and 23°. We must find what part of the way .3867 is from .3746 to .3907.

$$\begin{aligned} \sin 23^\circ &= .3907 \\ \sin 22^\circ &= \underline{.3746} \\ \text{difference} &= .0161 \end{aligned}$$

The difference between .3867 and .3746 is .0121. Then .3867 is $\frac{.0121}{.0161}$ of the way from .3746 to .3907. Then $\angle A$ is about $\frac{.0121}{.0161}$ of one degree more than 22°, or $\frac{121}{161}$ of 60′ more than 22°.

$$\tfrac{121}{161} \times 60' = 45'$$

Then, $\angle A = 22^\circ\, 45'$

Example 5. Find $\angle A$ if $\cos A = .4279$.

Solution. From the table we see that $\angle A$ is greater than 64° and less than 65°. We must find how many minutes must be added to 64° to make $\angle A$.

$$\begin{array}{r} \cos 64^\circ = .4384 \\ \underline{\cos 65^\circ = .4226} \\ \text{difference} = .0158 \end{array}$$

$.4384 - .4279 = .0105$. Then $\cos A$ is $\frac{.0105}{.0158}$ of the way from 64° to 65°. $\frac{.0105}{.0158} \times 60' = 40'$. Then $\angle A = 64^\circ\, 40'$.

EXERCISES [C]

Find $\angle A$ when

1. $\sin A = .2784$

2. $\cos A = .9088$

3. $\tan A = .1435$

4. $\cos A = .9805$

5. $\sin A = .2504$

6. $\tan A = .3574$

7. $\cos A = .9503$

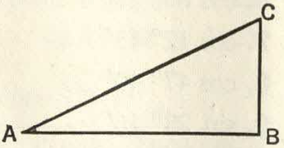
8. $\sin A = .3969$

9. $\tan A = 1.6386$

10. $\cos A = .2798$

11. $\tan A = 2.1159$

12. $\sin A = .7960$



Exercises 13–19 apply to $\triangle ABC$ in which $\angle B$ is the right angle.

13. Find AC when $BC = 2000$ feet and $\angle A = 21^\circ\, 10'$.

14. Find BC when $AB = 265.2$ feet and $\angle C = 54^\circ\, 15'$.

15. Find $\angle A$ when $BC = 195.3$ feet and $AC = 200.0$ feet.

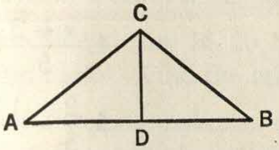
16. Find $\angle C$ when $BC = 1000$ and $AC = 1600$.

17. Find the area of $\triangle ABC$ when $\angle A = 18^\circ\, 30'$ and $AC = 500.0$.

18. If a hill has a slope of 8°, how far up the hill will a dam 62 feet high raise the water?

19. The Washington Monument is 555 feet high. What is the angle of elevation of its top when viewed from a point a mile away and on a level with the base of the monument?

20. $\triangle ABC$ is isosceles. Find the area of the triangle if AC and BC are 24 inches each and $\angle B = 35°$.



21. Find the angles of an isosceles triangle if the base is 20 inches and each of the other sides is 18 inches.

Historical Note [A]. Trigonometry had its beginnings in Egypt and Babylonia four thousand years ago, when heights and distances were found by indirect measurement. Thales, the founder of geometry, calculated the heights of the great pyramids of Egypt by indirect measurement about 550 B.C.

Hipparchus (180–125 B.C.), the greatest of the Greek astronomers, could rightly be called the father of trigonometry, for he was the first to study the subject on a scientific basis. He developed trigonometry to solve his astronomy problems. The accuracy of his observations and computations may be judged by the fact that he determined the length of the year to within six minutes of its real value.

The next contribution to trigonometry was made by the Hindus and the Arabs. They introduced the trigonometric functions we now have, and prepared tables of natural functions.

It was not until the fourteenth century that the trigonometry of the Arabs was introduced into Europe. Additions to the subject were made by Regiomontanus (1464), a German; by Vieta (1580), a Frenchman; by Napier (1614), a Scotsman; and by Sir Isaac Newton (1729), an Englishman.

WORD LIST

The following words should be understood. Can you spell each one?

acute angle (p. 515)
angle of depression (p. 521)
angle of elevation (p. 521)
approximate numbers (p. 509)
cosine (p. 526)
interpolation (p. 530)
numerical (p. 514)
obtuse angle (p. 515)
significant figures (p. 510)
sine (p. 524)
surveyor (p. 507)
tangent (p. 518)
transit (p. 516)
trigonometry (p. 514)

GENERAL REVIEW [A]

Solve:

1. $\frac{3x}{5} - \frac{x}{4} = 1$

2. $\frac{x}{2} - \frac{x}{6} = 10$

3. $\frac{y-3}{3} + 3 = \frac{y+4}{5}$

4. $\frac{x-3}{5} + \frac{2x-1}{3} = 6$

5. $\frac{2}{5} - \frac{1+2x}{5x} = \frac{7}{10}$

6. $\frac{4x-1}{2x+2} = \frac{2x-3}{x-1}$

Solve by completing the square:

7. $x^2 + 6x = 16$

8. $x^2 - 4x = 96$

9. $x^2 - 8x = 273$

10. $x^2 - 10x = 299$

11. $2x^2 - 5x - 3 = 0$

12. $3x^2 + 19x = -6$

13. $7x^2 - 20x - 3 = 0$

14. $3x^2 + x - 1 = 0$

15. Reduce $\frac{m^2 - 25}{2m^2 - 13m + 15}$ to lowest terms.

16. The denominator of a fraction is 3 greater than the numerator. Find the fraction if its value is $\frac{4}{5}$.

17. Find the number of degrees in each of two complementary angles if 5 times the larger exceeds 4 times the smaller by 270°.

18. How many pounds of 40-cent coffee and how many pounds of 70-cent coffee should be used to make a mixture of 60 pounds worth 60 cents a pound?

19. A number increased by 15% of itself equals 94.3. What is the number?

20. Solve for x and y:

$$5x - 3y = 46$$
$$2x + 5y = 6$$

21. Five times the square of a number is 79,380. Find the number.

22. Find the square root of 6.6049.

23. Complete $\sqrt{a}\sqrt{b} = \sqrt{?}$.

24. Solve $7c(2c - 1) + 2c(5 - 7c) - 18 = 0$.

25. Divide $x^3 - 3x^2 + 3x - 2$ by $x - 1$.

26. Separate 165 into two parts whose ratio is 4 : 7.

27. The sides of one triangle are 16, 20, and 27. The longest side of a similar triangle is 24. Find the other two sides.

28. A kite string 465 feet long makes an angle of 42° with the ground. Assuming that the string is straight, find the height of the kite.

29. What is the angle of elevation of an inclined plane that rises 1 inch in 10 feet?

30. Tell whether all rectangles are similar, and give the reason for your answer.

31. Are all squares similar? Why?

32. Add: $\frac{2c-4}{3} - \frac{6-5c}{4}$

33. Solve the equation $4x^2 + 7x = 2$ by all the methods that you know.

34. What are the three ways of finding the square root of $\frac{1}{3}$?

CHAPTER TEST[A]

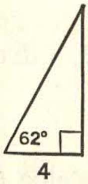
1. Round off the following numbers to the nearest tenth:

a. 7.86 **b.** 26.34 **c.** .09 **d.** 321.12

2. Round off the following numbers to three significant figures:

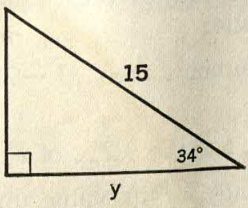
a. 2148 **b.** 62.17 **c.** .0324 **d.** 1.617

3.



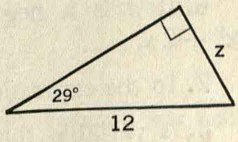
Find x.

4.



Find y.

5.



Find z.

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4. $\frac{x-3}{5} + \frac{2x-1}{3} = 6$

5. $\frac{2}{5} - \frac{1+2x}{5x} = \frac{7}{10}$

6. $\frac{4x-1}{2x+2} = \frac{2x-3}{x-1}$

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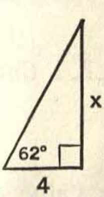
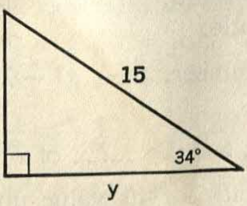


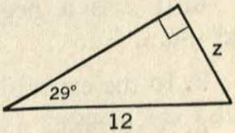
Find x.

4.



Find y.

5.

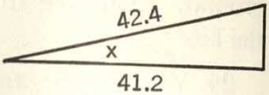


Find z.

6. An 18-foot ladder leaning against a house makes an angle of 55° with the level ground. How far from the house is the foot of the ladder?

7. When the angle of elevation of the sun is 27°, a telephone pole casts a shadow 36 feet long. How high is the telephone pole?

8. Find $\angle x$ in the diagram to the nearest degree.



CUMULATIVE TEST (CHAPTERS IX–XVIII). Time, 40 Minutes

See how many of these 30 problems you can do in 40 minutes. If you can do 10 problems correctly, you are just passing; if you can do 20 correctly, you are doing good work; and if you can do 25 or more correctly, you are doing excellent work.

In Exercises 1–5 write + for your answer if the statement is true and − for your answer if the statement is false.

1. If the numerator and denominator of a fraction are equal, the value of the fraction is zero.

2. One of the values of x that satisfies the equation $x^2 + 5x + 4 = 0$ is -4.

3. The product of any two numbers with like signs is a number having the same sign.

4. $A = \frac{1}{2}ab$, when a and b are both doubled, A is doubled.

5. $x^2 + y^2 = (x + y)^2$

Copy and complete statements 6–10, using in each case one of the following words: constant, greater, exponent, function, coefficient, variable, less, ratio.

6. If x is a negative number, then $x^2 - 2x$ is --?-- than $x^2 + 2x$.

7. In the expression $6x^3$, 6 is the --?-- of x^3.

8. A variable that depends for its value upon the value of some other variable is a --?-- of the other variable.

9. In the formula $p = 4s$, p and s are variables and 4 is a --?--.

10. $\frac{2}{3}$ is the --?-- of 12 to 18.

Factor:

11. $a^4 - 1$

12. $3a^2 - 2ay - 8a$

13. $4c^2 - 28c + 49$

14. $m^2 - 2m - 24$

15. Solve: $5a - a^2 = 7 - (a + a^2)$

16. Find the value of $15a^3 - 8a^2 + 2$ when $a = 2$.

17. If 10 is 4 more than $2a$, what is the value of $3a - 5$?

18. Simplify: $10 - 2(3a - 1)$

19. Find the value of $\frac{1}{x} + 3$ when $x = \frac{1}{3}$.

20. $f(x) = x^2 - 4x - 6$. Find $f(3)$.

21. A board is x feet long. When a piece a feet long is cut off one end and a piece b feet long is cut off the other end, how many inches long is the piece remaining?

22. $i = prt$. How long must $3000 be invested at 5% simple interest to produce $450 interest?

23. Each end of a rectangle is $3y - 1$. Find the top and bottom of the rectangle if its perimeter is $10y + 4$.

24. A merchant sold an automobile for $1575 at a margin of 25% of the cost. Find the cost.

25. Solve $3(x + 4)(x - 4) - x(3x - 5) = 12$.

26. Solve $x^2 - 20x = -91$.

27. Solve $x^2 + 6x = 11$.

28. A room is 14 feet long and 12 feet wide. Find its diagonal.

29. Solve $\frac{x}{4} - \frac{y}{6} = \frac{1}{2}$

$\frac{x}{3} - \frac{y}{2} = -1$

30. A tower casts a shadow of 20 feet when a man 6 feet tall casts a shadow of $1\frac{1}{2}$ feet. How high is the tower?

CHAPTER XIX

Review by Topics

COMBINING TERMS

Simplify by combining like terms:

1. $3a + 2b - 5a + 4c - 4b$

2. $x - y + z - 3x - 4y + 3z$

3. $3m - 4n + 4p - 6m - 8n - 4p$

4. $3h - 4 + 7h - 5h - 2 - h$

5. $2abc - 3b + 2c - 5a - abc - 2a - 2c$

6. $2.3a + 1.7b - 4.5 - 4.4a - 5.2b$

7. $\frac{1}{2}x - \frac{1}{3}y + \frac{1}{6}x - \frac{1}{2}y$

8. $2.3c + 4.1d + 5.2c - 8.6d + c$

9. $10x^3 + 5x^2 - 2x + 1 - x^3 - 10x^2 - 2x - 1$

10. $\frac{7}{9}c + \frac{1}{3}c + .4b - .15b$

ADDING POLYNOMIALS

Add:

1. $2x - 3y + 4z$, $5x - y - z$, and $-x - y + z$

2. $3x^2 - 2x + 1$, $4x^2 - x - 9$, and $-6x^2 + 7x + 1$

3. $4a - 3b + c$, $5a - c + b$, and $2b - 6a$

4. $2x - 1$, $-3x + 2$, $4x - 5$, and $10x - 11$

5. $3ab - 4bc + 5cd$, $-bc + ab$, and $8ab - cd$

6. $x^3 - 2x^2y - xy^2 + 2y^3$ and $-x^3 - x^2y + 5xy^2 + y^3$

7. $ab^2 - a^2b + ab$ and $-ab^2 - 3a^2b + 4ab$

8. $st + st^2 - t^3$, and $3\,st^2 - ts - 3\,t^3$

9. $.3\,x - 4.5\,y + 3.1\,z$ and $-\,.8\,x - 2.5\,y - .6\,z$

10. $\frac{1}{2}\,x - \frac{1}{3}\,y + \frac{1}{5}\,z$ and $\frac{1}{3}\,x - \frac{1}{4}\,y - \frac{2}{3}\,z$

SUBTRACTING POLYNOMIALS

1. From $3\,x - 4\,y + 2\,z$ take $x + y + z$.

2. From $5\,a - 4\,b + 6\,c$ take $6\,a - 7\,b - 3\,c$.

3. Take $2\,a + 3\,b - 7\,c$ from $-\,3\,a + 9\,b - 6\,c$.

4. Take $3\,x^3 - 2\,x^2 + 7$ from $5\,x^2 - 2\,x - 9$.

5. Find the remainder when the subtrahend is $3\,x^2 - xy - y^2$ and the minuend is $x^2 - 4\,xy + y^2$.

6. The remainder is $x - 5$ and the subtrahend is $x^2 + 2\,x + 1$. Find the minuend.

7. Subtract $ab - ac + 5\,ad$ from 0.

8. How much smaller is $x^2 - 3$ than $2\,x + 1$?

9. How much larger than $8\,a - 5\,b$ is $-\,3\,a + 5\,b$?

10. By how much does $4\,x - 5\,y + 1$ exceed $3\,y - x + 2$?

REMOVING PARENTHESES

Simplify:

1. $(3\,x - 4\,y + z) + (2\,x - y - z)$

2. $(4\,a - 2\,b + c) - (3\,a - 4\,b - c)$

3. $4 + (x - 1) - (3\,x + 1)$

4. $2\,k + (3\,k - 1) - [4\,k + 6]$

5. $(2\,x^3 - 3\,x^2 + x) - (x^2 - 1) + 4\,x^2$

6. $1 - x + (6\,x - 3) - (8 - 2\,x)$

7. $1 + 3(c - 1) - 4(2\,c - 4)$

8. $1 + 3(x - 4) - 5(2\,x - 5)$

9. $(3\,x - 1)4 - 5(6\,x + 7) + 3(x + 2)$

10. $x(x - 3) - (x^2 - 6) - [x^2 - 2\,x + 9]$

MULTIPLYING POLYNOMIALS

Multiply as indicated:

1. $(a + 3)(a - 2)$

2. $(c - 3)(c + 5)$

3. $(x - y)(x + y)$

4. $(x - 2y)(x + 2y)$

5. $(cd + 1)(cd - 3)$

6. $(3x - 5y)(2x + 3y)$

7. $(x^2 - 2x + 1)(x - 1)$

8. $(x - y + 1)(x + y - 1)$

9. $(c^2 - c + 3)(c + 2)$

10. $(x^2 - xy + y^2)(x + y)$

DIVIDING POLYNOMIALS

Divide:

1. $4m^3 - 8m$ by $4m$

2. $8x^2 - 12x$ by $-2x$

3. $c^2 - c$ by -1

4. $c^2 - c$ by $-c$

5. $4x^5 - 8x^4$ by $4x^3$

6. $6x^3 - 12x^2 - 30x$ by $-3x$

7. $24m^2 - 18m - 15$ by 3

8. $8x^3y^2 - 16x^2y^4$ by $4x^2y^2$

9. $10x^3y - 15x^2y^2$ by $5x^2y$

10. $8x^4 - 4x^2 - 4x$ by $-4x$

11. $(x^2 + 8x + 12) \div (x + 2)$

12. $(a^2 - 13a + 12) \div (a - 1)$

13. $(x^2 - 2x - 35) \div (x + 5)$

14. $(4a^2 + 7ab - 2b^2) \div (4a - b)$

15. $(x^3 - 3x^2 + 3x - 1) \div (x - 1)$

16. $(12a^3 + 34a^2 - 30) \div (3a + 4)$

17. $(2c^3 - 5c + 6 - 3c^2) \div (c^2 + 2 - 3c)$

18. $(8x^3 - y^3) \div (2x - y)$

EVALUATION

If $a = 6$, $b = 3$, and $c = 2$, find the value of

1. a^2

2. b^3

3. c^4

4. $3a^2$

5. abc

6. $4abc$

7. $a^2 - b^2$

8. $bc - 6$

9. $a^2 - bc$

10. $ab - 8bc + 2ac$

11. $a^3 + b^3 + c^3$

12. $5b - 4a + 3c$

If $x = 4, y = -2$, and $z = -1$, find the value of

13. $2x - 3y + 1$

14. $x^2 - 4x + y$

15. $xy + 8z - x$

16. $2xy - xy - y^2$

17. $y^2 + 10z + z^2$

18. $3x + 7y - 8z$

19. $-4x - 3y + 5z$

20. $3z - 5x + y$

21. $xyz - y^2$

22. $xy + 3yz$

If $a = 6, b = -5$, and $c = 0$, find the value of

23. $a^2 + b^2 + c^2$

24. $4abc - 10$

25. $bc + a^3$

26. $\frac{ab}{a+b}$

27. $28c + 4ab$

28. $2b^3 - 4a^2 + 80c$

29. $\frac{1}{a} + \frac{1}{b} + \frac{c}{1}$

30. $\frac{5a^2 + 30ab}{ab}$

31. $d = 2r$. Find d when $r = 15.1$.

32. $V = \pi r^2 h$. Find V when $\pi = 3.14$, $r = 20$, and $h = 30$.

33. $S = 180(n - 2)$. Find S when $n = 11$.

34. $A = \frac{h}{2}(b + b')$. Find A when $b = 8$, $b' = 14$, and $h = 1$.

35. $S = \frac{1}{2}gt^2$. Find S when $t = 5$ and $g = 32.16$.

36. $K = \frac{1}{2}mv^2$. Find K when $m = 840$ and $v = 16$.

37. $C = \frac{V}{R}$. Find C when $V = 832$ and $R = 40$.

SPECIAL PRODUCTS AND FACTORING

Write the products:

1. $x(a + b)$

2. $3m(a - b + c)$

3. $(x + 3)^2$

4. $(2x - 1)^2$

5. $(x + 4)(x - 4)$

6. $ab(3 - ab)$

7. $(3x + 5)^2$

8. $(a - 5)(a - 7)$

9. $(3x + 5)(4x - 5)$

10. $(x + 2y)^2$

11. $3a^2x(a^2 + ax + x^2)$

12. $(7x - y)(2x + y)$

13. $(3x - 2y)^2$

14. $-5a(a^2 - b^2)$

15. $(1 - a)(1 + a)$

16. $(a^2 - 3)(a^2 + 3)$

Find the prime factors:

17. $2x - 2y$

18. $x^4 - x$

19. $4xy - 8x^2y^2$

20. $P + Prt$

21. $a^2 - b^2$

22. $x^2 - 16$

23. $m^2 - 4n^2$

24. $a^2 + 2ab + b^2$

25. $1 + 4b + 4b^2$

26. $9 - c^2$

27. $x^2 + 3x + 2$

28. $x^2 - 6x - 7$

29. $a^2 + ab - 20b^2$

30. $2x^2 + x - 6$

31. $3a^2 - 4a - 4$

32. $6x^3 + x^2 - 5x$

33. $a^3 - a$

34. $4a - 4ab^2$

35. $x^3 - 4x$

36. $2b^2 + 12b + 18$

37. $2c^3 - 10c^2 + 12c$

38. $3x^2 + 12xy - 135y^2$

39. $4n^2 - 16n - 20$

40. $a^4 + 2a^3 + a^2$

41. $c^3 - 8c^2 + 10c$

42. $20a - 8ax - ax^2$

43. $a^4 - 81$

44. $2x^3 - 8x^3y^2$

45. $x^4 - 13x^2 + 36$

46. $24a^2b - 10ab - 4b$

LINEAR EQUATIONS

Solve:

1. $3x - 6 = x + 10$

2. $x + 6 = -5x + 12$

3. $-2c + 8 = c - 4$

4. $-8y - 5 = -2y + 1$

5. $x - 3 = 0$

6. $8 + x = -x + 5$

7. $10h + 1 = 8h + 2$

8. $4r - 10 = 7r + 5$

9. $x - 3(x + 4) = 2$

10. $7(m - 4) - (m - 4) = 0$

11. $3(x - 2) - 3 = 0$

12. $2(y - 6) = 3(y - 2)$

13. $x^2 + 3 = (x + 5)(x + 9)$

14. $(x + 4)^2 - (x + 3)^2 = 14$

15. $(x + 1)^2 - (x - 2)^2 = 21$

16. $4 - (x - 2)(x + 7) = 24 - x^2$

17. $\frac{x}{2} - \frac{x}{3} = 2$

18. $\frac{3n}{4} + \frac{2n}{5} = \frac{5n}{6} + \frac{19}{3}$

19. $3(4 - x) - 7(x - 2) = 9x + 7$

20. $\frac{5}{6}x + 21 = \frac{2}{3}x + 18$

21. $3 - \frac{3x}{2} = \frac{8 - 4x}{7}$

22. $\frac{x - 5}{2} + \frac{x + 5}{2} = 0$

23. $\frac{r+9}{6} - \frac{r+1}{5} = \frac{3}{2}$

24. $\frac{4x+3}{7} - \frac{x+5}{3} = 4$

25. $\frac{c-12}{10} = \frac{3(2c+1)}{5} - \frac{3c-6}{2}$

26. $\frac{5}{x+4} = \frac{4}{x-4}$

27. $\frac{c+5}{c-3} = \frac{1}{2}$

28. $\frac{x}{x-3} + \frac{3}{x+2} = \frac{x^2}{x^2-x-6}$

29. $\frac{1}{x} + \frac{1}{x+3} = \frac{2}{x-3}$

30. $3(y+2) - 2(y-3\frac{1}{2}) = 15$

31. $\frac{2}{x} + \frac{3}{x-1} = \frac{5}{x+1}$

32. $\frac{3}{x+4} + \frac{1}{x} = \frac{4}{x-2}$

33. $\frac{2}{m-3} + \frac{1}{m+1} = \frac{3}{m-4}$

34. $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5x+2}{x^2-1}$

35. $\frac{4}{x-5} - \frac{5}{x+5} = \frac{1-5x}{x^2-25}$

36. $\frac{4x+8}{2x-3} = \frac{2(x+3)}{x-1}$

37. $\frac{4c}{2c-1} = \frac{2c}{2c+1} + 1$

38. $\frac{2r-1}{r} - \frac{r}{r-1} = 1$

QUADRATIC EQUATIONS

Solve by factoring:

1. $x^2 - 4x = 0$

2. $x^2 - 36 = 0$

3. $x^2 - 3x + 2 = 0$

4. $x^2 - 4x = 96$

5. $x^2 + x = 42$

6. $2x^2 + x = 28$

7. $\frac{x^2}{3} = 27$

8. $4x^2 - 20x = 0$

9. $x + 7 = \frac{18}{x}$

Solve by completing the square or by formula:

10. $2x^2 - 3x + 1 = 0$

11. $y^2 - 6y = 1$

12. $x^2 - 7x + 2 = 0$

13. $y^2 - 3y - 10 = 0$

14. $2x^2 - 7x + 3 = 0$

15. $m^2 - 2m + 1 = 0$

16. $y^2 + \frac{5}{2}y = 6$

17. $3x^2 + 2x = 8$

18. $(x+1)^2 - 9 = 0$

19. $3h^2 - 10h + 3 = 0$

20. $\frac{4(x^2-5)}{x^2-9} = \frac{9}{x-3}$

21. $\frac{x+5}{x^2-3x} = \frac{4x}{x-3}$

22. $\frac{x-3}{2x} + 5 = \frac{9x+7}{3x-5}$

23. $\frac{1}{x} + \frac{2}{x-3} = \frac{5}{6}$

Solve for y:

24. $ay^2 + by + c = 0$

25. $my^2 - y + 3 = 0$

26. $y^2 + 2(b - 1) + b^2 = 0$

RADICAL EQUATIONS

Solve:

1. $2\sqrt{x} = 1$

2. $\sqrt{2x} - 6 = 0$

3. $\sqrt{x} - 7 = 0$

4. $\sqrt{x - 5} = 2$

5. $2\sqrt{y + 2} = 6$

6. $\sqrt{2x - 7} - 5 = 0$

7. $x - 7 = \sqrt{x^2 - 7}$

8. $4 + \sqrt{x + 1} = x - 1$

SETS OF EQUATIONS

Solve:

1. $2x + 3y = 7$
$4x - 5y = 3$

2. $5x - 3y = 23$
$x + y = 3$

3. $3x + y = 4$
$x + 3y = -4$

4. $4m - 5n = 8$
$2m + 3n = 10$

5. $3x = 5y + 22$
$5x - y = 0$

6. $c = d + 3$
$4d - c = 12$

7. $\frac{x}{4} - \frac{y}{5} = 0$
$x - y = -1$

8. $\frac{x}{3} + \frac{y}{5} = 11$
$\frac{x}{2} - \frac{y}{3} = 7$

9. $\frac{7y - 4}{5} + \frac{2x - 3}{2} = \frac{3}{2}$
$\frac{5x - 2}{3} + \frac{2y + 1}{5} = 2$

10. $\frac{2x - y}{2} + \frac{x + y}{4} = 8$
$3(x + y) - (x - y) = 4$

FRACTIONS

Reduce to lowest terms:

1. $\frac{4a}{6ab}$

2. $\frac{6xy}{9x^2y}$

3. $\frac{x + y}{x^2 - y^2}$

4. $\frac{(x - y)^2}{x^2 + xy - 2y^2}$

5. $\frac{a^2 - 4a}{a^2 - a - 12}$

6. $\frac{bx + b^2}{x^2 + 2bx + b^2}$

Multiply.

7. $\frac{15\,a^6}{8\,y^2} \cdot \frac{16\,y^3}{5\,a^4}$

8. $\frac{3\,y}{7\,x} \cdot \frac{21\,x}{9\,y^3} \cdot \frac{2\,y^2}{5\,x}$

9. $\frac{x^2 + 4\,x + 4}{x^2 - x - 6} \cdot \frac{x^2 - 6\,x + 9}{3\,x + 6}$

10. $\frac{7\,a - 14}{1 - x^2} \cdot \frac{2\,x + 2}{5\,a - 10}$

Divide:

11. $\frac{4\,x^2}{3\,y} \div \frac{x}{9\,y}$

12. $\frac{4\,b}{5\,m} \div \frac{6\,abc}{m^3}$

13. $\frac{x^2 - 4}{x^2 - 1} \div \frac{x + 2}{x - 1}$

14. $\frac{a^2 - 1}{a^2 - 4} \div \frac{3\,a - 3}{5\,a + 10}$

15. $\frac{a + 4}{a^2 + 2\,a - 3} \div \frac{3\,a + 12}{a^2 + 3\,a}$

16. $\frac{x^2 - 25}{x^2 + 10\,x + 25} \div \frac{x^2 + 5\,x}{x^2 + 2\,x - 15}$

Combine:

17. $\frac{x}{3} - \frac{2\,x}{7}$

18. $\frac{a}{b} + 1$

19. $\frac{c}{d} + \frac{d}{c}$

20. $\frac{2}{a} + \frac{3}{b} - \frac{4}{c}$

21. $\frac{x - 1}{2} - \frac{x + 3}{3}$

22. $\frac{a - 1}{a + 1} + \frac{a + 1}{a - 1}$

23. $\frac{4\,x}{x^2 - 9} + \frac{5}{x + 3}$

24. $\frac{a^2 + 3}{a^2 - 3\,a + 2} - \frac{5\,a}{a - 2}$

25. $\frac{x^2 + 2\,x}{x^2 + 7\,x + 10} - \frac{x - 3}{x + 2}$

26. $\frac{5\,x}{x - 3} + \frac{x^2 - 2\,x}{x^2 - 5\,x + 6}$

27. $\frac{a - b}{a + b} + \frac{a + b}{a - b}$

28. $\frac{c - 1}{c^2 - 3\,c} + \frac{1}{c}$

29. $\frac{a + 2}{a + 3} + \frac{a - 3}{a + 1} - \frac{a^2 - 6}{a^2 + 4\,a + 3}$

30. $\frac{5\,b - 1}{b^2 + 3\,b} + \frac{6\,b}{b^2 + 8\,b + 15} - \frac{2\,b + 1}{b^2 + 5\,b}$

Change to simple fractions:

31. $\dfrac{1 + \frac{1}{a}}{a - \frac{1}{a}}$

32. $\dfrac{\frac{a^2 - 1}{a}}{\frac{a - 1}{a^2}}$

POWERS AND ROOTS

Find the following powers:

1. 9^2

2. $(a^2)^3$

3. $(a^3)^2$

4. $(-2a)^2$

5. $(-4m)^3$

6. $(x^2y)^3$

7. $(.1c^2)^2$

8. $(\frac{1}{2}ab^2)^3$

9. $(1.2r^2)^3$

Find the indicated roots:

10. $\sqrt{100b^2}$

11. $\sqrt{64a^6}$

12. $\sqrt[3]{-8m^3}$

13. $\sqrt{x^6y^4}$

14. $-\sqrt{16}$

15. $\sqrt[3]{-125y^6}$

16. $\sqrt{16x^4y^2}$

17. $\sqrt{1.44c^8}$

18. $\sqrt[3]{.027h^9}$

19. $\sqrt{49y^2}$

20. $-\sqrt{a^{10}b^{12}}$

21. $\sqrt{a^2b^4c^6}$

Find the positive square root of

22. 1089

23. 116.64

24. 13.5776

25. 12387.69

26. .082369

27. .00136161

Simplify:

28. $\sqrt{32}$

29. $\sqrt{196}$

30. $\sqrt{50}$

31. $3\sqrt{27}$

32. $\sqrt{a^3}$

33. $2\sqrt{24}$

34. $5\sqrt{20}$

35. $\sqrt{a^4b}$

36. $2\sqrt{12}$

37. $\sqrt{\frac{2}{3}}$

38. $\sqrt{\frac{1}{2}}$

39. $\sqrt{\frac{4}{5}}$

Simplify:

40. $2\sqrt{3}-\sqrt{27}$

41. $\sqrt{18}+\sqrt{50}$

42. $\sqrt{20}-\sqrt{5}$

43. $\sqrt{27}-8\sqrt{\frac{1}{3}}$

44. $\sqrt{48}+4\sqrt{3}-5\sqrt{3}$

45. $\sqrt{36}-\sqrt{8}+\sqrt{12}$

46. $3\sqrt{8}+4\sqrt{\frac{1}{2}}-6\sqrt{2}$

47. $5\sqrt{12}+2\sqrt{27}-4\sqrt{3}$

48. $2\sqrt{50}-3\sqrt{98}+10\sqrt{\frac{1}{5}}-4\sqrt{125}$

Rationalize the denominators:

49. $\frac{4}{\sqrt{3}}$ **52.** $\frac{7}{\sqrt{7}}$ **55.** $\frac{2}{3\sqrt{3}}$

50. $\frac{1}{\sqrt{2}}$ **53.** $\frac{\sqrt{3}}{\sqrt{2}}$ **56.** $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}}$

51. $\frac{\sqrt{5}}{\sqrt{3}}$ **54.** $\frac{3}{\sqrt{8}}$ **57.** $\frac{2\sqrt{5}}{\sqrt{3}}$

NUMBER PROBLEMS

1. Two boys received $1.60 for mowing a lawn. If the older boy did three times as much work as the younger, how much money should each boy receive?

2. A rectangle is four times as long as it is wide, and the perimeter is 300 inches. Find the width and length of the rectangle. (Always make a drawing for a problem, if possible.)

3. The number of boys in an algebra class is one less than three times the number of girls. If there are 39 pupils in the class, how many of them are girls?

4. The sum of a certain number and 6 is 19. What is the number?

5. 42 is equal to a certain number increased by 14. What is the number?

6. Four times a certain number equals 96 increased by the number. What is the number?

7. If 20 is added to three times a certain number, the sum is 101. Find the number.

8. The sum of two numbers is 620 and the larger is 3 times the smaller. What are the numbers?

9. The sum of two numbers is 63 and one is 27 larger than the other. What are the numbers?

10. The sum of two numbers is 44 and one exceeds the other by 18. What are the numbers?

11. Four times a certain number, decreased by 5, equals 25 decreased by 6 times the number. What is the number?

12. The number of B grades in a class was 4 times the number of A grades, the number of C grades was 6 times the number of A grades, the number of D grades equaled the number of B grades, and the number of E grades equaled the number of A grades. If there were 32 grades in all, how many of each kind were there?

13. One number is 7 larger than another. If twice the larger is added to the smaller, the sum is 65. What are the numbers?

14. One truck can carry 4 tons of freight at a time, and a larger truck can carry 6 tons at a time. How many trips must each truck make if they make a total haul of 58 tons and the smaller truck makes 2 more trips than the larger?

15. The difference of two numbers is 10. If twice the larger is subtracted from five times the smaller, the remainder is 22. What are the numbers?

16. The sum of two numbers is 62. Three times the smaller exceeds twice the larger by one. Find the numbers.

17. The sum of two numbers is 45. If 3 times the larger is increased by 4 times the smaller, the sum is 155. Find the numbers.

18. The sum of two numbers is 47. What are the numbers if twice the larger added to three times the smaller makes a sum of 106?

19. Separate 1200 into two parts so that one part shall be 800 less than three times the other.

20. A farmer sold 425 bushels of wheat and oats for $526. How many bushels of each did he sell if he received $1.60 a bushel for the wheat and 72 cents a bushel for the oats?

21. The sum of two numbers is 108. If twice their difference is subtracted from the smaller, the remainder is 9. What are the numbers?

22. The sum of two numbers is 40. The difference of five times the larger and four times the smaller exceeds seven times the smaller by 8. Find the numbers.

23. The sum of two numbers is 48. If three times the smaller is subtracted from twice the larger, the remainder is 6. What are the numbers?

24. The perimeter of a rectangle is 156 inches. If the length is decreased by 13 inches and the width is increased by 13 inches, the figure becomes a square. What are the dimensions of the rectangle?

25. John thought of a number and multiplied it by 5. He then subtracted 92 from the result and found that the difference thus obtained lacked 11 of being twice the number he first chose. What was the original number?

26. One number is 5 less than 5 times another. If twice the larger is added to seven times the smaller, the sum is 92. What are the numbers?

27. A man gave to one son one third of his money, and to another son one fourth of his money. He had $10 left. How much had he at first?

28. If two pianos cost $600 more than four radios, and if five pianos cost $1350 more than 12 radios, find the cost of a piano and of a radio.

29. The sum of two numbers is 19, and the sum of their squares is 185. Find the numbers.

30. The difference of two numbers is 2 and the sum of the squares is 74. What are the numbers?

31. The denominator of a fraction exceeds its numerator by 3. If each term of the fraction is increased by 7, the value of the new fraction is $\frac{3}{4}$. What was the original fraction?

32. The numerator of a certain fraction is 2 less than half the denominator, and the value of the fraction is $\frac{1}{3}$. What is the fraction?

33. Twice the square of a number, decreased by 5 times the number, equals 3. Find the number.

34. If the square of a number is subtracted from 8 times the number, the remainder is 12. What is the number?

PERCENTAGE PROBLEMS

1. A man spends 18% of his monthly salary for rent. If his rent is $45 a month, what is his salary for a month?

2. How much money must a man invest at 4% to produce an annual income of $600?

3. A number decreased by 10% of itself equals 405. What is the number?

4. A number increased by 16% of itself equals 185.6. Find the number.

5. A merchant sold goods for $2592 and made a gain of 8% of the cost. Find the cost if there was no overhead.

6. How cheaply can a grocer sell berries that cost 32 cents a quart if he must make a profit of 20% of the selling price?

7. 500% of a number is 75. Find the number.

8. A man sold an article for $69, thereby making a profit of 15% on the cost of the article. What was the cost?

9. A man sold an automobile for $1295, which was 30% less than it cost. What did the car cost?

10. When wheat is ground into flour, 18% of the wheat is a by-product. How much wheat is needed to make 100 pounds of flour?

11. A dealer wishes to buy hats that can be retailed at $3.92. At what price must he buy them if the margin is 40% of the cost?

12. One of two complementary angles is 25% larger than the other. How large is each?

13. The population of a certain city has increased 15% in the last 10 years. If the present population is 109,200, what was the population 10 years ago?

14. A number less $12\frac{1}{2}$% of itself is 567. What is the number?

15. An article was priced to sell for $7.20, thus yielding a profit of 20% based on the cost. At what price must the article sell if a profit of 30%, based on the cost, is made?

16. A chair sold for \$10.08 after discounts of 30% and 20% were made. What was the list price?

17. Dick's father sold his farm for \$12,150. This was 10% less than the farm cost him. What was the cost of the farm?

18. A man bought a horse and sold it for \$96, gaining as many per cent of the cost as the cost of the horse in dollars. Find the cost of the horse.

COIN PROBLEMS

1. A man has two more nickels than dimes, and has \$1.15 in all. How many coins of each kind has he?

2. A boy has twice as many nickels as dimes, and 5 more half dollars than nickels. Find the number of each kind of coins if their total value is \$6.10.

3. A girl has two more nickels than dimes, and three more quarters than nickels, having in all \$3.35. How many coins of each kind has she?

4. A purse contains 21 coins, consisting of nickels and dimes. How many coins of each kind does it contain if their total value is \$1.65?

5. A safe contains 120 coins, the value of which is \$10. If the coins consist of nickels and dimes, how many of each kind are in it?

6. A sum of \$14 is made up of 92 coins, consisting of dimes and quarters. How many are there of each?

7. Dick gave the grocer \$3.60, consisting of 20 half dollars, quarters, and nickels. If there were 3 times as many nickels as half dollars, how many coins of each kind were there?

8. The value of some nickels and quarters is \$1.60. If there are one third as many nickels as quarters, how many nickels are there?

9. A boy's coins, consisting of nickels and dimes, amount to \$2.15. If the number of dimes exceeds 3 times the number of nickels by 4, find the number of each kind of coin.

10. A newsboy had \$2.65, consisting of nickels, dimes, and quarters. If the number of dimes exceeded the number of nickels by 1, and if the number of quarters was equal to the number of nickels decreased by 1, what was the number of each kind of coin?

AGE PROBLEMS

1. Henry is 10 years older than John. In 8 years twice Henry's age will equal 3 times John's age. How old is each now?

2. Roy is 5 years older than Bert. If the sum of their ages in six years will be 33 years, how old is each now?

3. The sum of the ages of Mary and her mother is 60 years. In 20 years twice Mary's age, increased by her mother's age then, will equal 138 years. How old is each?

4. Robert is 14 years old and his father is 38 years old. How many years ago was the father 7 times as old as the boy?

5. A man was 30 years old when his son was born. His age now exceeds 3 times his son's age by 6 years. How old is each?

6. Two years ago a man was 4 times as old as his son. In 3 years the father will be only 3 times as old as his son. How old is each?

7. Frank is 4 times as old as Karl. In 10 years he will be only twice as old as Karl. How old is each now?

8. The sum of the ages of a father and son is 49 years. Four years ago the father's age was one year more than 3 times the son's age. How old is each now?

9. John's age is 3 years more than twice Bill's age. Three years ago John was 4 times as old as Bill was then. How old is each now?

10. Ann is 7 years older than Jane. One year ago she was twice as old as Jane. How old is each now?

11. Frank is 5 times as old as Dick. Two years ago he was 7 times as old as Dick. How old is each now?

12. Jim is 10 years old and his father is 3 times as old. In how many years will Jim be half as old as his father?

GEOMETRY PROBLEMS

1. The perimeter of a rectangle is 170 feet. If the length exceeds 5 times the width by 1, what is the area of the rectangle?

2. One angle of a triangle exceeds another by 12°, and the third angle exceeds twice the smallest by 48°. How large is each angle?

3. Five times one of two complementary angles equals four times the other. How large is each?

4. Two angles are supplementary. If 7 times the smaller is subtracted from 3 times the larger, the difference is 190°. How large is each?

5. Find the angle whose supplement is three times its complement.

6. One angle of a triangle is 115° larger than another. The third angle exceeds twice the smallest by 9 degrees. How large is each angle?

7. One angle of a triangle exceeds another by 2 degrees. The sum of these two angles exceeds the third angle by 28°. How large is each angle?

8. In a parallelogram one angle is 30° more than twice one of the others. How large is each of the four angles?

9. How large a rectangular field, whose length is to exceed twice its width by 10 rods, can be enclosed by a mile of fence?

10. Find the circumference of a circle whose radius is 30 inches.

11. Find the radius of a circle whose circumference is 148 inches.

12. The first of the three angles of a triangle is one third of the second, and the third is $\frac{7}{6}$ of the second. How large is each?

13. How large is each angle of a triangle in which the second angle is 10° more than the first and the third is 5° less than three times the first?

14. What are the lengths of the sides of a triangle whose perimeter is 226 inches if the second side is three fourths as long as the first and the third side is 28 inches longer than the first?

15. The length of a rectangle is 2 feet more than twice its width, and its perimeter is 40 feet. Find its area.

16. Find the angle whose supplement is 6 degrees larger than 8 times its complement.

17. The length of a rectangle exceeds twice its width by 4 feet. The area of the rectangle is 144 square feet more than twice that of a square whose side equals the width of the rectangle. What are the dimensions of the rectangle?

18. One of two complementary angles exceeds the square of the other by 18°. How large are the two angles?

19. A tree 50 feet high casts a shadow 42 feet long at the same time that another tree casts a shadow 24 feet long. How high is the second tree?

20. The hypotenuse of a right triangle is 117 feet. Find the other two sides if their sum is 153 feet.

LEVER PROBLEMS

1. Jack, who sits 6 feet from the fulcrum, balances Carl who sits 8 feet from the fulcrum. If Jack weighs 70 pounds, how much does Carl weigh?

2. Mary, who weighs 135 pounds and sits 4 feet from the fulcrum, balances Imogen who weighs 67.5 pounds. How far from the fulcrum is Imogen?

3. A weight of 30 pounds balances a weight of 50 pounds on the other side of the fulcrum. If one weight is 2 feet farther from the fulcrum than the other, where is each weight located?

4. The weight on the short arm of a lever is 27 pounds more than the weight on the other arm. If the arms are 6 feet and 8 feet, find each weight.

5. Two weights whose sum is 128 pounds are placed at the ends of a balanced lever. If one arm is 7 feet long and the other arm is 9 feet long, find the two weights.

6. Where is the fulcrum of a lever 93 inches long if a weight of 70 pounds at one end balances a weight of 54 pounds at the other?

7. A lever is 33 feet long. How far from each end must a fulcrum be placed so that a $7\frac{1}{2}$-pound weight at one end balances a weight of 54 pounds at the other?

8. Charlene weighs 60 pounds and Jane 70 pounds. They have a ten-foot teeterboard. How far from Charlene must the fulcrum be placed in order that the two girls may balance each other?

9. Packages weighing 30 pounds and 35 pounds respectively are hung from the ends of a six-foot rod. How far from the 35-pound weight must the fulcrum be placed in order that the two packages may balance?

10. Robert and Arthur are playing seesaw. Their weights balance when Robert is 6 feet from the fulcrum and Arthur is 7 feet from the fulcrum. If Robert weighs 84 pounds, how much does Arthur weigh?

11. Edward and Roy have a 14-foot plank with which they wish to play seesaw. Edward weighs 96 pounds and Roy 72 pounds. If Roy is seated at one end of the plank, how far from the fulcrum must Edward sit if the boys balance each other?

12. Tom and Dick balance on a teeterboard when they are 6 feet and 8 feet respectively from the point of support. If Dick is replaced by Harry, who weighs 5 pounds more, it is found that Tom must sit $6\frac{1}{2}$ feet from the fulcrum. What is the weight of each boy?

MOTION PROBLEMS

1. A goes 8 miles an hour and B goes 20 miles an hour in the opposite direction. How many hours has B been traveling when they are 180 miles apart, if A starts 2 hours before B?

2. A and B, who are 566 miles apart, start in automobiles to meet each other, A driving 40 miles an hour and B driving 36 miles an hour. How soon will they meet if A has an accident and is delayed for 2 hours?

3. A boat starts from Baltimore for Norfolk at 10 miles an hour. Two hours later another boat starts from Baltimore for Norfolk at 14 miles an hour. How long will it take the second boat to overtake the first?

4. Train *A* travels at the rate of 40 miles an hour and train *B* at the rate of 34 miles an hour. Train *A* leaves Philadelphia for Harrisburg at 8 o'clock. Train *B* leaves Harrisburg for Philadelphia at 9 o'clock. The distance between the two cities is 100 miles. How long will train *B* travel before meeting train *A*?

5. Two trains start at the same time from towns 385 miles apart and meet in 5 hours. If the rate of one train is 7 miles an hour less than the rate of the other, what is the rate of each?

6. Two automobiles started toward each other at the same time from points 300 miles apart and met in 5 hours. If one traveled twice as rapidly as the other, what were their rates?

7. Two airplanes are traveling in opposite directions at the rates of 90 and 110 miles an hour, respectively. If they started at the same time from the same place, how soon after the start will they be 1500 miles apart?

8. An automobile makes a trip of 280 miles in 7 hours and 20 minutes. In making the trip it goes 28 miles through towns and the remainder of the distance on a national highway. If the speed through the towns is half of that on the highway, what is the speed through the towns?

9. An airplane can go 900 miles with the wind in the same time that it can go 720 miles against the wind. If the rate of the wind is 15 miles an hour, what is the rate of the airplane in still air?

10. A certain train makes a daily run of 260 miles. One day the train is one hour late in starting, and the engineer runs the train 13 miles an hour faster than usual to arrive on time. What is the usual rate of the train?

WORK PROBLEMS

1. *A* can do a piece of work in 10 days and *B* can do it in 8 days. If they work together, how many days are needed?

2. One pipe can fill a cistern in 12 hours and a smaller pipe can fill the cistern in 15 hours. If both pipes are used, how many hours are needed?

3. Frank can hoe the garden in 7 hours and Bill can hoe it in 9 hours. If both are at work how many hours are needed?

4. *A* can do a piece of work in 15 days and *B* can do it in 20 days. If *A* works 5 days alone on the job and then *B* helps him to complete it, how long does *B* work?

5. One printing press can produce 4000 evening papers in an hour, and a second press can produce 3000 of the papers in an hour. After the first press has been printing for $1\frac{1}{2}$ hours it must stop for repairs. How much time will be needed for the second press to run in order to complete the daily edition of 15,000 papers?

6. Two steam shovels working together can excavate a cellar in 80 hours. If one shovel can do the work $\frac{2}{3}$ as fast as the other, how long would it take each shovel to do the work alone?

7. One section gang can lay a mile of steel rails in 8 days. With the help of a second gang of men, the work can be done in 5 days. How long would it take the second gang to lay the rails working alone?

8. *A* can do a piece of work in 5 days, *B* can do it in 7 days, and *C* can do it in 10 days. How long will it take *A*, *B*, and *C* to do the work when working together?

MIXTURE PROBLEMS

1. A grocer wishes to make a mixture of 80 pounds of coffee to sell at 60 cents a pound. He wishes to make the mixture from 65-cent coffee and 45-cent coffee. How many pounds of each should he use?

2. A grocer has some tea worth $1.20 a pound and another brand of tea worth 90 cents a pound. How many pounds of each must he use to make a mixture of 100 pounds that can be sold for $1.08 a pound?

3. How many pounds of candy worth 50 cents a pound and how many worth 90 cents a pound must be used to make a 75-pound mixture which is worth 76 cents a pound?

4. How many pounds of nuts worth 54 cents a pound must be mixed with 50 pounds of nuts worth 90 cents a pound to make a mixture worth 75 cents a pound?

5. How much milk testing 4% butterfat should be used with milk testing 25% butterfat to make 200 pounds of milk testing 12% butterfat?

6. How much water must be added to a quart of 90% alcohol solution to make a mixture that contains 50% alcohol?

7. How much water must be added to 3 quarts of acid 95% pure to make a mixture containing 25% acid?

8. A radiator contains 2 gallons of a mixture of alcohol and water. If the mixture is 70% alcohol, how much pure alcohol must be added to make it 80% alcohol?

INVESTMENT PROBLEMS

1. A farmer invested $3000, part at 4% and the remainder at 5%. If his income on both investments was $138, how much did he invest at each rate?

2. A man invests $10,000, part at 6% and the remainder at 5%. The yearly income from both investments is $566. Find the amount of each investment.

3. Part of $7000 was invested at 6% and the other part at 5%. If the 6% investment yielded $90 more than the other, what was the amount of each investment?

4. A man invested some money at 4%, and $500 more than this sum at 6%. If the total income from these investments was $580, find how much was invested at each rate.

5. How can $8000 be invested, part at 4% and the remainder at 6%, so that both investments produce the same income?

6. A man lends some money at 5% and an equal amount at 3%. If the income from the 5% loan is $80 more than the income from the 3% loan, how much is loaned at each rate?

7. How can $6000 be divided so that one part can be loaned at 4% and the other at 7%, making an average of 6% on the $6000?

8. How much can a man afford to pay for eight \$1000 4% bonds so as to make $3\frac{1}{2}\%$ on his investment?

9. A man invests \$2000 at 4%, \$3000 at 5%, and \$1200 at 6%. What rate of interest is he receiving on the three investments combined?

10. In how many years will the simple interest on a principal invested at 6% equal the principal itself?

11. A man has \$4500 invested at 6% and a second sum invested at 7%. If the income from the second investment exceeds that from the first sum by \$80, what is the second sum?

12. A lady has one sum of money invested at 5%, and a second sum \$1500 larger than the first, invested at 6%. Her total income from these sums is \$200. How much has she invested at each rate?

VARIATION PROBLEMS

1. c varies directly as n, and $c = 45$ when $n = 25$. Find c when $n = 36$.

2. A formula for the area of a sphere is $V = \frac{\pi}{6} d^3$. How does V vary with respect to d?

3. $S = gt - \frac{1}{2} gt^2$. Does S vary directly as t? Does it vary directly as the square of t?

4. Write the formula for the following table:

x	1	5	9	13	17
y	0	2	4	6	8

5. How do c and d vary in $c = \pi d$?

6. How do I and R vary in $I = \frac{E}{R}$?

7. How do E and R vary in $I = \frac{E}{R}$?

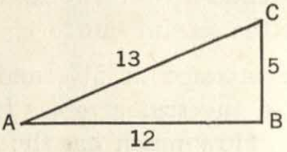
8. If 5 men can do a piece of work in 12 days, how many men are needed to do the work in 20 days?

9. If 24 yards of cloth cost \$3.60, find the cost of 30 yards of the cloth.

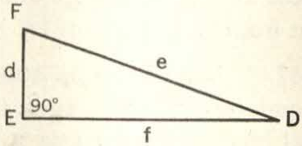
10. If 40 men in 42 days can dig a trench 8 miles long, how many days will be needed for 10 men to dig a trench 12 miles long?

TRIGONOMETRY PROBLEMS

1. Express in simple fractional form the sine, cosine, and tangent of $\angle A$ in $\triangle ABC$ below; of $\angle C$.



Ex. 1



Exs. 3–9

2. When an angle increases in size, how does its sine change? When the angle increases how does the cosine change?

Exercises 3–9 apply to triangle *DEF* above.

3. Find f when $\angle D = 52°$ and $d = 20$ feet.

4. Find $\angle D$ when $d = 40$ and $f = 62$.

5. Find e when $f = 118$ feet and $\angle D = 30°$.

6. Find $\angle F$ when $e = 100$ and $f = 60$.

7. Find f when $e = 175$ and $\angle D = 18° 30'$.

8. Find d when $f = 18$ feet and $\angle F = 45°$.

9. Find $\angle D$ when $\angle F = 10° 45'$.

10. The length of a kite string is 300 yards and the angle of elevation of the kite is 42°. How high is the kite?

11. What is the angle of elevation of the sun when a flagpole 45 feet high casts a shadow 18 feet long?

12. From the top of a lighthouse the keeper observes a floating barrel. How far away is the barrel if its angle of depression is 3° and the lighthouse is 85 feet high?

CHAPTER XX

Remedial Arithmetic

WHOLE NUMBERS

In arithmetic the integers are often referred to as whole numbers. Examples are 1, 2, 17, 55, and 360. The numbers that are not integers are either fractions or mixed numbers. There are two kinds of fractions: common fractions, as $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{7}{6}$; and decimal fractions, as 0.123 and 0.06. A mixed number is composed of a whole number and a fraction, as $3\frac{2}{5}$ and 7.035.

Addition

The numbers that are added are called *addends* and the answer is the *sum*.

324	addend
681	addend
748	addend
1753	sum

Subtraction

In a subtraction problem, the *subtrahend* is the number that is subtracted. It is subtracted from the *minuend*. The answer is the *remainder* or *difference*. To prove a subtraction problem, add the remainder to the subtrahend. This should give the minuend.

6420	minuend
1856	subtrahend
4564	remainder

Multiplication

The number that is being multiplied is the *multiplicand*. It is multiplied by the *multiplier*. The answer is the *product*.

437	multiplicand
28	multiplier
3496	
874	
12236	product

Division

In a division problem the number that is being divided is the *dividend.* The number that it is divided by is the *divisor.* The answer is the *quotient.* In this example, 3276 is the partial quotient; the complete quotient is 3276, R10 or $3276\frac{5}{12}$. To prove that a division is correct, multiply the divisor by the partial quotient and to the result add the remainder. This should give the dividend.

```
              3276  partial quotient
divisor 24)78634  dividend
           72
            66
            48
            183
            168
             154
             144
              10  remainder
```

BASIC ADDITION AND MULTIPLICATION COMBINATIONS

	a	b	c	d	e	f	g	h	i	j
1.	3	7	2	6	4	0	8	5	1	4
	5	1	0	8	5	0	4	7	6	7
2.	4	0	3	8	5	3	6	2	9	7
	4	2	9	7	6	2	3	4	4	0
3.	1	6	0	9	2	5	7	8	7	3
	8	7	4	3	7	8	8	3	9	3
4.	5	3	7	2	7	8	9	4	6	1
	2	4	7	6	3	1	6	1	0	2
5.	8	1	4	4	6	9	3	0	5	2
	0	3	8	9	4	5	1	8	1	5
6.	2	4	5	0	3	6	1	9	4	8
	1	6	3	1	8	6	5	0	2	9
7.	0	9	6	1	0	4	2	3	8	5
	3	9	5	4	9	3	2	6	8	4

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**
8.	9 7	2 8	8 6	5 5	1 0	7 4	5 9	6 2	3 7	9 8
9.	7 6	5 0	9 2	3 0	8 2	2 9	4 0	1 9	0 5	6 1
10.	6 9	8 5	1 1	7 2	9 1	1 7	0 7	7 5	2 3	0 6

PRACTICE EXERCISES

Add:

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**
1.	34 27	65 33	49 40	52 31	74 23	85 14	92 7	67 32
2.	69 8	26 55	27 47	14 69	48 32	37 28	87 14	39 52
3.	2 7 4 8 9	5 8 2 8 9	9 9 5 6 1	6 4 8 7 2	7 1 8 3 9	6 5 8 4 5	9 7 2 3 6	3 5 9 4 7

	a	**b**	**c**	**d**
4.	13562 16732 51723 64814	45462 38372 73861 257	69345 72108 26372 4163	76289 13724 52876 36218
5.	250545 614789 21672	673018 528991 63266	75193 42685 8505	645321 789063 954887

Multiply:

	a	b	c	d	e	f	g	h
6.	43	12	32	53	91	28	36	22
	2	8	3	4	5	6	7	9

	a	b	c	d	e	f
7.	542	671	928	265	226	925
	3	5	7	4	8	2
8.	628	119	864	973	216	486
	7	9	6	8	3	6
9.	203	608	209	516	701	809
	13	43	56	29	47	38
10.	529	364	791	265	902	647
	706	205	902	603	804	109

BASIC SUBTRACTION COMBINATIONS

	a	b	c	d	e	f	g	h	i	j
1.	7	10	11	5	11	4	12	3	9	12
	7	5	9	3	8	2	4	1	0	6
2.	6	5	10	7	12	8	15	10	18	9
	4	0	1	6	5	3	6	2	9	7
3.	5	7	6	1	10	6	5	15	13	10
	2	4	5	0	3	6	1	9	7	8
4.	7	11	6	14	10	4	9	7	1	8
	3	7	2	8	4	0	8	5	1	4
5.	9	11	13	10	5	8	9	8	10	8
	3	4	8	7	5	2	9	0	6	1

	a	b	c	d	e	f	g	h	i	j
6.	6 0	12 3	14 6	8 5	14 7	4 1	9 4	15 8	3 2	16 9
7.	2 0	13 9	9 6	6 1	15 7	13 4	2 2	4 3	12 8	13 5
8.	2 1	13 6	4 4	12 9	16 8	3 0	7 2	12 7	6 3	9 5
9.	17 8	9 1	5 4	16 7	8 6	10 9	11 3	8 7	14 5	11 2
10.	11 6	8 8	3 3	11 5	17 9	7 1	0 0	7 0	9 2	14 9

PRACTICE EXERCISES

Subtract:

	a	b	c	d	e	f	g	h
1.	39 3	97 2	46 5	78 6	29 9	49 7	23 1	86 4
2.	23 7	91 8	76 9	42 5	71 3	22 4	81 2	25 6

	a	b	c	d	e	f
3.	428 218	901 364	708 299	625 307	509 206	208 185

	a	b	c	d	e
4.	3900 3752	6200 2097	6066 3208	7189 3034	9036 6547
5.	55706 12987	67104 48356	21078 19206	71555 25725	31063 7892

BASIC DIVISION COMBINATIONS

	a	b	c	d	e	f	g	h	i
1.	$6\overline{)12}$	$5\overline{)25}$	$8\overline{)64}$	$3\overline{)3}$	$6\overline{)42}$	$2\overline{)6}$	$1\overline{)7}$	$5\overline{)35}$	$3\overline{)6}$
2.	$2\overline{)0}$	$4\overline{)36}$	$1\overline{)4}$	$5\overline{)40}$	$8\overline{)48}$	$6\overline{)18}$	$9\overline{)18}$	$2\overline{)2}$	$4\overline{)28}$
3.	$3\overline{)24}$	$7\overline{)0}$	$5\overline{)15}$	$2\overline{)10}$	$3\overline{)9}$	$9\overline{)81}$	$8\overline{)16}$	$9\overline{)45}$	$7\overline{)35}$
4.	$1\overline{)6}$	$2\overline{)8}$	$3\overline{)18}$	$6\overline{)54}$	$5\overline{)0}$	$7\overline{)14}$	$4\overline{)4}$	$6\overline{)36}$	$1\overline{)9}$
5.	$4\overline{)8}$	$8\overline{)40}$	$7\overline{)56}$	$9\overline{)36}$	$1\overline{)3}$	$4\overline{)0}$	$6\overline{)48}$	$7\overline{)21}$	$9\overline{)0}$
6.	$5\overline{)30}$	$3\overline{)12}$	$9\overline{)72}$	$8\overline{)0}$	$7\overline{)7}$	$8\overline{)72}$	$5\overline{)45}$	$1\overline{)0}$	$4\overline{)32}$
7.	$7\overline{)63}$	$1\overline{)5}$	$2\overline{)16}$	$7\overline{)28}$	$2\overline{)12}$	$9\overline{)27}$	$8\overline{)56}$	$8\overline{)24}$	$9\overline{)63}$
8.	$4\overline{)12}$	$6\overline{)24}$	$3\overline{)0}$	$5\overline{)5}$	$4\overline{)24}$	$1\overline{)2}$	$4\overline{)16}$	$6\overline{)30}$	$7\overline{)49}$
9.	$3\overline{)21}$	$6\overline{)0}$	$2\overline{)4}$	$3\overline{)27}$	$9\overline{)54}$	$4\overline{)20}$	$6\overline{)6}$	$1\overline{)8}$	$9\overline{)9}$
10.	$7\overline{)42}$	$8\overline{)32}$	$1\overline{)1}$	$5\overline{)20}$	$5\overline{)10}$	$2\overline{)18}$	$8\overline{)8}$	$3\overline{)15}$	$2\overline{)14}$

PRACTICE EXERCISES

Divide:

	a	b	c	d	e	f	g	h
1.	$2\overline{)64}$	$5\overline{)95}$	$3\overline{)69}$	$8\overline{)88}$	$4\overline{)48}$	$7\overline{)84}$	$6\overline{)96}$	$3\overline{)72}$

	a	b	c	d	e	f	g
2.	$3\overline{)963}$	$2\overline{)486}$	$8\overline{)488}$	$6\overline{)366}$	$5\overline{)585}$	$7\overline{)427}$	$4\overline{)488}$
3.	$7\overline{)504}$	$6\overline{)402}$	$5\overline{)305}$	$2\overline{)128}$	$9\overline{)207}$	$4\overline{)304}$	$3\overline{)201}$
4.	$5\overline{)525}$	$4\overline{)432}$	$6\overline{)654}$	$7\overline{)735}$	$3\overline{)321}$	$8\overline{)872}$	$9\overline{)963}$

	a	b	c	d	e
5.	$43\overline{)258}$	$75\overline{)525}$	$36\overline{)288}$	$24\overline{)264}$	$85\overline{)765}$
6.	$66\overline{)528}$	$32\overline{)928}$	$74\overline{)962}$	$31\overline{)434}$	$56\overline{)672}$

	a	b	c	d
7.	$152\overline{)11248}$	$273\overline{)22386}$	$467\overline{)25218}$	$891\overline{)40986}$

FRACTIONS

An Important Principle of Fractions

If both the numerator and denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not changed.

Example 1. $\frac{8}{32} = \frac{8 \div 8}{32 \div 8} = \frac{1}{4}$

Example 2. $\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$

Reducing Fractions to Lowest Terms

To reduce fractions to lowest terms, divide the numerator and denominator of the fraction by the largest number that can be contained exactly in both.

Example. Reduce $\frac{24}{30}$ to lowest terms.

Solution. The largest number that will divide 24 and 30 exactly is 6. 6 is called the *greatest common divisor* of 24 and 30. (Greatest common divisor is abbreviated G.C.D.)

Then $$\frac{24 \div 6}{30 \div 6} = \frac{4}{5}$$

EXERCISES

Reduce to lowest terms:

1. $\frac{2}{4}$ **4.** $\frac{25}{45}$ **7.** $\frac{50}{75}$ **10.** $\frac{68}{85}$

2. $\frac{3}{9}$ **5.** $\frac{18}{24}$ **8.** $\frac{56}{64}$ **11.** $\frac{81}{99}$

3. $\frac{10}{25}$ **6.** $\frac{8}{20}$ **9.** $\frac{26}{65}$ **12.** $\frac{24}{400}$

Changing Fractions to Higher Terms

To change a fraction to higher terms, first divide the new denominator by the denominator of the given fraction, and then multiply both the numerator and denominator of the given fraction by this quotient.

Example. Change $\frac{2}{3}$ to 18ths.

Solution. $18 \div 3 = 6.$

Then $\frac{2 \times 6}{3 \times 6} = \frac{12}{18}$.

EXERCISES

Change the following fractions to equal fractions having the denominators indicated:

1. $\frac{1}{3} = \frac{?}{15}$ **4.** $\frac{17}{24} = \frac{?}{48}$ **7.** $\frac{2}{7} = \frac{?}{28}$

2. $\frac{1}{8} = \frac{?}{48}$ **5.** $\frac{5}{12} = \frac{?}{36}$ **8.** $\frac{3}{5} = \frac{?}{25}$

3. $\frac{2}{3} = \frac{?}{12}$ **6.** $\frac{1}{8} = \frac{?}{64}$ **9.** $\frac{5}{6} = \frac{?}{54}$

Changing Mixed Numbers to Fractions

A mixed number is composed of a whole number and a fraction, as $7\frac{2}{5}$. To change $7\frac{2}{5}$ to a fraction, we multiply 7 by 5 to find the number of 5ths in 7. $7 \times 5 = 35$. Then there are 35 fifths in 7. Add the 35 fifths to 2 fifths and we have $\frac{37}{5}$. This may be written as follows:

$$7\frac{2}{5} = \frac{35 + 2}{5} = \frac{37}{5}$$

EXERCISES

Change to fractions:

1. $2\frac{1}{7}$
2. $3\frac{1}{3}$
3. $4\frac{1}{2}$
4. $6\frac{1}{4}$
5. $6\frac{2}{5}$
6. $7\frac{1}{8}$
7. $12\frac{1}{7}$
8. $10\frac{5}{6}$
9. $5\frac{1}{3}$
10. $14\frac{1}{4}$
11. $15\frac{2}{5}$
12. $6\frac{3}{7}$

Multiplying Fractions and Mixed Numbers

To Multiply Fractions or Mixed Numbers:

1. Change the mixed numbers to fractions.
2. Place the product of the numerators over the product of the denominators and reduce the resulting fraction to lowest terms.

Example 1. Multiply $\frac{2}{3}$ by $\frac{6}{5}$.

Solution. $\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$

Reducing to lowest terms, $\frac{12}{15} = \frac{4}{5}$.

The work may be shortened to

$$\frac{2}{\underset{1}{\cancel{3}}} \times \frac{\overset{2}{\cancel{6}}}{5} = \frac{4}{5}.$$

Example 2. Multiply $3\frac{1}{3}$ by $\frac{2}{5}$.

Solution.

1. $3\frac{1}{3} = \frac{10}{3}$

2. $\frac{\overset{2}{\cancel{10}}}{3} \times \frac{2}{\underset{1}{\cancel{5}}} = \frac{4}{3}$ or $1\frac{1}{3}$

EXERCISES

1. $\frac{7}{8} \times \frac{3}{4}$

2. $10 \times \frac{1}{5}$

3. $\frac{3}{2} \times 6$

4. $\frac{2}{7} \times \frac{7}{5}$

5. $\frac{7}{8} \times \frac{12}{5}$

6. $\frac{5}{18} \times \frac{54}{15}$

7. $\frac{1}{5} \times 50$

8. $\frac{2}{3} \times \frac{21}{10}$

9. $3\frac{1}{3} \times \frac{2}{5}$

10. $4\frac{1}{5} \times \frac{2}{7}$

11. $2\frac{3}{4} \times \frac{6}{11}$

12. $8\frac{2}{3} \times 13\frac{1}{3}$

13. $6\frac{1}{8} \times \frac{5}{21}$

14. $\frac{2}{3} \times \frac{6}{11}$

15. $\frac{9}{8} \times \frac{5}{18}$

Dividing Fractions and Mixed Numbers

To Divide Fractions or Mixed Numbers:

1. Change the mixed numbers to fractions.
2. Invert the divisor and multiply.

Example 1. Divide $\frac{3}{7}$ by $\frac{3}{5}$.

Solution.

$$\frac{\overset{1}{\cancel{3}}}{7} \times \frac{5}{\underset{1}{\cancel{3}}} = \frac{5}{7}$$

Example 2. Divide $\frac{2}{5}$ by 10.

Solution. 10 inverted is $\frac{1}{10}$

Then

$$\frac{\overset{1}{\cancel{2}}}{5} \times \frac{1}{\underset{5}{\cancel{10}}} = \frac{1}{25}$$

Example 3. Divide $6\frac{2}{3}$ by $\frac{2}{3}$.

Solution.

$$\frac{20}{3} \div \frac{2}{3} = \frac{\overset{10}{\cancel{20}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = 10$$

EXERCISES

Divide:

1. $8 \div \frac{1}{2}$
2. $16 \div \frac{4}{5}$
3. $\frac{5}{8} \div \frac{3}{16}$
4. $\frac{3}{4} \div \frac{3}{5}$
5. $3\frac{1}{3} \div 1\frac{2}{3}$
6. $5\frac{2}{5} \div 1\frac{1}{5}$
7. $8\frac{3}{4} \div 1\frac{2}{5}$
8. $\frac{2}{7} \div \frac{13}{7}$
9. $6\frac{1}{8} \div \frac{3}{8}$
10. $\frac{7}{15} \div \frac{14}{5}$
11. $\frac{9}{10} \div \frac{3}{5}$
12. $24 \div \frac{2}{3}$

Adding and Subtracting Fractions and Mixed Numbers

Before fractions can be added or subtracted, they must have the same denominator. If the fractions to be added or subtracted do not have the same denominator, change them to equivalent fractions having the same denominator.

Example 1. Add $\frac{2}{3}$ and $\frac{3}{5}$.

Solution. The lowest common denominator (L.C.D.) of $\frac{2}{3}$ and $\frac{3}{5}$ is 15. Then $\frac{2}{3}$ and $\frac{3}{5}$ must be raised to fifteenths. $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$.

Then $\frac{10}{15} + \frac{9}{15} = \frac{19}{15}$ or $1\frac{4}{15}$

Example 2. Subtract $\frac{1}{2}$ from $\frac{5}{8}$.

Solution. $\frac{5}{8} - \frac{1}{2}$. The L.C.D. is 8.

Then $\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

Example 3. Find the sum: $3\frac{2}{3} + \frac{5}{12}$

Solution.

$$3\frac{2}{3} + \frac{5}{12}$$
$$= \frac{11}{3} + \frac{5}{12}$$
$$= \frac{44}{12} + \frac{5}{12} = \frac{49}{12} \text{ or } 4\frac{1}{12}$$

Example 4. Find the difference: $\frac{7}{8} - \frac{5}{6}$

Solution. The L.C.D. is 24.

Then $\frac{7}{8} - \frac{5}{6} = \frac{21}{24} - \frac{20}{24} = \frac{1}{24}$

EXERCISES

Add or subtract as indicated:

1. $\frac{1}{3}+\frac{2}{3}$
2. $\frac{1}{3}+\frac{1}{5}$
3. $\frac{2}{4}+\frac{1}{2}$
4. $\frac{7}{8}-\frac{3}{4}$
5. $\frac{5}{9}-\frac{1}{3}$
6. $\frac{3}{4}+\frac{1}{2}$
7. $\frac{7}{15}+\frac{3}{5}$
8. $\frac{2}{7}-\frac{1}{7}$
9. $\frac{3}{4}+\frac{1}{4}$
10. $\frac{10}{11}+\frac{1}{11}$
11. $\frac{1}{8}-\frac{1}{9}$
12. $\frac{6}{7}-\frac{2}{21}$
13. $\frac{1}{20}+\frac{1}{10}$
14. $\frac{2}{100}+\frac{18}{100}$
15. $3\frac{1}{2}+\frac{3}{4}$
16. $3\frac{1}{4}-\frac{3}{4}$
17. $8\frac{1}{2}-7\frac{1}{4}$
18. $5\frac{1}{6}-\frac{5}{6}$
19. $3\frac{1}{3}-1\frac{5}{6}$
20. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
21. $\frac{1}{5}+\frac{1}{6}-\frac{1}{3}$

DECIMALS

Adding and Subtracting Decimals

To Add or Subtract Decimals:

1. Place the decimals in columns, being sure to keep the decimal points directly under each other.
2. Add or subtract as in whole numbers.
3. Place the decimal point in the result directly under the decimal points in the numbers.

Example 1. Add 4.7, 0.12, 3.42, and 21.63.

Solution.

$$\begin{array}{l} 4.7 \\ 0.12 \\ 3.42 \\ 21.63 \\ \hline 29.87 \text{ is the sum} \end{array}$$

Example 2. Subtract 17.64 from 32.8.

Solution. A zero may be annexed to 32.8 to make the same number of decimal places as in 17.64.

$$\begin{array}{l} 32.80 \\ 17.64 \\ \hline 15.16 \text{ is the difference} \end{array}$$

EXERCISES

Add or subtract as indicated:

1. $3.64 + $5.74 + $18.25 + $13.60
2. 0.15 + 0.03 + 0.62 + 4.51 + 6.32
3. 8 + 0.06 + 0.32 + 0.18 + 1.54
4. 39.73 − 14.82
5. 20.56 + 14.271 + 16.32 + 17.84
6. 0.15 − 0.07
7. 0.7 − 0.625
8. 4.2 − 0.83

Multiplying Decimals

To Multiply Decimals:

1. Do the multiplications as if the decimals were whole numbers.
2. Starting at the right, point off in the product as many places as there are in the multiplicand and multiplier combined.

Example. Multiply 4.32 by 5.6.

Solution. There are two decimal places in the multiplicand and one in the multiplier. Then there are 2 + 1 or 3 decimal places in the product.

$$\begin{array}{r} 4.32 \\ 5.6 \\ \hline 2592 \\ 2160 \\ \hline 24.192 \end{array}$$

EXERCISES

Multiply as indicated:

1. 0.2 × 0.5
2. 1.8 × 3.4
3. 0.35 × 1.9
4. 0.06 × 0.07
5. 32.5 × 1.84
6. 78.2 × 0.014
7. 9.63 × 1.45
8. 0.004 × 0.05
9. 7.2 × 8.46
10. 5.19 × 0.006
11. 0.56 × 0.18
12. 9.9 × 99

Dividing Decimals

First Method. In dividing decimals, the number of decimal places in the quotient is found by subtracting the number of decimal places in the divisor from the number of decimal places in the dividend. In the problem at the right there are 5 decimal places in the dividend and 2 in the divisor; $5 - 2 = 3$. Then there are 3 decimal places in the quotient.

$$\begin{array}{r} .065 \\ .03\,\overline{\smash{)}.00195} \end{array}$$

Second Method. Another method of placing the decimal point in the quotient is as follows: If the divisor is not a whole number, make it a whole number by using a caret (‸). Since the decimal point has been moved two places to the right in the divisor, the decimal point in the dividend must be moved two places to the right. In reality, both the numerator and denominator of the fraction $\frac{.00195}{.03}$ are multiplied by 100. The decimal point of the quotient is placed above the caret of the dividend. Since 3 will not divide into 1, a zero is placed above the 1. 3 divides into 19 six times, and 6 is placed above the 9, etc.

$$\begin{array}{r} .065 \\ .03_{\wedge}\,\overline{\smash{)}.00_{\wedge}195} \end{array}$$

EXERCISES

Copy the answers and insert the decimal point:

1. $7.382 \div 2 = 3691$

2. $68{,}436 \div .03 = 22{,}812$

3. $0.30 \div 0.02 = 15$

4. $0.9740 \div 0.005 = 1948$

5. $6814 \div 0.01 = 6814$

6. $3.00 \div 0.05 = 6$

7. $2.825 \div 2.5 = 113$

8. $0.003 \div 0.06 = 5$

Divide as indicated:

9. $1.125 \div 0.03$

10. $0.96 \div 24$

11. $37.5 \div 5$

12. $2 \div 0.08$

13. $2.1318 \div 19$

14. $4.5 \div 100$

15. $4.56 \div 2000$

16. $86.075 \div 27.5$

17. $0.00343 \div 3.43$

18. $0.006 \div 0.15$

19. $16.075 \div 0.5$

20. $4.734 \div 0.0003$

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